PREMBA ANALYTICAL PRIMER

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Essential Quantitative Concepts for
Business Math

Regina Treviño

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a mi esposo Paul

CONTENTS

<i>Preface</i>	xi
<i>Mathematical Symbols and Abbreviations</i>	xiii
1 Business Math: Functions and Graphs	1
1.1 Functions	1
<i>Application 1.1 The Revenue Function</i>	2
1.1.1 Multivariate Functions	3
<i>Application 1.2 The Demand and Supply Functions</i>	3
1.2 Graphing Functions	4
<i>Application 1.3 The Demand Curve</i>	5
1.3 Equations	7
1.4 Linear Functions	7
1.4.1 Solving Linear Equations	8
<i>Application 1.4 Linear Demand Curves</i>	9
1.4.2 Solving Linear Inequalities	10
1.5 Slopes and Intercepts of Linear Equations	12
1.5.1 Slope-Intercept Form of a Line	13
<i>Application 1.5 Graphing a Linear Demand Curve</i>	17
1.5.2 Finding the Equation of a Line	19
1.5.3 General Linear Form of a Line	20
<i>Application 1.6 The Consumer's Budget Constraint</i>	21
1.5.4 Horizontal and Vertical Lines	25
1.6 Solving Two-Variable Systems of Linear Equations	26
<i>Application 1.7 Market Equilibrium</i>	27
1.7 Properties of Functions	30
<i>Application 1.8 Kinked Budget Constraints</i>	31
1.8 Exponents	33
<i>Application 1.9 Cobb-Douglas Production Function</i>	33
1.8.1 Radicals and Fractional Exponents	34
1.8.2 Homogenous Functions	35
<i>Application 1.10 Constant Returns to Scale</i>	35
1.9 Changes and Growth Rates	36
1.9.1 Growth Rate Formulas	37
<i>Application 1.11 The Sources of Economic Growth</i>	38

	1.9.2	Index Numbers	39
		<i>Application 1.12 Consumer Price Index and Inflation</i>	39
		Practice Exercises	40
2		Business Math: Optimization	45
	2.1	Nonlinear Functions	45
	2.2	The Concept of a Derivative	45
	2.2.1	Notation of Derivatives	47
	2.2.2	Rules of Differentiation	47
		<i>Application 2.1 Marginal Functions</i>	49
	2.3	Graphical Interpretation of Derivatives	50
		<i>Application 2.2 Marginal Product of Labor</i>	52
	2.4	Additional Topics on Derivatives	54
	2.5	Second Derivatives	55
		<i>Application 2.3 Diminishing Marginal Utility</i>	57
	2.6	Partial Derivatives	59
		<i>Application 2.4 Partial Derivatives and the Ceteris Paribus Assumption</i>	60
		<i>Application 2.5 Income Elasticity of Demand</i>	60
	2.7	Optimization	61
	2.7.1	Unconstrained Optimization	62
		<i>Application 2.6 Profit Maximization</i>	64
	2.7.2	Constrained Optimization	66
		<i>Application 2.7 Consumer's Utility Maximization Problem</i>	67
	2.8	Exponential and Logarithmic Functions	69
		<i>Application 2.8 Continuously Compounded Interest Rate</i>	70
	2.8.1	Logarithmic Functions	71
	2.8.2	The Natural Logarithm	71
		<i>Application 2.9 Doubling Time for an Investment</i>	72
	2.8.3	Differentiation of e and $\ln$	73
	2.9	Measuring Areas under Linear Functions	73
		<i>Application 2.10 Consumer Surplus</i>	74
	2.10	Economic Models	76
	2.10.1	Flow and Stock Variables	77
		<i>Application 2.11 Balance Sheet and Income Statement</i>	77
	2.10.2	Nominal and Real Economic Variables	77
		<i>Application 2.12 Nominal and Real GDP</i>	77
	2.11	Comparative Statics Analysis	79
		<i>Application 2.13 Shifting the Demand and Supply Curves</i>	79
		Practice Exercises	81
3		Statistical Analysis Primer	83
	3.1	Elements of Statistics	83
	3.2	Sigma Notation	84

3.3	Measures of Central Tendency	85
3.4	Frequency Tables and Histograms	87
3.5	Measures of Variability	92
3.6	Measures of Association	93
3.7	Probability Essentials	97
3.8	Discrete Random Variables	98
	3.8.1 Discrete Random Variables and Expected Value	100
	3.8.2 Discrete Random Variables and Variance	101
	3.8.3 Decision Tree Basics	103
3.9	Continuous Random Variables	106
	3.9.1 The Normal Distribution	107
	3.9.2 Standardizing: z-Values	108
	3.9.3 Standard Normal Tables and z-Values	108
3.10	An Introduction to Estimation	112
	3.10.1 The Sampling Distribution of the Sample Mean	113
3.11	Regression Analysis	115
	3.11.1 Simple Linear Regression	115
3.12	Hypothesis Testing	121
	3.12.1 Types of Errors	121
	3.12.2 Significance from p-Values	122
	Practice Exercises	124
4	Mathematics of Finance	131
4.1	Time Value of Money	131
	4.1.1 Present Value	132
	4.1.2 Future Value	133
4.2	Nominal and Real Rates of Interest	134
4.3	Compound Interest	135
	4.3.1 Continuous Compounding	136
	4.3.2 Effective Interest Rate	137
4.4	Annuities	139
	4.4.1 Amortization of Loans	139
4.5	Perpetuities	140
	4.5.1 Growing Perpetuities	142
4.6	Net Present Value	143
	4.6.1 Discounted Cash Flow	144
	4.6.2 Risk-Adjusted Discount Rates	145
4.7	Bonds	146
	4.7.1 Zero-Coupon Bonds	148
4.8	Common Stocks	149
4.9	Portfolio Theory Primer	150

4.9.1	Portfolio Math	150
4.9.2	Diversification and Portfolio Risk	154
4.9.3	Measuring Risk	155
	Practice Exercises	158
<i>Appendix A: List of Rules</i>		163
<i>Appendix B: Answers to Practice Exercises</i>		169
<i>Notes</i>		193
<i>References</i>		197
<i>Index</i>		199

PREFACE

This book is a review of the analytical methods required in most of the quantitative courses taught at MBA programs. Students with no technical background, or who have not studied mathematics since college, may easily feel overwhelmed by the mathematical formalism that is typical of quantitative courses, such as economics, statistics, or finance. These students will benefit from a concise and focused review of the analytical tools that will become a necessary skill in their MBA classes. The objective of this book is to present the essential quantitative concepts and methods in a self-contained, nontechnical, and intuitive way.

I divide the contents of the book into three broad sections: business math, statistics, and finance. In the business math section (chapters 1 and 2), each analytical method is followed by an application that illustrates how such tool is typically used in MBA quantitative courses. In these applications, wherever possible, I provide the background and context to facilitate understanding and stimulate interest.

Chapter 3 is a statistical analysis primer and chapter 4 is a review of the mathematics of finance. The topics covered in both these chapters represent a knowledge basis that is essential before taking any MBA statistics, operations research, and finance courses. Simple examples are used to illustrate the topics covered in chapters 3 and 4.

THE STYLE OF THE BOOK

In choosing the contents of *PreMBA Analytical Primer: Essential Quantitative Concepts for Business Math*, I tried to err on the side of inclusion. My aim is to assure coverage of most of the analytical tools that students may be required to handle during their MBA courses.

I sought a style of presentation that is accessible yet rigorous. Wherever possible, I give precise definitions of the concepts covered. At the same time, I accompany this analysis with examples that illustrate the key points.

Topics that I consider either relatively advanced or too peripheral are marked with an asterisk (*). This allows students to skip them over easily in a first reading.

How To Use This Book

This book is intended to be a math refresher that serves students both as a primer, and as a reference along their courses. Students with little technical background will benefit from reading and working on the entire book before the start of their MBA classes. Others students, however, might use it as a reference tool for the review of particular topics on a need basis throughout their MBA courses.

The chapters have been written to be relatively self-contained. As a result, a student can alter the sequence in which the different topics (math, statistics, and finance) are reviewed. At the end of the book, I provide a listing with all the major rules covered in the chapters and I also include a brief summary of some basic algebra rules.

I include exercises that test students' understanding of the topics throughout the text as well as a set of practice problems at the end of each chapter. Working through as many of these exercises as possible is the surest way to master the material covered in each chapter. The answers to all the test-your-understanding exercises and practice problems are included at the end of the book.

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MATHEMATICAL SYMBOLS AND ABBREVIATIONS

The use of symbols and notation is explained in a self-contained discussion wherever these appear in each chapter, but for convenience I also list here the major symbol conventions that recur throughout the book.

Symbol Meaning

Δ	Greek letter delta. Denotes change in the value of a variable.
Π	Greek letter pi (uppercase). Denotes firm profits.
Σ	Greek letter sigma (uppercase). Denotes summation.
α	Greek letter alpha. Denotes significance level of a test.
β	Greek letter beta. Denotes sensitivity of a security's return to market movements.
ε	Greek letter epsilon. Denotes elasticity (chapter 2). Denotes random error (chapter 3).
λ	Greek letter lambda. Denotes Lagrange multiplier.
π	Greek letter pi (lowercase). Denotes inflation.
μ	Greek letter mu. Denotes mean.
ρ	Greek letter rho. Denotes correlation coefficient.
σ	Greek letter sigma (lowercase). Denotes standard deviation.
A	Generic constant. In a Cobb-Douglas production function, it denotes factor productivity.
a	Generic constant. Typically used to denote the vertical intercept of a line.
b	Generic constant. Typically used to denote the slope of a line.
C	Generic constant. Denotes cash flow (chapter 4).
$C(\cdot)$	Cost function.
cdf	Cumulative distribution function.
CPI	Consumer price index.
CS	Consumer surplus.
d	Generic constant. Next to a variable, it denotes small (infinitesimal) change in the value of the variable.
DCF	Discounted cash flow.
DIV	Dividend.
$E(\cdot)$	Expected value.
$f(\cdot)$	Function.

$f'(\cdot)$	First derivative of the function.
$f''(\cdot)$	Second derivative of the function.
F	Face value.
FOC	First order condition.
FV	Future value.
I	Consumer income.
g	Growth rate.
GDP	Gross domestic product.
K	Capital stock.
L	Labor.
$\ln$	Natural logarithm.
MC	Marginal cost.
MPL	Marginal product of labor.
MU	Marginal utility.
N	Normal distribution.
NPV	Net present value.
p	Price of a good.
p_a	Price of an alternative good.
p_m	Price of the materials used to produce a good.
p_t	Price of technologically related goods.
P	Principal.
pdf	Probability density function.
PV	Present value.
q	Quantity (individual level).
Q	Quantity (market level)
Q^d	Quantity demanded.
Q^s	Quantity supplied.
R	Firm revenue.
r	Interest rate.
r_e	Effective interest rate.
r_f	Risk-free interest rate.
r_{xy}	Sample correlation coefficient between variable x and variable y .
s	Sample standard deviation.
SOC	Second order condition.
t	Index variable for time (chapter 4).
$U(\cdot)$	Utility function.
x	Generic variable. Typically denotes the independent variable.
$\bar{x}$	Sample mean.
y	Generic variable. Typically denotes the dependent variable.
Υ	Generic variable. In the aggregate production function, it denotes total output.

$ $	Absolute value.
∞	Infinity.
∂	Partial derivative.
$L(\cdot)$	Lagrangian.

Asterisks, Hats, and Primes

Asterisks: An asterisk is attached to a variable to denote optimal, equilibrium, or solution quantity (e.g., x^*).

Hats: A hat is attached to a variable to denote predicted value (e.g., $\hat{y}$).

Primes: A prime attached to a function of a single variable denotes first derivative (e.g., $f'(x)$).

A prime attached to a variable, as opposed to a function, denotes a specific value of that variable (e.g., p').