

Inequalities

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Michael J. Cloud Byron C. Drachman

Inequalities

With Applications to Engineering

With 14 Figures



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Michael J. Cloud
Department of Electrical Engineering
Lawrence Technological University
Southfield, MI 48075
USA

Byron C. Drachman
Department of Mathematics
Michigan State University
East Lansing, MI 48824-1027
USA

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Preface

We might wonder why it is necessary to study inequalities. Many applied science and engineering problems, for instance, can be pursued without their explicit mention. Nevertheless, a facility with inequalities seems to be necessary for an understanding of much of mathematics at intermediate and higher levels. Inequalities serve a natural purpose of comparison, and they sometimes afford us indirect routes of reasoning or problem solving when more direct routes might be inconvenient or unavailable.

This small guide to inequalities was originally written with engineers and other applied scientists in mind. Comments from those mathematicians who have seen the manuscript lead us to hope that some mathematicians will find some of the applications interesting, and that students of mathematics will also find the book useful. It is intended to help fill the gap between college-algebra treatments of inequalities and the formidable treatises on the subject that exist in the mathematics literature. Important techniques are all reinforced through the exercises that appear at the end of each chapter, and hints are included to expedite the reader's progress. We review a few topics from calculus, but make no attempt at a thorough review. In order to simplify the discussion, we use a stronger hypothesis than is necessary in some of the statements or proofs of theorems and in some of the exercises. For a review of calculus, we recommend the fine classic by Landau [37]. Among the many good books on analysis, we can recommend Stromberg [57].

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