The Behavior of Structures Composed of Composite Materials

SOLID MECHANICS AND ITS APPLICATIONS

Volume 105

Series Editor: G.M.L. GLADWELL
Department of Civil Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?*
The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.

The Behavior of Structures Composed of Composite Materials

Second Edition

by

JACK R. VINSON

*H.Fletcher Brown Professor of Mechanical & Aerospace Engineering,
The Center for Composite Materials and The College of Marine Studies,
Department of Mechanical Engineering,
University of Delaware,
Newark, Delaware, U.S.A.*

and

ROBERT L. SIERAKOWSKI

*Chief Scientist,
AFRL/MN Eglin AFB,
Florida, U.S.A.*

Library of Congress Control Number: 2008921377

ISBN 978-1-4020-0904-4
ISBN 978-0-306-48414-8 (e-book)

Published by Springer,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

www.springer.com

Printed on acid-free paper

First print 2002
Reprinted 2008

All Rights Reserved

© 2008 Springer Science+Business Media B.V.

No part of this work may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission from the Publisher, with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work.

To my beautiful wife, Midge, for providing the wonderful environment, love, patience and encouragement to complete this text - JRV

In loving memory of Nina, and also to my wonderful children, Sandy and Steve - RLS

Preface to the Corrected Second Edition

Since the introduction of the second printing, a number of exciting advances has taken place within the materials community. Historically, we have accepted the inherent properties of materials as given for any specified material; that is, metals, polymers, ceramics and so on. Introduction of composite materials technology has allowed researchers to consider multi-constituent systems resulting in properties that can be superior to each of the constituents independently. Now we are on the cusp of a new era in materials technology which can be coined as the engineered materials revolution. That is, we are now developing the capacity to tailor a material to perform a specific or representative set of functions from the basic atomic level. This technical development allows for the potential design of microsystems with nano-components. The advent of this technology offers a number of creative future applications in advancing miniaturized systems including structural nano-electronic devices, circuits, and computers as but a few examples. These advances are based upon the extraordinary properties of carbon nanotubes including a Young's modulus approaching that of diamond. These properties are ideal for reinforced composites, and this third edition provides the student or researcher with the necessary tools to advance the state of the art in structural design of new and potentially exciting multifunctional, self-repairing, self-duplicating, nano-structural materials.

Preface to the Second Edition

The purpose of this text is to educate the engineering reader in the various aspects of mechanics for using composite materials in the design and analysis of composite structures and products.

In Chapter 1, the text acquaints the reader with the description of a composite material, and its constituents. Then, methods by which to manufacture composite materials are discussed, followed by a description of the uses of composite materials early in the twenty-first century.

Chapter 2 provides the fundamentals of anisotropic elasticity, and the methods to characterize and mathematically describe composite laminae and laminates which are the “building blocks” of composite structures. Also discussed are thermal, hygrothermal, high strain rate and piezoelectric considerations in modern composites.

Chapter 3 then deals exclusively with the static and dynamic response of composite plates and panels subjected to a variety of mechanical and environmental loads in great detail. This includes stresses, deformations, buckling loads, natural frequencies and response to blast loads. Chapter 4 analogously treats a special case of the above, namely beams, columns and rods.

In Chapter 5, cylindrical composite shells are discussed, both in determining the stresses and deformations due to static loads, but in treating the buckling of these shells under various loads and their combinations. The peculiar behavior of shells, (such as the bending boundary layer) compared to plates and beams is discussed in detail.

Because so many practical structural problems are too difficult or complex to obtain analytical solutions, Chapter 6 provides in-depth knowledge of attacking real life structural design problems using energy principles and variational methods. Thus, the engineer can always obtain a solution to a problem.

Chapter 7 provides various strength and failure theories widely used today, and their comparison. Chapter 8 provides suggested ways to analyze and design adhesive bonded joints and mechanically fastened joints.

Chapter 9 has been added to provide a needed introduction to composite design philosophy.

Appendix 1 provides a discussion of micromechanics basics; Appendix 2 lists all or most of the test standards for polymer matrix composite and Appendix 3 lists the mechanical properties of many composite in use today.

At the end of each chapter are numerous problems, which can be useful as homework problems or modified for examination problems. Professors may contact the authors for solutions to these problems.

Appreciation is hereby expressed to James T. Arters, an engineering student at the University of Delaware, who meticulously typed the text through its evolution. His accuracy, stamina and diligence are greatly appreciated. Appreciation is also expressed to Dr. Gregg Schoeppner for his contributions to Chapter 1 and the Appendices, and to Ms. Jill O'Donnell for her manuscript reading.

Jack R. Vinson
Robert L. Sierakowski

Preface to the First Edition

While currently available tests dealing with the subject of high performance composite materials touch upon a spectra of topics such as mechanical metallurgy, physical metallurgy, micromechanics and macromechanics of such systems, it is the specific purpose of this text to examine elements of the mechanics of structural components composed of composite materials. This text is intended for use in training engineers in this new technology and rational thought processes necessary to develop a better understanding of the behavior of such material systems for use as structural components. The concepts are further exploited in terms of the structural format and development to which the book is dedicated. To this end the development progresses systematically by first introducing the notion and concepts of what these new material classes are, the fabrication processes involved and their unique features relative to conventional monolithic materials. Such introductory remarks, while far too short in texts of this type, appear necessary as a precursor for engineers to develop a better understanding for design purposes of both the threshold limits to which the properties of such systems can be pushed as well as the practical limitations on their manufacture.

Following these introductory remarks, an in-depth discussion of the important differences between composites and conventional monolithic material types is discussed in terms of developing the concepts associated with directional material properties. That is, the ideas of anisotropic elasticity for initially homogeneous bodies in the phenomenological sense are described and presented. The use of such analytical tools is then presented through exemplification of selected problems for a number of classical type problems of various geometric shapes including plane stress, plane strain and the bending of a simply supported beam.

These ideas are carried forward and developed for continuous fiber composites in Chapter Two which discusses both single ply laminae and multi-ply laminate theory. This is then followed by a series of chapters, each of which deals with functional aspect of structural design in which the basic building blocks of a structural system are made. That is, plates and panels; beams, columns and rods; and cylindrical and spherical shells are each discussed within the framework of their potential use in a functional environment. Thus the traditional topics of conventional monolithic (isotropic) material structural elements such as structures subjected to static loads, thermal and other environmental loads, structural instability and vibratory response are included along with chapters on energy methods and failure theories of composite materials.

Energy methods have been included to present a tool for solving difficult problems of various types encountered in practice. Indeed, in many instances closed form solutions are not possible and approximate solutions must be sought. Energy methods thus provide both an alternative for the formulation of such problems plus a means of generating approximate solutions.

The chapter on failure theories is a generic presentation in the senses that any and/or all of the above structural components consisting of various multi-ply construction can fail when subjected to a sufficiently large loading combination. It is emphasized that the failure of composites is a complicated, changing issue because of the diverse ways in which such structural systems can fail due both to the geometric ply arrangement of the

components, complicated load paths, and the diversity of failure mechanisms which can be activated. Therefore, this chapter should serve in a global sense at best as a guide to the prediction of structural integrity, while more common and acceptable phenomenological failure theories are being developed.

Finally, a chapter on joining is included to discuss to some detail the two methods by which composite material structural components can be joined: namely, adhesive bonding and mechanical fastening. Again, the material presented is an introduction to the subject which is rapidly changing and developing.

At the end of each chapter are several problems, characteristic of the material covered which can be used. Some answers are given in an appendix.

Knowing that nothing is perfect, the authors welcome any notification of errors and ambiguities, and if addresses are provided, authors will forward errata sheets periodically.

Appreciation is hereby expressed to many students at the University of Delaware, University of Florida, Ohio State University, The Ballistics Research Laboratory, and the Argentine Air Force who have helped directly or indirectly in refining, improving and correcting the text, as well as working various problems and examples. In addition appreciation is expressed to Dr. W.J. Renton, Vought Corporation, who has used portions of the text at the University of Texas-Arlington, and made suggestions and corrections.

Jack R. Vinson
Robert L. Sierakowski

Contents

Preface to the Corrected Second Edition	VII
Preface to the Second Edition	VIII
Preface to the First Edition	IX
1. Introduction to Composite Materials	1
1.1. General History	1
1.2. Composite Material Description	2
1.3. Types of Composite Materials	6
1.4. Constituent Properties	8
1.5. Composite Manufacturing, Fabrication and Processing	11
1.6. Uses of Composite Materials	21
1.7. Design and Analyses with Composite Materials	33
1.8. References	36
1.9. Journals	36
1.10. Problems	37
2. Anisotropic Elasticity and Composite Laminate Theory	39
2.1. Introduction	39
2.2. Derivation of the Anisotropic Elastic Stiffness and Compliance Matrices	40
2.3. The Physical Meaning of the Components of the Orthotropic Elasticity Tensor	46
2.4. Methods to Obtain Composite Elastic Properties from Fiber and Matrix Properties	50
2.5. Thermal and Hygrothermal Considerations	53
2.6. Time-Temperature Effects on Composite Materials	57
2.7. High Strain Rate Effects on Material Properties	58
2.8. Laminae of Composite Materials	59
2.9. Laminate Analyses	66
2.10. Piezoelectric Effects	76
2.11. References	77
2.12. Problems	79
3. Plates and Panels of Composite Materials	87
3.1. Introduction	87
3.2. Plate Equilibrium Equations	87
3.3. The Bending of Composite Material Laminated Plates: Classical Theory	91
3.4. Classical Plate Theory Boundary Conditions	94
3.5. Navier Solutions for Rectangular Composite Material Plates	95
3.6. Navier Solution for a Uniformly Loaded Simply Supported Plate – An Example Problem	98
3.7. Levy Solution for Plates of Composite Materials	102

3.8. Perturbation Solutions for the Bending of a Composite Material Plate With Mid-Plane Symmetry and No Bending-Twisting Coupling	106
3.9. Quasi-Isotropic Composite Panels Subjected to a Uniform Lateral Load	109
3.10. A Static Analysis of Composite Material Panels Including Transverse Shear Deformation Effects	111
3.11. Boundary Conditions for a Plate Using the Refined Plate Theory Which Includes Transverse Shear Deformation	114
3.12. Composite Plates on an Elastic Foundation	115
3.13. Solutions for Plates of Composite Materials Including Transverse-Shear Deformation Effects, Simply Supported on All Four Edges	116
3.14. Dynamic Effects on Panels of Composite Materials	119
3.15. Natural Flexural Vibrations of Rectangular Plates: Classical Theory	120
3.16. Natural Flexural Vibrations of Composite Material Plate Including Transverse-Shear Deformation Effects	122
3.17. Forced-Vibration Response of a Composite Material Plate Subjected to a Dynamic Lateral Load	124
3.18. Buckling of a Rectangular Composite Material Plate – Classical Theory	130
3.19. Buckling of a Composite Material Plate Including Transverse-Shear Deformation Effects	132
3.20. Some Remarks on Composite Structures	135
3.21. Methods of Analysis for Sandwich Panels With Composite Material Faces, and Their Structural Optimization	138
3.22. Governing Equations for a Composite Material Plate With Mid-Plane Asymmetry	138
3.23. Governing Equations for a Composite Material Plate With Bending-Twisting Coupling	139
3.24. Concluding Remarks	140
3.25. References	141
3.26. Problems and Exercises	143
4. Beams, Columns and Rods of Composite Materials	155
4.1. Development of Classical Beam Theory	155
4.2. Some Composite Beam Solutions	160
4.3. Composite Beams With Abrupt Changes in Geometry or Load	165
4.4. Solutions by Green's Functions	171
4.5. Composite Beams of Continuously Varying Cross-Section	173
4.6. Rods	177
4.7. Vibration of Composite Beams	179
4.8. Beams With Mid-Plane Asymmetry	183
4.9. Advanced Beam Theory for Dynamic Loading Including Mid-Plane Asymmetry	184
4.10. Advanced Beam Theory Including Transverse Shear Deformation Effects	193
4.11. Buckling of Composite Columns	197
4.12. References	200
4.13. Problems	200

5. Composite Material Shells	215
5.1. Introduction	215
5.2. Analysis of Composite Material Circular Cylindrical Shells	215
5.3. Some Edge Load and Particular Solutions	222
5.4. A General Solution for Composite Cylindrical Shells Under Axially Symmetric Loads	228
5.5. Response of a Long Axi-Symmetric Laminated Composite Shell to an Edge Displacement	230
5.6. Sample Solutions	232
5.7. Mid-Plane Asymmetric Circular Cylindrical Shells	239
5.8. Buckling of Circular Cylindrical Shells of Composite Materials Subjected to Various Loads	243
5.9. Vibrations of Composite Shells	252
5.10. Additional Reading On Composite Shells	253
5.11. References	253
5.12. Problems	254
6. Energy Methods For Composite Material Structures	259
6.1. Introduction	259
6.2. Theorem of Minimum Potential Energy	260
6.3. Analysis of a Beam Using the Theorem of Minimum Potential Energy	261
6.4. Use of Minimum Potential Energy for Designing a Composite Electrical Transmission Tower	268
6.5. Minimum Potential Energy for Rectangular Plates	272
6.6. A Rectangular Composite Material Plate Subjected to Lateral and Hygrothermal Loads	274
6.7. In-Plane Shear Strength Determination of Composite Materials in Laminated Composite Panels	276
6.8. Use of the Theorem of Minimum Potential Energy to Determine Buckling Loads in Composite Plates	282
6.9. Trial Functions for Various Boundary Conditions for Composite Material Rectangular Plates	285
6.10. Reissner's Variational Theorem and its Applications	286
6.11. Static Deformation of Moderately Thick Beams	289
6.12. Flexural Vibrations of Moderately Thick Beams	293
6.13. Flexural Natural Frequencies of a Simply Supported Beam Including Transverse Shear Deformation and Rotatory Inertia Effects	295
6.14. References	299
6.15. Problems	299
7. Strength and Failure Theories	303
7.1. Introduction	303
7.2. Failure of Monolithic Isotropic Materials	306
7.3. Anisotropic Strength and Failure Theories	309
7.3.1. Maximum Stress Theory	310
7.3.2. Maximum Strain Theory	310

XIV

7.3.3. Interactive Failure Theories	311
7.4. Lamina Strength Theories	315
7.5. Laminate Strength Analysis	328
7.6. References	331
7.7. Problems	332
8. Joining of Composite Material Structures	333
8.1. General Remarks	333
8.2. Adhesive Bonding	333
8.3. Mechanical Fastening	348
8.4. Recommended Reading	354
8.5. References	354
8.6. Problems	357
9. Introduction to Composite Design	361
9.1. Introduction	361
9.2. Structural Composite Design Procedures	368
9.3. Engineering Analysis	371
Appendices	375
A-1 Micromechanics	375
A-2 Test Standards for Polymer Matrix Composites	391
A-3 Properties of Various Polymer Composites	393
Author Index	397
Subject Index	401