

HANDBOOK OF METAHEURISTICS

**INTERNATIONAL SERIES IN
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edited by

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To our wives, Diane and Ann, whose meta-patience and meta-support have sustained us through this effort!

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PREFACE

Metaheuristics, in their original definition, are solution methods that orchestrate an interaction between local improvement procedures and higher level strategies to create a process capable of escaping from local optima and performing a robust search of a solution space. Over time, these methods have also come to include any procedures that employ strategies for overcoming the trap of local optimality in complex solution spaces, especially those procedures that utilize one or more neighborhood structures as a means of defining admissible moves to transition from one solution to another, or to build or destroy solutions in constructive and destructive processes.

The degree to which neighborhoods are exploited varies according to the type of procedure. In the case of certain population-based procedures, such as genetic algorithms, neighborhoods are implicitly (and somewhat restrictively) defined by reference to replacing components of one solution with those of another, by variously chosen rules of exchange popularly given the name of “crossover.” In other population-based methods, based on the notion of path relinking, neighborhood structures are used in their full generality, including constructive and destructive neighborhoods as well as those for transitioning between (complete) solutions. Certain hybrids of classical evolutionary approaches, which link them with local search, also use neighborhood structures more fully, though apart from the combination process itself. Meanwhile, “single thread” solution approaches, which do not undertake to manipulate multiple solutions simultaneously, run a wide gamut that not only manipulate diverse neighborhoods but incorporate numerous forms of strategies ranging from thoroughly randomized to thoroughly deterministic, depending on the elements such as the phase of search or (in the case of memory-based methods) the history of the solution process.¹

A number of the tools and mechanisms that have emerged from the creation of metaheuristic methods have proved to be remarkably effective, so much so that metaheuristics have moved into the spotlight in recent years as the preferred line of attack for solving many types of complex problems, particularly those of a combinatorial nature. While metaheuristics are not able to certify the optimality of the solutions they find, exact procedures (which theoretically can provide such a certification, if allowed to run long enough)² have often proved incapable of finding solutions whose

¹Methods based on incorporating collections of memory-based strategies, invoking forms of memory more flexible and varied than those used in approaches such as tree search and branch and bound, are sometimes grouped under the name Adaptive Memory Programming. This term, which originated in the tabu search literature where such adaptive memory strategies were first introduced and continue to be the primary focus, is also sometimes used to encompass other methods that have more recently adopted memory-based elements.

²Some types of problems seem quite amenable to exact methods, particularly to some of the methods embodied in the leading commercial software packages for mixed integer programming. Yet even by these approaches the “length of time” required to solve many problems exactly appears to exceed all reasonable measure, including in some cases measures of astronomical scale. It has been conjectured that metaheuristics

quality is close to that obtained by the leading metaheuristics—particularly for real world problems, which often attain notably high levels of complexity. In addition, some of the more successful applications of exact methods have come about by incorporating metaheuristic strategies within them. These outcomes have motivated additional research and application of new and improved metaheuristic methodologies.

This handbook is designed to provide the reader with a broad coverage of the concepts, themes and instrumentalities of this important and evolving area of optimization. In doing so, we hope to encourage an even wider adoption of metaheuristic methods for assisting in problem solving, and to stimulate research that may lead to additional innovations in metaheuristic procedures.

The handbook consists of 19 chapters. Topics covered include Scatter Search, Tabu Search, Genetic Algorithms, Genetic Programming, Memetic Algorithms, Variable Neighborhood Search, Guided Local Search, GRASP, Ant Colony Optimization, Simulated Annealing, Iterated Local Search, Multi-Start Methods, Constraint Programming, Constraint Satisfaction, Neural Network Methods for Optimization, Hyper-Heuristics, Parallel Strategies for Metaheuristics, Metaheuristic Class Libraries, and A-Teams. This family of metaheuristic chapters, while not exhaustive of the many approaches that have sprung into existence in recent years, encompasses the critical strategic elements and their underlying ideas that represent the state-of-the-art of modern metaheuristics.

This book is intended to provide the communities of both researchers and practitioners with a broadly applicable, up to date coverage of metaheuristic methodologies that have proven to be successful in a wide variety of problem settings, and that hold particular promise for success in the future. The various chapters serve as stand alone presentations giving both the necessary underpinnings as well as practical guides for implementation. The nature of metaheuristics invites an analyst to modify basic methods in response to problem characteristics, past experiences, and personal preferences and the chapters in this handbook are designed to facilitate this process as well.

The authors who have contributed to this volume represent leading figures from the metaheuristic community and are responsible for pioneering contributions to the fields they write about. Their collective work has significantly enriched the field of optimization in general and combinatorial optimization in particular. We are especially grateful to them for agreeing to provide the first-rate chapters that appear in this handbook. We would also like to thank our graduate students, Gyung Yung and Rahul Patil, for their assistance. Finally, we would like to thank Gary Folven and Carolyn Ford of Kluwer Academic Publishers for their unwavering support and patience throughout this project.