

QRD-RLS Adaptive Filtering

José Antonio Apolinário Jr.
Editor

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Foreword by Prof. John G. McWhirter

 Springer

Editor

José Antonio Apolinário Jr.
Instituto Militar de Engenharia (IME)
Rio de Janeiro
Brazil
apolin@ime.eb.br

ISBN 978-0-387-09733-6 e-ISBN 978-0-387-09734-3
DOI 10.1007/978-0-387-09734-3

Library of Congress Control Number: 2008936641

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To Ana, Isabela, and Eduardo.

Foreword

I feel very honoured to have been asked to write a brief foreword for this book on QRD-RLS Adaptive Filtering – a subject which has been close to my heart for many years. The book is well written and very timely – I look forward personally to seeing it in print. The editor is to be congratulated on assembling such a highly esteemed team of contributing authors able to span the broad range of topics and concepts which underpin this subject.

In many respects, and for reasons well expounded by the authors, the LMS algorithm has reigned supreme since its inception, as the algorithm of choice for practical applications of adaptive filtering. However, as a result of the relentless advances in electronic technology, the demand for stable and efficient RLS algorithms is growing rapidly – not just because the higher computational load is no longer such a serious barrier, but also because the technological pull has grown much stronger in the modern commercial world of 3G mobile communications, cognitive radio, high speed imagery, and so on.

This book brings together under one cover, and with common notation, the key results from many different strands of research relating to QRD-RLS adaptive filtering over recent years. It succeeds in putting this research into a clear historical perspective which highlights the underpinning theory and common motivating factors that have shaped the subject. This is achieved in the course of providing a very thorough and comprehensive account of the various key topics including numerous up-to-date algorithms in easily accessible form. As such, it should serve as a very good reference text whilst having considerable tutorial value.

Chapter one provides an excellent tutorial review of the fundamental topics in linear algebra which are essential in the context of developing and applying QRD-RLS algorithms. It starts with a very useful historical review and goes on to bring the concept of matrix triangularization and QR decomposition right up-to-date. The Gram–Schmidt orthogonalization technique is included for comparison and it was great to see a clear explanation of the difference between the Gram–Schmidt and modified Gram–Schmidt (MGS) techniques. For this chapter alone, and its extensive bibliography, the book is likely to be very high on the essential reading list for most of my post-graduate students in future. But there is much more to follow.

The second chapter provides a very good overview of adaptive filtering techniques ideal for someone fairly new to the subject. It gives a clear account of the least mean square (LMS) and normalized LMS algorithms before going on to introduce the basic recursive least-square (RLS) algorithm. On the way, it cleverly presents data-reusing versions of the LMS algorithm, typified by the affine projection method. These help to bridge the gap between the LMS and RLS algorithms and provide useful intermediate options. The LMS and data-reusing LMS algorithms are thus seen to be special, simplified cases of the RLS technique.

The QRD approach to adaptive filtering is clearly explained and presented in detail in Chapter 3 where the use of Givens rotations is assumed throughout. Unusually, and very sensibly, it also introduces the inverse QRD technique (based on Givens rotations). This is closely related to the basic QRD technique and best explained at this stage of the book since it is required in later chapters.

The core content of the book is presented in Chapters 4 and 5, which introduce and give a detailed exposition of the fast QRD-RLS algorithms and closely related QRD least squares lattice algorithms. A useful classification of the various QRD-RLS algorithms in Chapter 5 helps to unify and clarify the different variations which have emerged over the years. Similarly, explaining the key distinction between the QRD-RLS and QRD least squares lattice algorithms helps to put the latter class into context. It is worth noting that the author of Chapter 5 adopts a less conventional, but very interesting, approach to deriving QRD lattice algorithms. He does it in the more general context of linear interpolation, from which the conventional linear prediction methods may be deduced whilst other novel algorithms are also derived. A wealth of specific algorithms is presented throughout these two chapters.

Subsequent chapters of the book introduce and develop other important techniques such as multi-channel fast QRD-RLS algorithms (including the generalization to channels with different orders of prediction), QRD-RLS algorithms based on Householder transformations, linearly constrained QRD-RLS algorithms, and techniques for explicit weight extraction from fast QRD-RLS algorithms. The book also moves on to consider some vitally important practical aspects such as numerical stability (a difficult topic which is expertly presented in Chapter 8), the practical effect of finite-precision arithmetic, and the design of pipelined processing architectures to exploit the potential power of parallel computation for higher speed implementation.

In all, this is a very worthwhile text for anyone working, or planning to work, on adaptive filtering or adaptive beamforming. I have thoroughly enjoyed reading it and have no doubt that most readers will find it equally useful and enjoyable.

Wales, UK
September 2008

Prof. John G. McWhirter, FRS FREng
Distinguished Research Professor
School of Engineering
Cardiff University

Preface

The fast growth of the technological resources observed nowadays has triggered the development of new DSP techniques to cope with the requirements of modern industry. The research of efficient algorithms to be used in the ever-increasing applications of adaptive filters has therefore developed tremendously. In such a scenario, the QRD-RLS-based algorithms are a good option in applications where speed of convergence is of paramount importance and an efficient, reliable, and numerically robust adaptive filter is needed.

However, I believe that the nice features of this family of algorithms, in many occasions, are not used simply due to the fact that their matrix equations are not easy to understand. On the other hand, students, researchers, and practitioners need to be constantly up-to-date with the recent developments, not only by attending conferences and reading journal papers, but also by referring to a comprehensive compendium, where all concepts were carefully matured and are presented in such a way as to provide easy understanding. This is the main goal of this book: To provide the reader with the necessary tools to understand and implement a variety of QRD-RLS algorithms suitable to a vast number of applications.

This publication gathers some of the most recent developments as well as the basic concepts for a complete understanding of the QRD-RLS-based algorithms. Although this work does not cover all fronts of research in the field, it tries to bring together the most important topics for those who need an elegant and fast-converging adaptive filter.

QR decomposition has been a pearl in applied mathematics for many years; its use in adaptive filtering is introduced in the first chapter of this book in the form of an annotated bibliography.

The fundamental chapters materialized from lecture notes of a short course given at Helsinki University of Technology in the winter of 2004–2005, a number of conference and journal publications, and some theses I supervised. I was also lucky to receive contributions from many prominent authorities in the field.

This book consists of 12 chapters, going from fundamentals to more advanced aspects. Different algorithms are derived and presented, including basic, fast, lattice, multichannel, and constrained versions. Important issues, such as numerical

stability, performance in finite-precision environments, and VLSI oriented implementations are also addressed. All algorithms are derived using Givens rotations, although one chapter deals with implementations using Householder reflections.

I hope the readers will find this book a handy guide to most aspects of theory and implementation details, quite useful in their professional practice. Upon request to the editor, a set of MATLAB[®]¹ codes for the main algorithms described in this book would be available.

Finally, I express my deep gratitude to all authors for their effort and competence in their timely and high quality contributions. I also thank the people from Springer, always very kind and professional. I am particularly grateful to my former DSc supervisor, Paulo S. R. Diniz, for his support and ability to motivate his pupils, and Marcello L. R. de Campos, the dear friend who, in the middle of a technical meeting on a sunny Friday, suggested this book.

Rio de Janeiro, Brazil
September 2008

José A. Apolinário Jr. D. Sc.
apolin@ieee.org

¹ MATLAB is a registered trademark of The MathWorks, Inc.

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List of Contributors

José Antonio Apolinário Jr. (Editor)

Department of Electrical Engineering (SE/3)

Military Institute of Engineering (IME)

Praça General Tibúrcio 80, Rio de Janeiro, RJ, 22290-270 – Brazil

e-mail: apolin@ieee.org

Richard C. Le Borne

Department of Mathematics

Tennessee Technological University

Box 5054, Cookeville, TN 38505 – USA

e-mail: rleborne@tntech.edu

Marcello L. R. de Campos

Electrical Engineering Program, COPPE

Federal University of Rio de Janeiro (UFRJ)

P. O. Box 68504, Rio de Janeiro, RJ, 21941-972 – Brazil

e-mail: campos@lps.ufrj.br

Shiunn-Jang Chern

Department of Electrical Engineering

National Sun-Yat Sen University

70 Lienhai Road, Kaohsiung, Taiwan 80424 – R.O.C.

e-mail: chern@mail.ee.nsysu.edu.tw

Paulo S. R. Diniz

Electrical Engineering Program, COPPE

Federal University of Rio de Janeiro (UFRJ)

P. O. Box 68504, Rio de Janeiro, RJ, 21941-972 – Brazil

e-mail: diniz@lps.ufrj.br

Jun Ma

School of Microelectronics
Shanghai Jiaotong University
800 Dongchun Road, Shanghai 200240 – China
e-mail: majun@ic.sjtu.edu.cn

Maria D. Miranda

Department of Telecommunications and Control
University of São Paulo (USP)
Avenida Prof. Luciano Gualberto 158, São Paulo, SP, 05508-900 – Brazil
e-mail: maria@lcs.poli.usp.br

Mohammed Mobien

Department of Signal Processing and Acoustics, SMARAD CoE
Helsinki University of Technology
P.O. Box 3000 TKK, FIN-02015 – Finland
e-mail: mobien@signal.tkk.fi

Sergio L. Netto

Electrical Engineering Program, COPPE
Federal University of Rio de Janeiro (UFRJ)
P. O. Box 68504, Rio de Janeiro, RJ, 21941-972 – Brazil
e-mail: sergioln@lps.ufrj.br

Keshab K. Parhi

Department of Electrical and Computer Engineering
University of Minnesota
200 Union Street SE, Minneapolis, MN 55455 – USA
e-mail: parhi@umn.edu

Antônio L. L. Ramos

Department of Technology (ATEK)
Buskerud University College (HIBU)
P. O. Box 251, 3603 Kongsberg – Norway
e-mail: antonio.ramos@hibu.no

Phillip Regalia

Department of Electrical Engineering and Computer Science
Catholic University of America
620 Michigan Avenue NE, Washington, DC 20064 – USA
e-mail: regalia@cua.edu

Athanasios A. Rontogiannis

Institute for Space Applications and Remote Sensing
National Observatory of Athens
Metaxa and Vas. Pavlou Street, Athens 15236 – Greece
e-mail: tronto@space.noa.gr

Marcio G. Siqueira

Cisco Systems
170 West Tasman Drive, San Jose, CA 95134-1706 – USA
e-mail: mgs@cisco.com

Gilbert Strang

Department of Mathematics
Massachusetts Institute of Technology (MIT)
77 Massachusetts Avenue, Cambridge, MA 02139-4307 – USA
e-mail: gs@math.mit.edu

Sergios Theodoridis

Department of Informatics and Telecommunications
University of Athens
Panepistimiopolis, Ilissia, Athens 15784 – Greece
e-mail: stheodor@di.uoa.gr

Stefan Werner

Department of Signal Processing and Acoustics, SMARAD CoE
Helsinki University of Technology
P.O. Box 3000 TKK, FIN-02015 – Finland
e-mail: stefan.werner@tkk.fi

Jenq-Tay Yuan

Department of Electrical Engineering
Fu Jen Catholic University
510 Chung Cheng Road, Hsinchuang, Taiwan 24205 – R.O.C.
e-mail: yuan@ee.fju.edu.tw