

Glossary

Agronomy The science of agriculture.

Bridge process A continuous-time stochastic process, originated from a given process X and defined only on the time interval $[t_0, t_1]$, whose probability distribution is the conditional probability distribution of X given that it takes fixed values at time t_0 and t_1 , where $t_i < \infty$.

Diffusion process A Markov process in continuous time with continuous sample paths.

First passage time The random variable representing the time until a given stochastic process first hits a set of its state space. For a one-dimensional process it can for example represent the time at which a boundary level is first reached starting from the origin.

Green's function A Green's function or impulse response function is a function used to solve ordinary or partial differential equations with given initial and/or boundary conditions. Green's functions are named after the mathematician George Green who first used them in 1830.

Hodgkin–Huxley spatial model The Hodgkin–Huxley spatial model is a system of nonlinear partial differential equations originally employed in 1952 to study the propagation of action potentials in a squid axon.

Inference Statistical inference is the process of drawing conclusions from data that are subject to random variation, for example measurement noise or sampling variation.

Interspike interval Time between two successive spikes of a neuron. When the neuron membrane potential reaches a critical threshold value it triggers an action potential, commonly called a spike.

Jump diffusion process A stochastic process consisting of a continuous part driven by a Wiener process or in general by a diffusion process and a jump part with jumps driven by a Poisson process and either fixed or random jump sizes.

Large deviations theory A branch of probability theory concerning the asymptotic behaviour of the tails of sequences of probability distributions.

Leaky Integrate and Fire models Formal spiking neuron models. The basic circuit of a leaky integrate and fire model consists of a capacitor C in parallel with a resistor R driven by a current $I(t)$. The voltage $v(t)$ across the capacitance is compared to a threshold θ . If $v(t_i) = \theta$ an output pulse $\delta(t - t_i)$ is generated.

Markov process A stochastic process in which the conditional expectation of the next value, given the current and preceding values, only depends on the current value, i.e. conditionally on the present state of the system, its future and past are independent.

Martingale A stochastic process in which the conditional expectation of the next value, given the current and preceding values, is the current value.

Neuron membrane potential The potential difference between the inside and outside of the cell. At rest, neurons have a membrane potential typically about -70 mV.

Oncology The science of cancer.

Ornstein–Uhlenbeck process A stochastic process that is solution of an SDE with linear drift and constant diffusion term. It is Gaussian and Markov. The process tends to drift towards its long-term mean: this is called mean-reverting. It describes the velocity of a Brownian particle under the influence of friction.

Reaction–diffusion system One or more differential equations, at least one of which is a diffusion partial differential equation, with source and sink terms which determine the dynamics of the system. The simplest example is the Fisher's equation.

SPDE Stochastic partial differential equation. A partial differential equation such as the heat equation or wave equation, with random forcing terms, or parameters or boundary conditions which contain random processes.

Spreading depression A slowly moving surface-negative wave which occurs in many brain structures in which there is at first excitation of neurons followed by silence and then recovery. It is accompanied by massive changes in ionic and other concentrations and is believed to be primarily pathological.

Stochastic (non-deterministic) A stochastic process is one whose behavior contains random elements and which can be described through its probability distributions.

Stochastic differential equation (SDE) A differential equation in which one or more terms are stochastic processes, often Wiener processes, thus resulting in a solution which is itself a stochastic process. However, other types of random fluctuations are possible, such as compound Poisson processes, resulting in jump processes.

Stroke A stroke occurs if blood supply to a part of the brain is disrupted by either blockage or by loss of blood. The disabling effects depend on the part of the brain affected. It is the second leading cause of death worldwide.

Two-parameter white noise The second mixed partial derivative of two-parameter Brownian motion is a random process called two-parameter white noise. The process is delta-correlated in both parameters.

Wiener process or Brownian motion A continuous time random (diffusion) process with independent normally distributed (Gaussian) increments with mean zero and variance proportional to the elapsed time.

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