Intertemporal Macroeconomic Models, Money and Rational Choices

Also by Giuseppe Chirichiello

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Preface

Intertemporal macroeconomics relies on microeconomics and general equilibrium analysis to describe choices of agents over period of time, perhaps infinitely long. In particular, the formal modelling of individual choices uses the methods of dynamic optimization or optimal control theory. To this, methods of dynamic macroeconomics, that is of growth theory, are linked and the effects of policy interventions are then examined as the interaction between decisions of agents and policy interventions. This way of approaching macroeconomics sustains a fundamental change of perspective, with respect to the conventional macroeconomics originated from the IS/LM–AD/AS Keynesian foundations. The idea is that macroeconomics is not only concerned with short-run and the state of the effective demand but also is concerned with long-run and intertemporal allocation of economic resources. Economic policy interventions, therefore, are reconsidered in relation to the effects on rate of growth and capital accumulation. Two basic approaches can be pointed out: models of infinitely-lived agents (Cass-Ramsey-Koopmans approach) and models of overlapping-generations (Allais-Fisher-Samuelson approach). This book explores both the approaches, by first presenting the major issues of the ‘real’ models of growth and the role of fiscal policies. However, whereas results about real models of growth and fiscal policies are well-established, more controversial are the questions concerning monetary models and monetary policies. Indeed, money may be or may not be a source of instability and changes in growth rate of money stock may be or may not be ‘superneutral’ according to the way monetary economy and/or monetary policy are modelled. The book offers a framework and a systematic exploration of these and related questions in both basic models, and also in models of endogenous growth. The book has originated from advanced courses I taught to students of a research group of which I was co-ordinator in the 1990s. The original Italian manuscript was finished in the 1996; the book is the result of a revised translation of that work. I want to thank Cora Hahn for her professional job of thoroughly revising my translation and turning the manuscript into a more readable work. I dedicate the book to my mother Nigro Antonia.

Introduction

Intertemporal macroeconomics provides a completely new paradigm with respect to conventional AD/AS macroeconomics, originating from Keynesian models. Since new answers to traditional questions continue to appear in the professional journals, this book offers an assessment and a step toward standardization of intertemporal macroeconomics. It ideally consists of two parts. A first part covers Chapter 1 to Chapter 3 and is concerned with basic intertemporal models. The second part covers Chapter 4 to Chapter 6 and is concerned with more complex and controversial subjects, such as monetary models and endogenous growth models, the latter also extended to monetary economies. The book is organized as follows. Chapter 1 contains a review of descriptive models of growth. Section 1.1 examines the Harrod-Domar 'Keynesian' model of growth, which could be considered a bridge between conventional macroeconomics and actual dynamic macroeconomics. Sections 1.2 to 1.6 discuss neoclassical (Solow's) model by revisiting the issues of existence, stability and efficiency of steady state, and the golden rule and its importance for the efficiency of steady state. Two of these sections are devoted to the comparative analysis of changes in propensity to save and to the comparison of stylized facts and prescriptions of the neoclassical model.

Chapter 2 deals with aggregate intertemporal microfounded models. It sheds light on the fundamental change in macroeconomics: the view that macroeconomics, like microeconomics, deals with interactions over time between rational agents. Section 2.1 demonstrates Ramsey's optimal saving rule for a single 'pure-saver' agent. As in Ramsey, this problem is initially formulated as a variational problem, but it is then reformulated as an optimal control problem. Optimal control is the methodology applied to dynamic choices in the rest of the book.

In section 2.2 Ramsey's optimal saving rule is extended to the economy as a whole, by proposing a peculiar model where income is not distributed to agents by the market. This model serves as tool to examine in sections 2.3 and 2.4 the existence, stability and efficiency of optimal steady state paths. The remainder of Chapter 2 examines the dynamics, efficiency and the effects of fiscal policy in a decentralized competitive economy.

Chapter 3 deals with overlapping generations models. Section 3.1 presents Fisher's optimal saving rule for a single agent. Sections 3.2 and 3.3 are

devoted to a close examination of the properties of the saving function from the microeconomic point of view. Basic microeconomic methodology, even the dual approach, is applied here to highlight the micro-foundations of the aggregate saving function on which the pure-exchange aggregate model and the growth model of competitive economy with overlapping generations are built. Sections 3.4 to 3.15 examine the dynamics, efficiency and the effects of fiscal policies. In particular sections 3.9 to 3.15 offer an extensive analysis of taxation, social security, and public debt and deficit.

Chapter 4 deals with intertemporal monetary models with an infinite horizon. Section 4.1 contains a basic descriptive monetary model. In three subsections (4.1.1–4.1.3), dynamics, stability and the existence of a steady state are examined using various hypotheses of forming expectations. Section 4.2 revises Cagan's model of monetary instability and prepares the ground for subsequent issues. Section 4.3 considers the monetary model with optimizing agents (Sidrauski). In this framework three issues are considered: the superneutrality of money (section 4.4), the welfare effects of inflation (section 4.5), and self-generating hyperinflation phenomena (section 4.6). Finally, the appendix to Chapter 4 presents alternative models of monetary economy with an infinite horizon.

Chapter 5 examines alternative approaches to a monetary economy with overlapping-generations. Section 5.1 considers the basic pure exchange model, which offers a framework for discussing issues regarding the foundations of money. Section 5.2 examines the dynamics of the basic model through the 'reflected offer curve'. Section 5.3 extends the model to the production of goods and the accumulation of physical capital, and it examines dynamics. The remaining sections are devoted to problems of monetary theory. Section 5.4 considers an economy with storage technology and the non-dominance of money in nominal interest rate when there is a market for private credit. Section 5.5 shows the inefficiency of seignorage policy. Section 5.6 reconsiders the optimum quantity of money problem in the overlapping generations model when money enters into the utility function. Sections 5.7 to 5.10 examine alternative overlapping generations models of a money economy (cash-in-advance constraint; transaction costs approach; legal restrictions approach) and the effects of monetary policy.

Chapter 6 offers an introduction to endogenous growth models. Sections from 6.2 to 6.4 examine a basic framework for endogenous growth derived from (i) models with constant-returns-of-scale and human capital; (ii) models with externalities; (iii) models of non-competitive economies. Sections 6.5 and 6.6 examine taxation and public spending

policies. Sections 6.7 to 6.11, devoted to monetary economies and to an examination of the effects of monetary policy on growth, offer an innovative synthesis of issues. The book makes extensive use of mathematics. However, it presumes only a basic knowledge of calculus. Essential tools such as difference and differential systems of equations, stability and saddle-path stability–instability and the theory of optimal control are presented in mathematical appendixes at the end of the book.