Undergraduate Texts in Mathematics

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Foulds: Optimization Techniques: An Introduction.
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Robin Hartshorne

Geometry: Euclid and Beyond

With 550 Illustrations

Springer

Robin Hartshorne
Department of Mathematics
University of California
Berkeley, CA 94720
USA
robin@math.berkeley.edu

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For
Edie, Ben, and Joemy

and

In Loving Memory of
Jonathan Churchill Hartshorne
1972–1992

I have not found anything in Lobatchevski's work that is new to me, but the development is made in a different way from the way I had started and to be sure masterfully done by Lobatchevski in the pure spirit of geometry.

– letter from Gauss to Schumacher (1846)

Preface

In recent years, I have been teaching a junior–senior-level course on the classical geometries. This book has grown out of that teaching experience. I assume only high-school geometry and some abstract algebra. The course begins in Chapter 1 with a critical examination of Euclid's *Elements*. Students are expected to read concurrently Books I–IV of Euclid's text, which must be obtained separately. The remainder of the book is an exploration of questions that arise naturally from this reading, together with their modern answers. To shore up the foundations we use Hilbert's axioms. The Cartesian plane over a field provides an analytic model of the theory, and conversely, we see that one can introduce coordinates into an abstract geometry. The theory of area is analyzed by cutting figures into triangles. The algebra of field extensions provides a method for deciding which geometrical constructions are possible. The investigation of the parallel postulate leads to the various non-Euclidean geometries. And in the last chapter we provide what is missing from Euclid's treatment of the five Platonic solids in Book XIII of the *Elements*.

For a one-semester course such as I teach, Chapters 1 and 2 form the core material, which takes six to eight weeks. Then, depending on the taste of the instructor, one can follow a more geometric path by going directly to non-Euclidean geometry in Chapter 7, or a more algebraic one, exploring the relation between geometric constructions and field extensions, by doing Chapters 3, 4, and 6. For me, one of the most interesting topics is the introduction of coordinates into an abstractly given geometry, which is done for a Euclidean plane in Section 21, and for a hyperbolic plane in Section 41.

Throughout this book, I have attempted to choose topics that are accessible to undergraduates and that are interesting in their own right. The exercises are meant to be challenging, to stimulate a sense of curiosity and discovery in the student. I purposely do not indicate their difficulty, which varies widely.

I hope this material will become familiar to every student of mathematics, and in particular to those who will be future teachers.

I owe thanks to Marvin Greenberg for reading and commenting on large portions of the text, to Hendrik Lenstra for always having an answer to my questions, and to Victor Pambuccian for valuable references to the literature. Thanks to Faye Yeager for her patient typing and retyping of the manuscript. And special thanks to my wife, Edie, for her continual loving support.

> Of all the works of antiquity which have been transmitted to the present times, none are more universally and deservedly esteemed than the *Elements of Geometry* which go under the name of Euclid. In many other branches of science the moderns have far surpassed their masters; but, after a lapse of more than two thousand years, this performance still maintains its original preeminence, and has even acquired additonal celebrity from the fruitless attempts which have been made to establish a different system.
>
> – from the preface to
> Bonnycastle's Euclid
> London (1798)

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