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Valen E. Johnson
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Ordinal Data Modeling

With 73 illustrations



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Preface

This book was written for graduate students and researchers in statistics and the social sciences. Our intent in writing the book was to bridge the gap between recent theoretical developments in statistics and the application of these methods to ordinal data. Ordinal data are the most common form of data acquired in the social sciences, but the analysis of such data is generally performed without regard to their ordinal nature.

The first two chapters of the book provide an introduction to the principles and practice of Bayesian inference, and the modern computational techniques that have made Bayesian inference possible in the models that are described later in the text. Together with a previous course in introductory statistics and a knowledge of multiple regression, these two chapters provide the background necessary for studying the remainder of the book. Readers familiar with this material can safely skip to the third chapter, where models for binary data are introduced.

The third chapter describes binomial regression models from both classical and Bayesian perspectives. Considerable attention is given to residual analyses and goodness-of-fit criteria. Latent variable interpretations of binomial regression models are also emphasized.

Chapter 4 extends the results of the previous chapter to regression models for ordered categorical data, or ordinal data. Motivation for these models is again drawn from a latent variable interpretation of categorical data, thus exposing the close connections between the underlying model structures and computational algorithms common to both binomial and ordinal regression models. As in Chapter 3, considerable attention is paid to residual analyses from both classical and Bayesian perspectives.

Multirater ordinal data and regression models are introduced in Chapter 5. To effectively handle the complications introduced by the implicit rater variability associated with such data, a Bayesian inferential structure is assumed throughout. The multirater ordinal regression models developed in this chapter provide a general framework for a large number of psychometric models, including the item response models of Chapter 6 and the graded response models in Chapter 7. ROC analysis is discussed within the general context of multirater ordinal regression data in the latter sections of this chapter.

Chapters 6 and 7 describe item response and graded response models. Item response models are special cases of multirater ordinal data models in which responses are binary. Basic terminology associated with item response models is introduced, and a hierarchical Bayesian model that represents a compromise between one-parameter Rasch-type models and more general two-parameter models is proposed. Numerous diagnostic plots for item response models are illustrated. Chapter 7 closes with a brief description of graded response models and illustrates the principles of these models in a case study of undergraduate grade data.

Much of the data described in the book and many of the computational algorithms used in the analyses are available electronically from the authors or Mathworks. On the world-wide web, this material may be obtained from

www-math.bgsu.edu/~albert/ord_book

or

<ftp://ftp.mathworks.com/pub/books/johnson>.

We would like to thank several individuals for their assistance in preparing the manuscript. David Draper provided very helpful comments on an early draft. John Kimmel, our editor, was also most helpful in bringing closure to the process. We would also like to thank a number of students who contributed to proofreading: Sining Chen, Maria De Iorio, Courtney Johnson, Dae Young Kim, Scott Lynch, Randy Walsh, and Hongjun Wang. We are also grateful to Dr. Robert Terry and Brian Skotko for their permission to use previously unpublished data.

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