CHAPTERS IN GAME THEORY

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CHAPTERS IN GAME THEORY

In honor of Stef Tijs

Edited by

PETER BORM

*University of Tilburg,
The Netherlands*

and

HANS PETERS

*University of Maastricht,
The Netherlands*

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Preface

On the occasion of the 50th birthday of Stef Tijs in 1987 a volume of surveys in game theory in Stef's honor was composed¹. All twelve authors who contributed to that book still belong to the twenty-nine authors involved in the present volume, published fifteen years later on the occasion of Stef's 65th birthday. Twenty-five of these twenty-nine authors wrote—or write, in one case—their Ph.D. theses under the supervision of Stef Tijs. The other four contributors are indebted to Stef Tijs to a different but hardly less decisive degree.

What makes a person deserve to be the honorable subject of a scientific *liber amicorum*, and that on at least two occasions in his life? If that person is called Stef Tijs then the answer includes at least the following reasons. First of all, until now Stef has supervised about thirty Ph.D. students in game theory alone. More importantly than sheer numbers, most of these students stayed in academics; for instance all those who contributed to the 1987 volume. It is beyond doubt that this fact has everything to do with the devotion, enthusiasm and deep knowledge invested by Stef in guiding students. Moreover, the number of his internationally published papers has increased from about sixty in 1987 to about two hundred now. His papers cover every field in game theory, and extend to related areas as social choice theory, mathematical economics, and operations research. Last but not least, Stef's numerous coauthors come from and live in all parts of this world: he has been a true missionary in game theory, and the contributors to this volume are proud to be among his apostles.

PETER BORM

HANS PETERS

Tilburg/Maastricht

February 2002

¹H.J.M. Peters and O.J. Vrieze, eds., *Surveys in Game Theory and Related Topics*, CWI Tract 39, Amsterdam, 1987.

About Stef Tijs

The first work of Stef Tijs in game theory was his Ph.D. dissertation *Semi-infinite and infinite matrix games and bimatrix games* (1975). He took his Ph.D. at the University of Nijmegen, where he had held a position since 1960. His Ph.D. advisors were A. van Rooij and F. Delbaen. From 1975 on he gradually started building a game theory school in the Netherlands with a strong international focus. In 1991 he left Nijmegen to continue his research at Tilburg University. In 2000 he was awarded a doctorate honoris causa at the Miguel Hernandez University in Elche, Spain.

About this book

The authors of this book were asked to write on topics belonging to their expertise and having a connection with the work of Stef Tijs. Each contribution has been reviewed by two other authors. This has resulted in fourteen chapters on different subjects: some of these can be considered surveys while other chapters present new results. Most contributions can be positioned somewhere in between these categories. We briefly describe the contents of each chapter. For the references the reader should consult the list of references in each chapter under consideration.

Chapter 1, *Stochastic cooperative games: theory and applications* by Peter Borm and Jeroen Suijs, considers cooperative decision making under risk. It provides a brief survey on three existing models introduced by Charnes and Granot (1973), Suijs *et al.* (1999), and Timmer *et al.* (2000), respectively. It also compares their performance with respect to two applications: the allocation of random maintenance cost of a communication network tree to its users, and the division of a stochastic estate among the creditors in a bankruptcy situation.

Chapter 2, *Sequencing games: a survey* by Imma Curiel, Herbert Hamers, and Flip Klijn, gives an overview of the start and the main developments in the research area that studies the interaction between sequencing situations and cooperative game theory. It focuses on results related to balancedness and convexity of sequencing games.

In Chapter 3, *Game theory and the market* by Eric van Damme and Dave Furth, it is argued that both cooperative and non-cooperative game models can substantially increase our understanding of the functioning of actual markets. In the first part of the chapter, by going back to the

work of the founding fathers von Neumann, Morgenstern, and Nash, a brief historical sketch of the differences and complementarities between the two types of models is provided. In the second part, the main point is illustrated by means of examples of bargaining, oligopolistic interaction and auctions.

In Chapter 4, *On the number of extreme points of the core of a transferable utility game* by Jean Derks and Jeroen Kuipers, it is derived from a more general result that the upper core and the core of a transferable utility game have at most $n!$ different extreme points, with n the number of players. This maximum number is attained by strict convex games but other games may have this property as well. It is shown that $n!$ different extreme core points can only be obtained by strict exact games, but that not all such games have $n!$ different extreme points.

In Chapter 5, *Consistency and potentials in cooperative TU-games: Sobolev's reduced game revived* by Theo Driessen, a consistency property for a wide class of game-theoretic solutions that possess a potential representation is studied. The consistency property is based on a modified reduced game related to Sobolev's. A detailed exposition of the developed theory is given for semivalues of cooperative TU-games and the Shapley and Banzhaf values in particular.

In Chapter 6, *On the set of equilibria of a bimatrix game: a survey* by Mathijs Jansen, Peter Jurg, and Dries Vermeulen, the methods used by different authors to write the set of equilibria of a bimatrix game as the union of a finite number of polytopes, are surveyed.

Chapter 7, *Concave and convex serial cost sharing* by Maurice Koster, introduces the concave and convex serial rule, two new cost sharing rules that are closely related to the serial cost sharing rule of Moulin and Shenker (1992). It is shown that the concave serial rule is the unique rule that minimizes the range of cost shares subject to the excess lower bounds. Analogous results are derived for the convex serial rule. In particular, these characterizations show that the serial cost sharing rule is consistent with diametrically opposed equity properties, depending on the nature of the cost function: the serial rule equals the concave (convex) serial rule in case of a concave (convex) cost function.

In Chapter 8, *Centrality orderings in social networks* by Herman Monsuur and Ton Storcken, a centrality ordering arranges the vertices in a social network according to their centrality position in that network.

Centrality addresses notions like focal points of communication, potential of communicational control, and being close to other network vertices. In social network studies they play an important role. Here the focus is on the conceptual issue of what makes a position in a network more central than another position. Characterizations of the cover, the median and degree centrality orderings are discussed.

In Chapter 9, *The Shapley transfer procedure for NTU-games* by Gert-Jan Otten and Hans Peters, the Shapley transfer procedure (Shapley, 1969) is extended in order to associate with every solution correspondence for transferable utility games satisfying certain regularity conditions, a solution for nontransferable utility games. An existence and a characterization result are presented. These are applied to the Shapley value, the core, the nucleolus, and the τ -value.

Chapter 10, *The nucleolus as equilibrium price* by Jos Potters, Hans Reijnierse, and Anita van Gellekom, studies exchange economies with indivisible goods and money. The notions of a stable equilibrium and regular prices are introduced. It is shown that the nucleolus concept for TU-games can be used to single out specific regular prices. Algorithms to compute the nucleolus can therefore be used to determine regular price vectors.

Chapter 11, *Network formation, costs, and potential games* by Marco Slikker and Anne van den Nouweland, studies strategic-form games of network formation in which an exogenous allocation rule is used to determine the players' payoffs in various networks. It is shown that such games are potential games if the cost-extended Myerson value is used as the exogenous allocation rule. The question is then studied which networks are formed according to the potential maximizer, a refinement of Nash equilibrium for potential games.

Chapter 12, *Contributions to the theory of stochastic games* by Frank Thuijsman and Koos Vrieze, presents an introduction to the history and the state of the art of the theory of stochastic games. Dutch contributions to the field, initiated by Stef Tijs, are addressed in particular. Several examples are provided to clarify the issues.

Chapter 13, *Linear (semi-)infinite programs and cooperative games* by Judith Timmer and Natividad Llorca, gives an overview of cooperative games arising from linear semi-infinite or infinite programs.

Chapter 14, *Population uncertainty and equilibrium selection: a maximum likelihood approach* by Mark Voorneveld and Henk Norde, intro-

duces a general class of games with population uncertainty and, in line with the maximum likelihood principle, stresses those strategy profiles that are most likely to yield an equilibrium in the game selected by chance. Under mild topological restrictions, an existence result for maximum likelihood equilibria is derived. Also, it is shown how maximum likelihood equilibria can be used as an equilibrium selection device for finite strategic games.

About the authors

PETER BORM (p.e.m.borm@kub.nl) is affiliated with the Department of Econometrics of the University of Tilburg. He wrote his Ph.D. thesis, *On game theoretic models and solution concepts*, under the supervision of Stef Tijs.

IMMA CURIEL (curiel@math.umbc.edu) is affiliated with the Department of Mathematics and Statistics of the University of Maryland, Baltimore County. She wrote her Ph.D. thesis, *Cooperative game theory and applications*, under the supervision of Stef Tijs.

ERIC VAN DAMME (eric.vandamme@kub.nl) is affiliated with CentER, University of Tilburg. He wrote his master's thesis under the supervision of Stef Tijs and his Ph.D. thesis, *Refinements of the Nash equilibrium concept*, under the supervision of Jaap Wessels and Reinhard Selten.

JEAN DERKS (jean.derks@math.unimaas.nl) is affiliated with the Department of Mathematics of the University of Maastricht. His Ph.D. thesis, *On polyhedral cones of cooperative games*, was written under the supervision of Stef Tijs and Koos Vrieze.

THEO DRIESSEN (t.s.h.driessen@math.utwente.nl) is affiliated with the Department of Mathematical Sciences of the University of Twente. He wrote his Ph.D. thesis, *Contributions to the theory of cooperative games: the τ -value and k -convex games*, under the supervision of Stef Tijs and Michael Maschler.

DAVE FURTH (dfurth@fee.uva.nl) is affiliated with the Faculty of Economics and Econometrics of the University of Amsterdam. He wrote his Ph.D. thesis on oligopoly theory with Arnold Heertje and has been a regular guest of the game theory seminars organized since 1983 by Stef Tijs.

ANITA VAN GELLEKOM (anita.v.gellekom@mail.cadans.nl) works for a nonprofit institution. Her Ph.D. thesis, *Cost and profit sharing in a cooperative environment*, was written under the supervision of Stef Tijs.

HERBERT HAMERS (h.j.m.hamers@kub.nl) is affiliated with the Department of Econometrics of the University of Tilburg. Stef Tijs supervised his Ph.D. thesis, *Sequencing and delivery situations: a game theoretic approach*.

MATHIJS JANSEN (m.jansen@ke.unimaas.nl) is affiliated with the Department of Quantitative Economics of the University of Maastricht. His Ph.D. supervisors were Frits Ruymgaart and T.E.S. Raghavan, and his thesis *Equilibria and optimal threat strategies in two-person games* was written in close cooperation with Stef Tijs.

PETER JURG (peter@jurg.nl) works for a private software company. He wrote his thesis, *Some topics in the theory of bimatrix games*, under the supervision of Stef Tijs.

FLIP KLIJN (fklijn@uvigo.es) is affiliated with the Department of Statistics and Operations Research of the University of Vigo, Spain. His Ph.D. thesis, *A game theoretic approach to assignment problems*, was written under the supervision of Stef Tijs.

MAURIC KOSTER (mkoster@fee.uva.nl) is affiliated with the Department of Economics and Econometrics of the University of Amsterdam. Stef Tijs supervised his thesis *Cost sharing in production situations and network exploitation*.

JEROEN KUIPERS (jeroen.kuipers@math.unimaas.nl) is affiliated with the Department of Mathematics of the University of Maastricht. His Ph.D. thesis, *Combinatorial methods in cooperative game theory*, was supervised by Stef Tijs and Koos Vrieze.

NATIVIDAD LLORCA (nllorca@umh.es) is a Ph.D. student, under the supervision of Stef Tijs, at the Department of Statistics and Applied Mathematics of the University of Elche, Spain.

HERMAN MONSUUR (h.monsuur@kim.nl) is affiliated with the Royal Netherlands Naval College, section International Security Studies. His Ph.D. thesis, *Choice, ranking and circularity in asymmetric relations*, was supervised by Stef Tijs.

HENK NORDE (h.norde@kub.nl) is a member of the Department of Econometrics of the University of Tilburg, where his main research is in the area of game theory. He wrote a Ph.D. thesis in the field of differential equations, supervised by Leonid Frank at the University of Nijmegen.

ANNE VAN DEN NOUWELAND (annev@oregon.uoregon.edu) is affiliated with the Department of Economics of the University of Oregon, Eugene. She wrote her Ph.D. thesis, *Games and graphs in economic situations*, under the supervision of Stef Tijs.

GERT-JAN OTTEN (g.j.otten@kpn.com) works for KPN Telecom. His Ph.D. thesis, *On decision making in cooperative situations*, was supervised by Stef Tijs.

HANS PETERS (h.peters@ke.unimaas.nl) is affiliated with the Department of Quantitative Economics of the University of Maastricht. He wrote his Ph.D. thesis, *Bargaining game theory*, under the supervision of Stef Tijs.

JOS POTTERS (potters@sci.kun.nl) is affiliated with the Department of Mathematics of the University of Nijmegen. He wrote a Ph.D. thesis on a subject in geometry at the University of Leiden and cooperates with Stef Tijs in the area of game theory since the beginning of the eighties.

HANS REIJNIERSE (j.h.reijnierse@kub.nl) is affiliated with the Department of Econometrics of the University of Tilburg. He wrote his Ph.D. thesis, *Games, graphs, and algorithms*, supervised by Stef Tijs.

MARCO SLIKKER (m.slikker@tm.tue.nl) is affiliated with the Department of Technology Management of the Eindhoven University of Technology. Stef Tijs supervised his Ph.D. thesis *Decision making and cooperation restrictions*.

TON STORCKEN (t.storcken@ke.unimaas.nl) is affiliated with the Department of Quantitative Economics of the University of Maastricht. His Ph.D. thesis, *Possibility theorems for social welfare functions*, was supervised by Pieter Ruys, Stef Tijs, and Harrie de Swart.

JEROEN SUIJS (j.p.m.suijs@kub.nl) is affiliated with the CentER Accounting Research Group. He wrote his Ph.D. thesis, *Cooperative decision making in a stochastic environment*, under the supervision of Stef Tijs.

FRANK THUIJSMAN (frank@math.unimaas.nl) is affiliated with the Department of Mathematics of the University of Maastricht. He wrote his Ph.D. thesis, *Optimality and equilibria in stochastic games*, under the supervision of Stef Tijs and Koos Vrieze.

JUDITH TIMMER (j.b.timmer@math.utwente.nl) is affiliated with the Department of Mathematical Sciences of the University of Twente. Stef

Tijs supervised her Ph.D. thesis *Cooperative behaviour, uncertainty and operations research*.

DRIES VERMEULEN (d.vermeulen@ke.unimaas.nl) is affiliated with the Department of Quantitative Economics of the University of Maastricht. His Ph.D. thesis, *Stability in non-cooperative game theory*, was written under the supervision of Stef Tijs.

MARK VOORNEVELD (mark.voorneveld@hhs.se) works at the University of Stockholm and wrote a Ph.D. thesis, *Potential games and interactive decisions with multiple criteria*, supervised by Stef Tijs.

KOOS VRIEZE (o.j.vrieze@math.unimaas.nl) is affiliated with the Department of Mathematics of the University of Maastricht. His Ph.D. thesis, *Stochastic games with finite state and action spaces*, was written under the supervision of Henk Tijms and Stef Tijs.

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