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List of Symbols

Symbols

- $[\pm, \pm, \pm, \pm]$ 78
 $*$, Hodge star operator 94
 $\Lambda^+(N)$ 56
 $\Lambda^+(N)_{\text{even}}$ 58
 $\Lambda^+(N)_{\text{BD}}$ 58
 $\Delta = -(dd^* + d^*d)$, Hodge Laplacian 95
 $\Delta_{\mathbb{R}^{n-1}}$ 5, 101
 $\Delta_{\mathbb{C}^n}$, holomorphic Laplacian 42
 \mathcal{E} , light cone 14, 132
 $\Pi_{\ell, \delta}$, irreducible unitary representation of $O(n+1, 1)$ 19
 Π_{n-1} , projection onto $\text{Ker}(\iota_{\frac{\partial}{\partial x_n}})$ 103, 108, 122
 $\Phi_{u, \delta}^* \equiv \left(\Phi_{u, \delta}^{(i)}\right)^*$ 93
 $\Omega(h, x)$, conformal factor 1
 $\gamma(\mu, a)$ 5, 157, 165, 166, 175
 $\varepsilon_n(I)$, signature of index set I 94
 ι , conformal compactification 15, 136
 $\iota_{\lambda}^{(i)}$, map to flat picture 16, 126
 $\iota_{N_Y(X)}$ 2, 99
 $\iota_{\frac{\partial}{\partial x_n}}$, interior multiplication 5, 61, 101, 104, 108
 $[\lambda]$, $O(N)$ -modification rule 59
 $\lambda \setminus \nu$, skew diagram 57
 λ/ν 57
 $\mu^b \equiv \mu^b(i, \alpha)$, small K -type 19
 $\mu^{\#} \equiv \mu^{\#}(i, \alpha)$, small K -type 19
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 $[\xi^{\pm}] (\in \mathcal{E}/\mathbb{R}^{\times} = S^n)$ 14, 21
 $[\xi^{-}]$, north pole in S^n 15, 136
 $\varpi_{u, \delta}^{(i)}$, conformal representation on i -forms 1, 17, 20, 93, 94
 $\pi_{(\sigma, \lambda)}$, principal series 33
 $\pi_{(\sigma, \lambda)^*}$ 33
 ρ 18, 33
 ρ_G 18
 $\sigma_{\lambda} := \sigma \boxtimes \mathbb{C}_{\lambda}$ 15, 33, 35, 41
 $\sigma_{\lambda}^* := \sigma^{\vee} \boxtimes \mathbb{C}_{2p-\lambda}$ 33
 $\sigma_{\lambda, \alpha}^{(i)}$, representation of P on $\wedge^i(\mathbb{C}^n)$ 15, 63, 67, 88, 121, 144
 $\tau_V \equiv \tau \boxtimes \mathbb{C}_V$ 34, 41
 $\tau_{\nu, \beta}^{(j)}$, representation of P' on $\wedge^j(\mathbb{C}^{n-1})$ 21, 63, 67, 88, 121
 $\vartheta_z = z \frac{d}{dz}$ 79, 175
 $\chi_{\pm\pm}$, one-dimensional representation of $O(n+1, 1)$ 16, 20, 27
 χ_{--} 16, 21, 126
A
 A , split torus ($\simeq \mathbb{R}$) 14, 17, 21, 32
 $A_{II'}$, matrix coefficient of A_{σ} 47, 52
 $A_{II'}$, matrix component of A_{σ} 74, 75, 146

A# 31 A_σ , vector part of $\widehat{d\pi_{(\sigma,\lambda)^*}}$ **37, 45, 47****B** $B^{(k)}$, bilinear map **53, 107****C** $C_\ell^+ (= 2N_\ell^+)$, basis of $\mathfrak{n}_+(\mathbb{R})$ **13** $C_\ell^- (= N_\ell^-)$, basis of $\mathfrak{n}_-(\mathbb{R})$ **13** $C_\ell^\mu(t)$, Gegenbauer polynomial **173** $\tilde{C}_\ell^\mu(t)$, renormalized Gegenbauer polynomial **5, 22, 67, 69, 112, 174** $\text{Conf}(X)$ **2, 98, 100, 131** $\text{Conf}(X; Y)$ **2, 98, 100, 131, 133** \mathbb{C}_λ , one-dimensional representation of A **15** \mathbb{C}_{2p} **18, 33** $\tilde{\mathbb{C}}_{\lambda, \nu} \left(= \text{Rest}_{x_n=0} \circ \mathcal{D}_{\nu-\lambda}^{\lambda-\frac{n-1}{2}} \right)$, Juhl's operator **22** $\mathbb{C}_{\lambda, \nu}^{i,j} \left(= \mathcal{D}_{\lambda-i, \nu-\lambda}^{i \rightarrow j} \right): \mathcal{E}^i(\mathbb{R}^n) \rightarrow \mathcal{E}^j(\mathbb{R}^{n-1})$, (unnormalized) differential symmetry breaking operator **23** $\mathbb{C}_{\lambda, \nu}^{i,i-1}$ **23** $\mathbb{C}_{\lambda, \nu}^{i,i}$ **23** $\tilde{\mathbb{C}}_{\lambda, \nu}^{i,j} \left(= \tilde{\mathcal{D}}_{\lambda-i, \nu-j}^{i \rightarrow j} \right)$, normalized differential symmetry breaking operator **23** $\tilde{\mathbb{C}}_{\lambda, \nu}^{i,i-2}$ **126** $\tilde{\mathbb{C}}_{n-i, n-i+1}^{i,i-2}$ **24, 129** $\tilde{\mathbb{C}}_{\lambda, 1}^{n, n-2}$ **24, 129** $\tilde{\mathbb{C}}_{\lambda, \nu}^{i,i-1}$ **23, 122, 126** $\tilde{\mathbb{C}}_{\lambda, \nu}^{i,i}$ **23, 126** $\tilde{\mathbb{C}}_{\lambda, \nu}^{i,i+1}$ **24, 126** $\tilde{\mathbb{C}}_{i, i+1}^{i, i+1}$ **24** $\tilde{\mathbb{C}}_{\lambda, 1}^{0,1}$ **24****D** d , differential **101** d^* , codifferential **4, 97, 101** $\mathcal{D}(E)$, Weyl algebra **31, 34** \mathcal{D}_ℓ^μ **5, 22, 112** $\text{Diff}^{\text{const}}$ **34, 107** $\text{Diff}_{G'}(\mathcal{E}^i(X)_{u,\delta}, \mathcal{E}^j(Y)_{v,\varepsilon})$ **2** $\mathcal{D}_{u,a}^{i \rightarrow j} \left(= \mathbb{C}_{u+i, u+i+a}^{i,j} \right): \mathcal{E}^i(\mathbb{R}^n) \rightarrow \mathcal{E}^j(\mathbb{R}^{n-1})$, (unnormalized) differential symmetry breaking operator **22** $\mathcal{D}_{u,a}^{i \rightarrow i-1}$ **5, 23, 68, 157** $\mathcal{D}_{u,a}^{i \rightarrow i}$ **6, 23, 116, 157** $\tilde{\mathcal{D}}_{u,a}^{i \rightarrow j} \left(= \tilde{\mathbb{C}}_{u+i, u+i+a}^{i,j} \right): \mathcal{E}^i(\mathbb{R}^n) \rightarrow \mathcal{E}^j(\mathbb{R}^{n-1})$, normalized differential symmetry breaking operator **22** $\tilde{\mathcal{D}}_{u,a}^{i \rightarrow i-2}$ **8** $\tilde{\mathcal{D}}_{u,a}^{i \rightarrow i-1}$ **7, 68, 168** $\tilde{\mathcal{D}}_{u,a}^{i \rightarrow i}$ **7, 168** $\tilde{\mathcal{D}}_{u,a}^{i \rightarrow i+1}$ **8, 24, 125** $\widehat{d\pi}_\lambda^*$ **37, 46** $\widehat{d\pi_{(\sigma,\lambda)^*}}$ **33, 38** $\widehat{d\pi_{(\sigma,\lambda)^*}}$, algebraic Fourier transform of principal series **34, 35** $d\pi_{(\sigma,\lambda)^*}^{\text{scalar}}$ **38** $d\pi_{(\sigma,\lambda)^*}^{\text{vect}}$ **38** $\widehat{d\pi_{(i,\lambda)^*}}$ **52, 64, 72, 146****E** $\mathcal{E}^i(X)$ **1** $\mathcal{E}^i(X)_{u,\delta}$, conformal representation on i -forms on X **1, 2, 98** $\mathcal{E}^i(S^n)_{u,\delta}$ **3, 142** E_ζ , Euler homogeneity operator **45****G** (Gj) **180** G_ℓ^λ , Gegenbauer differential operator **173, 175** $G = O(n+1, 1)$ **16**

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- $h_{i \rightarrow j}^{(k)}$ **61**, 107
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 $\bar{I}(i)_\alpha^\#$, irreducible subquotient **19**
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- $J(j, \nu)_\beta$, principal series of $O(n, 1)$ **21**, 24, 64, 87, 121

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- p , stereographic projection 4, **15**, 136
 P , parabolic subgroup of $O(n+1, 1)$ **14**, 17, 35
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