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Unit Root Tests in Time Series

Volume 1

Key Concepts and Problems

Kerry Patterson

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To Bella and a continuing treasure at Auton Farm

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Symbols and Abbreviations

$\Rightarrow_D$	convergence in distribution (weak convergence)
$\rightarrow_P$	convergence in probability
$\rightarrow$	tends to; for example, ε tends to zero, $\varepsilon \rightarrow 0$
$\mapsto$	mapping
$\Rightarrow$	implies
$\sim$	is distributed as
$\equiv$	definitional equality
$\neq$	not equals
$\Phi(z)$	the cumulative distribution function of the standard normal distribution
$\Re$	the set of real numbers; the real line ($-\infty$ to ∞)
$\Re^+$	the positive half of the real line
$\mathbb{N}^+$	the set of non-negative integers
ε_t	white noise, unless explicitly excepted
$\prod_{j=1}^n x_j$	the product of x_j , $j = 1, \dots, n$
$\sum_{j=1}^n x_j$	the sum of x_j , $j = 1, \dots, n$
0^+	approach zero from above
$_0$	approach zero from below
L	the lag operator, $L^j y_t \equiv y_{t-j}$
Δ	the first-difference operator, $\Delta \equiv (1 - L)$
Δ_s	the s -th difference operator, $\Delta_s \equiv (1 - L^s)$
Δ^s	the s -th multiple of the first difference operator, $\Delta^s \equiv (1 - L)^s$
$\subset$	a proper subset of
$\subseteq$	a subset of
$\cap$	intersection of sets
$\cup$	union of sets
$\in$	an element of
$ a $	the absolute value (modulus) of a
iid	independent and identically distributed
m.d.s	martingale difference sequence
$N(0, 1)$	the standard normal distribution, with zero mean and unit variance
niid	independent and identically normally distributed
$B(t)$	standard Brownian motion; that is, with unit variance
$W(t)$	nonstandard Brownian motion

Preface

The purpose of this book is to provide a review and critical assessment of some key procedures related in one way or another to the problem of testing for a unit root in a stochastic process. As is now well known, the presence of a unit root implies a form of nonstationarity that is considered to be relevant for economic time series, so that a non-standard inferential and distributional framework is required.

The research and literature on this topic has grown almost exponentially since Nelson and Plosser's seminal article published in 1982. Therein they applied the framework due to Dickey (1976) and Fuller (1976) to testing for a unit root in a number of macroeconomic time series. Subsequent key articles by Dickey and Fuller (DF) (1979, 1981) developed some aspects of the initial testing framework. The basic set-up for a DF unit root test is now familiar enough, being taught in most intermediate, if not introductory, courses in econometrics; however, the underlying distribution theory is somewhat more advanced, and the many complications that have arisen in practice has meant the development of a voluminous literature that, because of its extent, is difficult to comprehend, especially for the non-specialist. Indeed, it is probably the case that a simple survey of the field of methods and applications is virtually infeasible; indeed, the topic is so extensive that even some 20 years ago Diebold and Nerlove (1990) noted the scale of the literature on this topic.

The articles on unit root tests are amongst the most cited in economics and econometrics and have clearly influenced the direction of economic research at a much wider level than simply testing for a unit root. A citation summary for articles based on univariate processes is presented in Table P.1. The numbers shown here clearly indicate that there has been a sustained interest in the topic over the last 30 years or so and, looking at the wider influence, the unit root literature led to the concept of cointegration and to some of the most cited of econometric articles, including Engle and Granger (1987), Johansen (1988, 1991) and Johansen and Juselius (1990) (see the note to Table P.1).

The appropriate prerequisites for this book include some knowledge of econometric theory at an intermediate or graduate level as, for example, in Davidson (2000), Davidson and Mackinnon (2004) or Mittlehammer et al. (2000), and, possibly, with some additional directed study, as in good introductory books such as Gujarati (2006), Dougherty (2007), Ramanathan (2002) and Stock and Watson (2007). It would also be helpful to have had an introduction to the

Table P.1 Number of citations of key articles on unit roots.

Author(s)	Number of citations	
Dickey and Fuller (1979)	1	7,601
Phillips and Perron (1988)	2	4,785
Dickey and Fuller (1981)	3	4,676
Perron (1989)	4	3,371
Kwiatkowski, Phillips, Schmidt, Shin (1992)	5	3,280
Nelson and Plosser (1982)	6	3,035
Phillips (1987a)	7	1,881
Zivot and Andrews	8	1,694
Elliott, Rothenberg and Stock (1996)	9	1,556
Said and Dickey (1984)	10	1,342

Notes: Articles relate to univariate unit root tests. Prominent articles, on a citation basis, involving largely multivariate methods are: Engle and Granger (1987): 12,366; Johansen (1988): 8,236; Johansen and Juselius (1990): 4,886; Johansen (1991): 4,150; and in econometric methods more generally, White (1980), 12,359. (The last article is the most highly cited on the basis of several citation methods.) On the index of economics articles since 1970, compiled by Han Kim et al. (2006), Dickey and Fuller (1979) ranks 7, whilst Engle and Granger (1987) and Johansen (1988) rank 4 and 8, respectively.

Source: Google Scholar, accessed 22 February 2010.

methods of maximum likelihood and generalised least squares (GLS), for example as provided in Greene (2006). An introduction to time series analysis and mathematical statistics would also be useful; for example, for the former at the level of Chatfield (2004) and the latter along the lines covered by Mittelhammer (1996). A book designed especially as a primer for this one is Patterson (2010). Some familiarity with the application of unit root tests would also be helpful to set the context.

I have taken the brief of this book to include issues that are related to but theoretically separate from the central concern of testing for a unit root. For example, one of these is the problem of the bias in estimating the coefficients in an autoregressive model; whilst this is strictly a finite sample effect, it is of practical interest and serves as a 'lead in' to the problems associated with unit testing. This book is, therefore, not about listing tests for a unit root. Not only would there not be enough space for such an enterprise, it is not the best way to indicate which test statistics, and methods more generally, have been taken up by practitioners. The main tests are of course presented and their rationale explained, together with examples to illustrate how they are used. However, the problem suggested by the presence of a unit root or near-unit root is more than just the design of a test statistic. There are two other important practical issues that a researcher has to face. The first is to consider what the appropriate alternative hypothesis is. In the Dickey-Fuller paradigm, followed by many in practice, the alternative to a stochastic trend is a deterministic trend, usually

characterised as a low order polynomial in time. However, the choice of the order of the polynomial, typically representing reversion to a constant or a linear trend, has a critical effect on the power of the test if over-specified; on the other hand, if under-specified, a test will have no power to detect a true alternative. The representation of the ‘attractor’ as being generated by a low order polynomial trend is likely to be a shorthand, or reduced form, for a far more complex process. In part, the pre-eminent role of a simple deterministic trend in providing the mean or trend reverting alternative to a non-reverting process (nonstationary by way of a unit root or roots), is historical and, of itself, deserves further study and evaluation.

A second key, practical aspect arises from the usual need to choose some form of a truncation parameter. In the context of the familiar augmented Dickey-Fuller tests, this is a lag truncation parameter and in the context of the semi-parametric tests, which require an estimate of the long-run variance, a parameter limiting the bandwidth in the formation of a sum of autocovariances. Whilst familiar criteria, such as the AIC, BIC and general-to-specific (g-t-s), rules are in frequent use, the combination of each one of these with a test statistic defines test procedures with potentially differing characteristics; their use in combination then leads to the accumulation of type I error.

Some of the developments covered in this book are as follows.

- The distinction between difference stationary and trend stationary processes and the implications of this distinction for the permanence or otherwise of shocks.
- An outline of the autoregressive moving average (ARMA) modelling framework and its role in testing for a unit root.
- The finite sample bias in estimating models and its implications for inference even when seemingly well into the region of stationarity.
- Forming confidence intervals that are robust to the problem of quantiles that are not constant.
- The DF unit root tests and developments of them to account for weak serially correlated processes.
- Bootstrapping confidence intervals and unit root tests.
- Tests that:
 - are based on a direct maximum likelihood approach;
 - are based on a GLS, or quasi-GLS, approach, including the influential tests by Elliott, Rothenberg and Stock (1996);
 - combine the backward and forward recursions of a random walk;
 - are based on recursive estimation of the mean or trend;
 - are robust to the initial condition;
 - allow for more than one unit root;

- are based on stationarity as the null hypothesis;
- allow for unit roots in seasonal data.

The results of a number of Monte Carlo studies are reported in various chapters. Indeed, simulation is a key tool that is used throughout to provide some guidance on finite sample issues. Consider, for example, the problems caused by the presence of weakly dependent errors when testing for a unit root. Then under fairly weak assumptions, the asymptotic properties of several frequently used test statistics are unaffected by such errors, but typically, the finite sample properties do not reflect the asymptotic properties, an example being the difficulty caused by the near cancellation of a root, especially a near-unit root, in the AR and MA components of an ARMA model. To understand the finite sample nature of such problems, many more simulations were run than are reported in the various chapters; the results are then typically illustrated for one or two sample sizes where they are representative of a wider range of sample sizes.

There are a number of developments and problems not covered in this volume, but which are included in Volume 2. These include the following.

Nonparametric tests: the tests that have been considered in this volume, such as the family of Dickey-Fuller tests, are parametric tests in the sense that they are concerned with direct estimation in the context of the parametric structure of an AR or ARMA model. Nonparametric tests use less structure in that no such explicit parametric framework is required and inference is based on other information in the data, such as ranks, signs and runs. Semi-parametric tests use some structure, but it falls short of a complete parametric setting; an example here is the rank score based test, which is based on ranks.

Fractional integration: this considers the case of fractional values of the integration parameter. That is, suppose that a stochastic process generates a time series that is integrated of order d , where d is a fractional number. What meaning can we attribute to such an operation and how can the parameter d be estimated? There are two general approaches to the analysis and estimation of fractional $I(d)$ process, as they may be either analysed in the time domain or the frequency domain.

Bounded random walks: the application of random walk models to some economic time series can be inappropriate, as where there are natural bounds or limits to the values that the series can take, such as in the case of unemployment rates and nominal interest rates. One way of modelling this is to allow unit root behavior; for example, persistence and the absence of mean reversion over a range of possible values but reversion at other values. These models have

in common that they involve some form of nonlinearity. Perhaps the simplest form of nonlinearity actually arises from piecewise linearity; that is, an overall model comprises two or more linear models for subperiods where the component models differ not in their form, for example all are $AR(p)$, but in their parameters. A popular class of such models is the smooth transition autoregressive – or STAR – class, of which the exponential and logistic members are the most frequent in application, giving rise to the acronyms STAR and LSTAR.

Structural breaks: Perron's (1989) seminal article began another thread of the unit root literature. What if, instead of a unit root process generating the data, there was a trend subject to a break due to 'exceptional' events? How would standard unit root tests perform? For example, what would be their power characteristics, if the break was ignored in the alternative hypothesis? The idea of regime change that could affect led to a fundamental re-evaluation of the simplicity of the simple 'opposing' mechanisms of a unit root process, on the one hand, and a trend stationary process, on the other. In practice, although there are likely to be some contemporaneous and, later, historical indications of regime changes, there is almost inevitably likely to be uncertainty not only about the dating of such changes but also the nature of the changes. This poses another set of problems for econometric applications. If a break is presumed, when did it occur? Which model captures the nature of the break? If multiple breaks occurred, when did they occur?

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The graphs in this book were prepared with MATLAB www.mathworks.co.uk, which was also used, together with TSP (www.tspintl.com) and RATS (www.estima.com), for the numerical examples. Martinez and Martinez (2002) provide an invaluable guide to statistics with many MATLAB examples; guides to MATLAB include Hanselman and Littlefield (2004), Moler (2004) and Hahn and Valentine (2007).

If you have comments on any aspects of the book, please contact me at my email address given below.

Author's email address: k.d.patterson@reading.ac.uk

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Palgrave Macmillan email address: orders@palgrave.com