THEORY OF DIDACTICAL SITUATIONS IN MATHEMATICS

DIDACTIQUE DES MATHÉMATIQUES, 1970–1990

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VOLUME 19

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THEORY OF DIDACTICAL SITUATIONS IN MATHEMATICS

DIDACTIQUE DES MATHÉMATIQUES, 1970–1990

by

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KLUWER ACADEMIC PUBLISHERS
NEW YORK / BOSTON / DORDRECHT / LONDON / MOSCOW

eBook ISBN: 0-306-47211-2
Print ISBN: 0-792-34526-6

Created in the United States of America

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TABLE OF CONTENTS

EDITORS' PREFACE

On the occasion of the celebration of "Twenty Years of *Didactique* of Mathematics" in France, Jeremy Kilpatrick commented that though the works of Guy Brousseau are known through texts referring to them or mentioning their existence, the original texts are unknown, or known only with difficulty, in the non-French-speaking world. With very few exceptions, what has been available until now have been interpretations of the works of Brousseau rather than the works themselves. It was in response to this need that two of us, in the euphoria of an unforgettable Mexican evening at the time of the 1990 PME conference, decided to undertake the task of translating into English most of the works of Guy Brousseau.

The œuvre is immense, and once past the initial moments of enthusiasm, with the accompanying ambition to produce the entire of it, we recognized the need to choose both the texts and a method of proceeding. As far as the texts go, we chose to take the period from 1970 to 1990, in the course of which it seemed to us that Brousseau had forged the essentials of the Theory of Didactical Situations. But even there the collection is huge. So, after an initial translation of most of the publications of the period, we carved out a selection, retaining the texts which gave the best presentation of the principles and key concepts of the Theory. At the heart of the book we put two works which demonstrate in detail the articulation between theoretical work and experimental research which is the source of the richness of the Theory of Didactical Situations.

The texts we chose, which came from a variety of sources, occasionally overlapped. In the interests of creating a book rather than a collection of papers, we have permitted ourselves to recompose some of them to avoid redundancies and fuse some previously distinct texts. It was, however, out of the question to rewrite absolutely everything, and the attentive reader will observe that a few redundencies remain. Perhaps, though, they will prove to be an aid to comprehension. We have composed preludes and interludes to situate the chosen texts and clarify the construction of the book. And finally, footnotes here and there fill out references from the original texts or elucidate for the reader certain points which seem to us particularly specific to the French educational context of the research presented.

In the domain which interests us, language plays an essential rôle, and at times words resist being translated. Thus we have had to make some choices to take into account distinctions which exist in French but are difficult to translate—as, for instance, that of "savoir" and "connaissance". The most important of these choices are pointed out in the course of the text. But the English language itself has variations depending on the country in which it is spoken and the culture in which it has developed. To take this into account, the initial duo invited in two "first readers" who swiftly turned into full-fledged editors. The work of translation was thus illuminated by the English of Great Britain, Australia, and the United States.

Finally, this work could not have been accomplished without the collaboration of Nadine Brousseau, who accompanied us in our research and found the answers to many questions, and Guy Brousseau, who produced for this translation a number of amendments which a connoisseur may spot.

BIOGRAPHY OF GUY BROUSSEAU

Guy Brousseau was born at Taza, Morocco, on February 4th, 1933. As the son of a soldier, he had an early education marked by frequent changes from one school to another. In 1948, he received his secondary school diploma and earned admission to *l'École Normale d'lnstituteurs* (Normal School) at Agen (in *Lot et Garonne*). There he completed his studies for the first level of the *Baccalauréat.* This in turn earned him admission to *l'École Normale* in Montpellier where he was awarded a *Baccalauréat* with distinction in elementary mathematics in 1950. He was fascinated by both mathematics and physics. At that point, he also obtained a scholarship to study higher mathematics at Toulouse in order to prepare for entrance to the *École Normale Supérieure* of St. Cloud. At the end of one academic year, however, he decided to abandon his studies and return to the *École Normale* at Agen to undertake a year of professional education. He knew already what interested him and what he wanted to study: the way in which children learn mathematics. He explained this to his mathematics professor, Mr. Duclos, who told him, "Then it is not mathematics that you must do, but psychology!". Brousseau replied, "No, it is not psychology!"

In 1953, Brousseau was appointed teacher in a small village in the *Lot et Garonne* region, in a one-class school in which he taught all subjects to students whose ages ranged from 5 to 14. During the year, he married Nadine Labeyue, whom he had met on his first day at the *École Normale* at Agen. (He reports with relish that he may have been only a fifteen-year-old in short pants, but he had great powers of discernment!)

At the beginning of the next school year, in October 1954, he and Nadine were appointed to a "double position" in a two-class school where he taught students in the "Middle Class" (9–10 year-olds) and those in their final year (14 year-olds). It was during this period that his reflections on the acquisition of mathematics and the teaching of mathematics really started.

In 1953, Brousseau took part in a controversy about the "teaching of calculation". The issue was the comparison of Freinet's modern school of thought with traditional methods. During the next two years (from 1954 to 1956), he taught, observed children, prepared sheets of lessons, analyses and problems and continued to learn mathematics. This activity was interrupted in October 1956, when he was called up for military service. He was stationed briefly in Paris, where he was able to take some courses at the Sorbonne from Mr. Pisot (in which the theory of sets and of structures was revealed to him) and at the *Conservatoire Nationale des Arts et Métiers* (CNAM) from Mr. Hocquemgheim. In Algeria, in 1958, he created worksheets which he was anxious to try out in his class on his return.

In January 1959, having completed his military obligations, Brousseau returned to the same village, and to his work as a teacher. From January, 1959 to October, 1961, he experimented and created new texts, all the while carrying out the tasks of all sorts which were expected of teachers at that period. He wrote in a series of grey notebooks (his *cahiers gris*), sometimes all through the night: lesson plans, problems, reflections. The grey notebooks became a kind of talisman for him. As of April, 1996, he figured to have filled some 5750 pages ("with whatever, with everything, and especially with nothing", as he put it.)

In October, 1960, he was encouraged by an article in *Sciences et Avenir,* in which he learned that in Belgium G. Papy was teaching Bernstein's Theorem to future kindergarten teachers (with students 3 to 6 years old).

A more important, determinant event took place in May 1961: Brousseau's encounter with Lucienne Félix[1] , who had written an article in the pedagogical review, *l'Education Nationale.* The content of this article and of her works dedicated to elementary education so completely agreed with Brousseau's beliefs that he decided to write to the author. She replied, asking him to send her copies of his texts to look at. He had, at that time, about three hundred lesson-sheets. Mme. Félix strongly encouraged him to continue this work, and following the exchange of some letters she suggested that he participate in the conference of CIEAEM, which was being held in Switzerland that year. There he had minor disagreements with G. Choquet, who was leaving the presidency, but met G. Papy and W. Servais. He presented the results of his observations of a class where "a more modern teaching of mathematics" was being attempted and returned from the meeting full of enthusiasm.

He continued his writing, producing a textbook for fourth and fifth grades. The primary-school inspector whom he consulted before conducting a larger experiment in the class had himself to obtain authority from the General Inspector, Mr. Degeorge, who was very reluctant and replied that this teacher (G. Brousseau) ought to start by studying mathematics (which he had, in fact, started and which he continued to do).

During this year, Brousseau also became interested in the mathematical problems posed by the management of agricultural business, which he worked on with the farmers in the village. At issue were problems of optimisation (applications of operational calculation) or of representations and treatment of numerous variables which must be considered in order to modify the practices of polyculture. He became aware at that point of the importance and the difficulty of the diffusion of mathematical knowledge in the population at large.

In January, 1962, teachers holding an elementary-mathematics *Baccalauréat* were invited to apply for one year's training to teach mathematics in a general teachers' college in which there was a shortage of teachers. Brousseau was accepted and granted leave for the following school year (1962–63) to attend this course in Bordeaux. At the same time, he applied to the university to resume officially his mathematical studies (interrupted in 1952). In June 1963, he was admitted to the IPES and the MGP[2] . He continued his studies at the university without ceasing in his efforts to disseminate new mathematical ideas. In this way he was introduced to Professor Colmez, who took an interest in him, then, by Lucienne Félix, to Professor Lichnèrowicz who introduced him to the editor, Georges Dunod. Dunod's publishing company published his manual for teachers of the *Cours Préparatoire*[3] , which appeared in 1965.

In 1964, Professor Lichnèrowicz proposed to Brousseau a thesis topic: "the limit conditions of an experiment in mathematical pedagogy", and introduced him to Pierre Greco, who co-directed the work. In order to treat the subject correctly, Brousseau proposed to the director of the *Centre Régional de Documentation Pédagogique* (CRDP) of Bordeaux, to create within his establishment a *Centre de Recherche sur l'Enseignement Mathématique* (CREM). The director, Mr. la borderie, accepted. With the help of a number of people from the university and teachers from the *École Normale d'Instituteurs,* Brousseau published a large number of "notebooks" intended to suggest innovations to teachers, but also to spell out the conditions for the emergence of real research.

Research on these

- *technical conditions* (theories, methodology, fields of experience),
- *sociological conditions* (administrative organisation, contacts among researchers, subjects, domains of reference, fundamental concepts), and
- *pedagogical conditions* (acceptable forms of teaching, ethical aspects)

led him to deepen his knowledge in various domains, and to seek the guidance of specialists whom he encountered at the University of Bordeaux or thanks to the activities of the CRDP: cinematic linguistics and semiology with Christian Metz, later the Science of Education with J. Wittwer, etc.

In February, 1968, at a colloquium at Amiens, Brousseau presented, with J. Becker and J. Colmez, the results his reflections on Professor Lichnérowicz's topic. What had developed was a project for the creation of an *Institut de Recherche sur l'Enseignement des Mathématiques* (IREM) made up of three components: a colloquium open to all teachers of mathematics; teaching, information and documentation of professors of mathematics (of all levels and in all materials); research on the teaching of mathematics. The component "research on the teaching of mathematics" was itself composed of two activities:

– the first which one could today call "engineering and workshops of *didactique*" (and which has been called, at different points, "innovation", "action research", "production of materials, projects and curriculum"...)

– the second which was dedicated to experimental and theoretical research on the teaching of mathematics and was to obey the academic rules of research. In addition to the "application" of the fundamental domains of teaching, it was necessary to create a theoretical instrument for the integration and co-ordination of these efforts. The project was studied in a systemic perspective. In order to permit it to function, its relations with all the organisations concerned were examined. Academic research on what was to become "experimental epistemology" and then "*didactique* of mathematics" (after nearly having been called "didactology of mathematics") was at the time completely non-existent—even inconceivable. It was necessary to propose one or more initial theoretical models (to be demolished, but consistent and large enough), research methods, an initial group of researchers ... and systems for re-opening the discussion. The most delicate point was imagining a relationship between the researchers and their object of study—teaching—which would neither compromise the academic research nor be detrimental to the teaching. The creation for this purpose of a *Centre pour l'Observation de l'Enseignement des Mathématiques* (COREM), distinct from the experimental schools and pilot schools of the period, became the kernel of the project. The COREM would provide the milieu in contact with which a composite team would be able to carry out the first whorls of the spiraling development of this academic research.

In July, 1968, the International Commission on Mathematics Education (ICME) held its first congress. The congress was at Lyon, where Maurice Glayman and his group had also prepared a form of IREM which was to be created in January, 1970 (with Paris and Strasbourg). The IREM of Bordeaux was created in October, 1970. Guy Brousseau, a licensed teacher of mathematics, was recruited by the University as an Assistant in Mathematics to participate in the realisation of the announced project.

The first elements of Brousseau's Theory of Didactical Situations were communicated in a session of the Congress of the *Association des Professeurs de Mathématiques de l'Enseignement Public* (APMEP) at Clermont-Ferrand in 1970. The first article on methodology (on quasi-implications) was published in the bulletin of psychology of the University of Paris in 1969. The first example of a predictive mathematical model relative to a modification of teaching, experimentally verified (the teaching of the calculation of multiplication and division), was communicated in 1973 at the Sixth International Congress on the Science of Education.

1972 marked the creation of' a school for observing children learning mathematics. This Jules Michelet School at Talence developed progressively into a school for the observation of the teaching of mathematics (the COREM).

Research was carried out there (1970–74) on the teaching of the natural numbers, the operations on the natural numbers and fundamental structures. Other topics were the teaching of probability and statistics, in collaboration with P. L. Hennequin, statistical studies under the direction of H. Rouanet (1973–74), the teaching of rationals and decimals (1973–80),...

In 1975 a doctoral program in *didactique* was created at Paris, Strasbourg and Bordeaux. Brousseau was in charge of theses and several principal courses. He played a leading rôle in the late seventies in the development of *Didactique des Mathématiques* as a scientific discipline. Among the essential events of this period we should mention especially the creation of a National Seminar in Paris held three times a year, a Research Summer School and the establishment in 1980 of the international journal *Recherches en Didactique des Mathématiques.*

Member of the Department of Mathematics of the University of Bordeaux 1 since 1970, Guy Brousseau, after having twice decided against presenting different works as theses in areas which he did not favour, submitted his *Thèse d'Etat* and was granted his doctorate in 1986. He wrote his thesis with constant interaction with Professor Bernard Malgrange who supported him and, says Brousseau, “allowed him to clarify his ideas and to improve his texts”.

Brousseau has been a Professor at the IUFM of Bordeaux since 1990. He is currently Director of the *Laboratoire Aquitain de Didactique des Sciences et des Technologies* of the University of Bordeaux.

NOTES

1. Lucienne Félix (1901-1 994) was the author of numerous works of mathematics, including, among many others: “Mathé*matiques modernes et enseignement élémentaire*” (Modern mathematics et elementary teaching), *“L'exposé moderne des mathématiques élémentaires”* (Modern presentation of elementary mathematics), Geometry textbooks for all levels of high school, an Analysis textbook for the most advanced level of high school and a number of articles in journals on the teaching of mathematics. She designed a course on Henri Lebesgue (whose assistant she had been), “*Constructions Géométriques”,* and wrote a book *“Message d'un mathématicien*—Henri Lebesgue” on the occasion of the centennial of his birth.
2. IPES was a kind of institute allowing students access, through a competition, to a state position while they prepared themselves to become secondary school teachers. The training in IPES was essentially in the discipline, in this case mathematics. MGP, *“Muthématiques Générales et Physique”,* was at this time the diploma obtained after the first two years of studies at the university.
3. First year of schooling.