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Kenneth R. Meyer
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Introduction to Hamiltonian Dynamical Systems and the N-Body Problem

Second edition

Springer

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Preface to the Second Edition

This new edition expands on some old material and introduces some new subjects. The expanded topics include: parametric stability, logarithms of symplectic matrices, normal forms for Hamiltonian matrices, spacial Delaunay elements, pulsating coordinates, Lyapunov–Chetaev stability applications and more. There is a new section on the Maslov index and a new chapter on variational arguments as applied to the celebrated figure-eight orbit of the 3-body problem.

Still the beginning chapters can serve as a first graduate level course on Hamiltonian dynamical systems, but there is far too much material for a single course. Instructors will have to select chapters to meet their interests and the needs of their class. It will also serve as a reference text and introduction to the literature.

The authors wish to thank their wives and families for giving them the time to work on this project. They acknowledge the support of their universities and various funding agencies including the National Science Foundation, the Taft Foundation, the Sloan Foundation, and the Natural Sciences and Engineering Research Council through the Discovery Grants Program.

This second edition in manuscript form was read by many individuals who made many valuable suggestions and corrections. Our thanks go to Hildeberto Cabral, Scott Dumas, Vadin Fitton, Clarissa Howison, Jesús Palacián, Dieter Schmidt, Jaume Soler, Qiudong Wang, and Patricia Yanguas.

Nonetheless, it is the readers responsibility to inform us of additional errors. Look for email addresses and an errata on MATH.UC.EDU/~MEYER/.

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Preface to the First Edition

The theory of Hamiltonian systems is a vast subject that can be studied from many different viewpoints. This book develops the basic theory of Hamiltonian differential equations from a dynamical systems point of view. That is, the solutions of the differential equations are thought of as curves in a phase space and it is the geometry of these curves that is the important object of study. The analytic underpinnings of the subject are developed in detail. The last chapter on twist maps has a more geometric flavor. It was written by Glen R. Hall. The main example developed in the text is the classical N-body problem; i.e., the Hamiltonian system of differential equations that describes the motion of N point masses moving under the influence of their mutual gravitational attraction. Many of the general concepts are applied to this example. But this is not a book about the N-body problem for its own sake. The N-body problem is a subject in its own right that would require a sizable volume of its own. Very few of the special results that only apply to the N-body problem are given.

This book is intended for a first course at the graduate level. It assumes a basic knowledge of linear algebra, advanced calculus, and differential equations, but does not assume knowledge of advanced topics such as Lebesgue integration, Banach spaces, or Lie algebras. Some theorems that require long technical proofs are stated without proof, but only on rare occasions. The first draft of the book was written in conjunction with a seminar that was attended by engineering graduate students. The interest and background of these students influenced what was included and excluded.

This book was read by many individuals who made valuable suggestions and many corrections. The first draft was read and corrected by Ricardo Moena, Alan Segerman, Charles Walker, Zhangyong Wan, and Qiudong Wang while they were students in a seminar on Hamiltonian systems. Gregg Buck, Konstantin Mischaikow, and Dieter Schmidt made several suggestions for improvements to early versions of the manuscript. Dieter Schmidt wrote the section on the linearization of the equation of the restricted problem at the five libration points. Robin Vandivier found copious grammatical errors by carefully reading the whole manuscript. Robin deserves a special thanks. We hope that these readers absolve us of any responsibility.

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