Appendix A
Periodic Maps Which Are Homotopic

The purpose of this appendix is to give a proof of Theorem 2.2. We will state the theorem again.

Theorem A.1. *Let f and f' be (orientation-preserving) periodic maps of a compact surface Σ each component of which has negative Euler characteristic. Suppose f and $f' : (\Sigma, \partial\Sigma) \to (\Sigma, \partial\Sigma)$ are homotopic as maps of pairs. Then there exists a homeomorphism $h : \Sigma \to \Sigma$ isotopic to the identity, such that $f = h^{-1} f' h$.*

Proof. First we consider the case when Σ is *connected*. Note that the quotient space $M = \Sigma/f$ is an orbifold with negative orbifold-Euler characteristic, (cf. [62]).

Case 1. The underlying space $|M|$ of M is not S^2 nor D^2.

Let $DM = M \cup \overline{M}$ be the double of M. (If $\partial M = \emptyset$, set $DM = M$.) We put a hyperbolic metric on DM so that DM admits a decomposition by a finite number of simple closed geodesics

$$G_1,\ G_2, \dots,\ G_l$$

which satisfy the following conditions:

1. No G_i passes through a cone point (of course, this can never happen, because the cone angle is less than 2π),
2. the boundary curves of M are closed geodesics, and are members of
$$\{G_1,\ G_2, \dots,\ G_l\},$$
3. the intersections of
$$G_1,\ G_2, \dots,\ G_l$$
are only double points,
4. each component of $DM - \bigcup_{i=1}^{l} G_i$ is an open cell whose closure is a polygon with more than 3 edges, and

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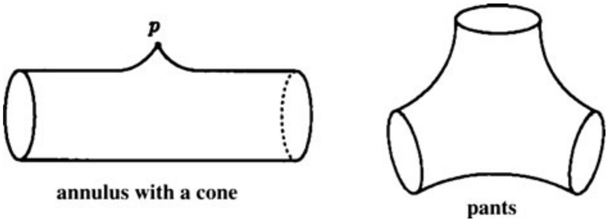

Fig. A.1 Hyperbolic parts ("annulus with a cone" and "pants")

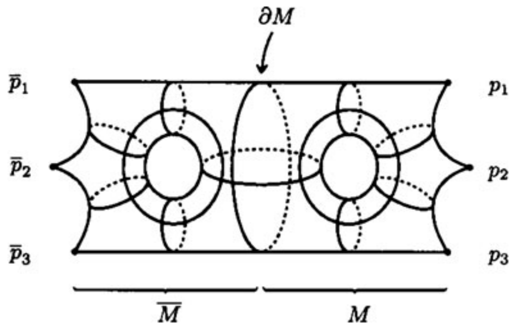

Fig. A.2 Decomposing the double DM by simple closed geodesics

5. each component of $DM - \bigcup_{i=1}^{l} G_i$ contains at most one cone point.

   Such a decomposition is certainly possible. In fact, one can start with two kinds of hyperbolic parts; annuli with one cone point and pants, both having geodesic boundaries of length 1. See Fig. A.1. Glue them along the boundaries and construct a hyperbolic orbifold homeomorphic to DM. Add further simple closed geodesics to get a desired decomposition. See Fig. A.2.

Let $D\Sigma = \Sigma \cup \overline{\Sigma}$ denote the double of Σ. The periodic map $f : \Sigma \to \Sigma$ symmetrically extends to a periodic map $D\Sigma \to D\Sigma$, which will be denoted by f again. Lift the metric on DM just constructed, to $D\Sigma$ to make the latter a (smooth) hyperbolic surface. The periodic map $f : D\Sigma \to D\Sigma$ preserves this metric σ.

Let

$$\Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_m$$

be the lifts to $D\Sigma$ of the simple closed geodesics

$$G_1,\ G_2, \ldots,\ G_l.$$

The total number m of the Γ_i's might be different from the total number l of the $G_i' s$. The simple closed geodesics

$$\Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_m$$

satisfy the following conditions:

(i) No Γ_i passes a multiple point of f,
(ii) the boundary curves of Σ are members of

$$\{\Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_m\},$$

(iii) the intersections of

$$\Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_m$$

are only double points,
(iv) each component of $D\Sigma - \bigcup_{i=i}^{m} \Gamma_i$ is an open cell whose closure is a polygon with more than 3 edges, and
(v) each component of $D\Sigma - \bigcup_{i=1}^{m} \Gamma_i$ contains at most one cone point.
Moreover, we have the following:
(vi) No pair Γ_i, Γ_j $(i \neq j)$ are freely homotopic (because they are distinct closed geodesics. Cf. [16].)
(vii) Γ_i and Γ_j $(i \neq j)$ have minimal intersection. Cf. [16].
(viii) f preserves the configuration $\Gamma_1 \cup \Gamma_2 \cup \cdots \cup \Gamma_m$;

$$f(\bigcup_{i=1}^{m} \Gamma_i) = \bigcup_{i=1}^{m} \Gamma_i$$

due to the construction of

$$\Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_m.$$

So far we have only considered the periodic map f. Now we consider the other

$$f' : \Sigma \to \Sigma.$$

Extend f' symmetrically to the double $D\Sigma$ and denote the resulting periodic map by $f' : D\Sigma \to D\Sigma$ also. Put a hyperbolic metric σ' on $D\Sigma$ which is invariant under f' and such that the boundary curves of Σ are closed geodesics.

The simple closed curves

$$\Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_m$$

are no longer geodesics with respect to σ', in general. But they are freely homotopic to simple closed geodesics

$$\Gamma_1',\ \Gamma_2', \ldots,\ \Gamma_m'$$

with respect to σ'. [16, Lemma 2.3.] These curves are distinct thanks to property (vi) of the Γ_i's.

We will construct an isotopy

$$g_\tau : D\Sigma \to D\Sigma, \quad 0 \leq \tau \leq 1,$$

such that

(a) $g_0 = id_{D\Sigma}$,
(b) if $\Gamma_i = \Gamma_i'$ (for instance, a boundary curve of Σ), then $g_\tau(\Gamma_i) = \Gamma_i'$, and
(c) $g_1(\Gamma_i) = \Gamma_i', i = 1, 2, \ldots, m$.

The construction is only a mimic of Lemmmas 2.4, 2.5 of Casson's lecture notes, [16], where the case $m = 2$ is treated, and is done essentially by an innermost arc argument. We proceed by induction. Suppose $\Gamma_i = \Gamma_i'$ $(i = 1, 2, \ldots, k)$ for some $k < m$. We will find an isotopy which starts with the identity, setwise preserves $\Gamma_i (= \Gamma_i')$ for $i = 1, 2, \ldots, k$, and at the final stage sends Γ_{k+1} to Γ_{k+1}'. (It will be helpful to consider

$$\Gamma_1',\ \Gamma_2', \ldots,\ \Gamma_m'$$

as patterns drawn on a floor, that remain fixed, and

$$\Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_m$$

as loops laid on it, that can move.) First we isotop Γ_{k+1} to separate it from Γ_{k+1}', and then using the annulus between Γ_{k+1} and Γ_{k+1}' move Γ_{k+1} onto Γ_{k+1}', as we will sketch now. A typical move is performed through the shaded disk in Fig. A.3.

The geodesics

$$\Gamma_1', \ldots,\ \Gamma_k' \ \ (\Gamma_1, \ldots,\ \Gamma_k)$$

have minimal intersection with the geodesic Γ_{k+1}' (cf. [16, Lemma 2.5]). Also they have minimal intersection with Γ_{k+1} because of property (vii) of Γ_i's. Thus as in Casson's Lemma 2.5, $\Gamma_i' \cap$ (the shaded disk) is a family of arcs passing "through" the disk from "top to bottom", for $i = 1, 2, \ldots, k$. But if there were a situation as shown in Fig. A.4, we would have an obstacle; we could not move Γ_{k+1} "along" $\Gamma_1', \ldots, \Gamma_k'$.

This "bad" situation is, however, prohibited by property (iv) of the Γ_i's. Therefore, we have the situation of Fig. A.3, and can move Γ_{k+1} through the shaded disk "along"

$$\Gamma_1',\ \Gamma_2', \ldots,\ \Gamma_k' \ \ (= \Gamma_1,\ \Gamma_2, \ldots,\ \Gamma_k).$$

This completes the inductive step, and we have obtained an isotopy

$$g_\tau : D\Sigma \to D\Sigma, \quad 0 \le \tau \le 1$$

satisfying conditions (a), (b), (c) stated above.

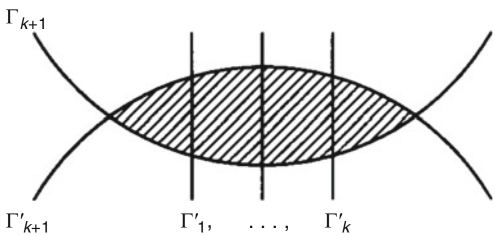

Fig. A.3 A move of Γ_{k+1}

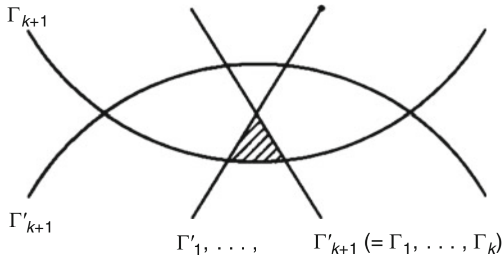

Fig. A.4 A situation in which the move of Γ_{k+1} is obstructed

Let $g : D\Sigma \to D\Sigma$ be the final stage g_1 of the isotopy. Then

$$g(\Gamma_1 \cup \Gamma_2 \cup \cdots \cup \Gamma_m) = \Gamma_1' \cup \Gamma_2' \cup \cdots \cup \Gamma_m'.$$

Lemma A.1. $f'(\Gamma_1' \cup \Gamma_2' \cup \cdots \cup \Gamma_m') = \Gamma_1' \cup \Gamma_2' \cup \cdots \cup \Gamma_m'.$

Proof. First note that

$$\begin{aligned} f'(\Gamma_i') &\simeq f'(\Gamma_i) && (\text{because} \quad \Gamma_i' \simeq \Gamma_i) \\ &\simeq f(\Gamma_i) && (\text{because} \quad f' \simeq f) \\ &= \Gamma_j && (\text{for some } j\text{, because of property (viii) of the } \Gamma_i\text{´s}) \\ &\simeq \Gamma_j' && (\text{because } \Gamma_j \simeq \Gamma_j'), \end{aligned}$$

where "$\simeq$" denotes "is freely homotopic to".

The metric σ' on $D\Sigma$ is invariant under f', so $f'(\Gamma_i')$ is a simple closed geodesic as well as Γ_i's. In a free homotopy class of simple closed curves, there is only one simple closed geodesic, (cf. [16, Lemma 2.4]). Therefore, $f'(\Gamma_i') = \Gamma_j'$. □

Let $\mathbf{H}^2$ be the hyperbolic plane which is the universal covering of $(D\Sigma, \sigma')$. Let $D = \mathbf{H} \cup S_\infty$ be its compactification to the unit disk. Let

$$\tilde{g}_\tau : \mathbf{H}^2 \to \mathbf{H}^2, \quad 0 \le \tau \le 1$$

be the lifted isotopy of g_τ starting from $\tilde{g}_0 = id_{\mathbf{H}^2}$. Then

$$\tilde{g} := \tilde{g}_1 : \mathbf{H}^2 \to \mathbf{H}^2$$

is a lift of

$$g : D\Sigma \to D\Sigma.$$

By Nielsen [52], the isotopy

$$\tilde{g}_\tau : \mathbf{H}^2 \to \mathbf{H}^2$$

extends to an isotopy

$$(\tilde{g}_\tau)\hat{}: D \to D,$$

and being a lift of an isotopy of a *compact* surface $D\Sigma$, the restriction of $(\tilde{g}_\tau)\hat{}$ to S_∞ is *constant* because $\tilde{g}_\tau$ varies equivariantly with respect to the group of superpositions of the covering $\mathbf{H}^2 \to D\Sigma$, and the variation is determined by the restriction of $\tilde{g}_\tau$ to a compact set (because $D\Sigma$ is compact). Since the euclidean size of a fundamental domain for $D\Sigma$ tends to zero if it goes to infinity, then $\tilde{g}_\tau$ at S_∞ does not depend on τ. Thus $(\tilde{g}_\tau)\hat{}|S_\infty = id|S_\infty$.

Let

$$\tilde{f}: \mathbf{H}^2 \to \mathbf{H}^2$$

be a lift of

$$f: D\Sigma \to D\Sigma,$$

and

$$(\tilde{f})\hat{}: D \to D$$

its extention. Since

$$f \simeq f': D\Sigma \to D\Sigma,$$

$\tilde{f}$ is homotopic, through a lifted homotopy, to a lift $\tilde{f}'$ of f'. For the same reason as above, we have $(\tilde{f})\hat{}|S_\infty = (\tilde{f}')\hat{}|S_\infty$.

Now let us regard

$$\Gamma_1 \cup \Gamma_2 \cup \cdots \cup \Gamma_m$$

and

$$\Gamma_1' \cup \Gamma_2' \cup \cdots \cup \Gamma_m'$$

as finite graphs Γ and Γ', respectively, drawn on $D\Sigma$. They decompose $D\Sigma$ into finite cell complexes. Consider their lifts $\tilde{\Gamma}$, $\tilde{\Gamma}'$ in $\mathbf{H}^2$. $\tilde{\Gamma}$ consists of lifts of the Γ_i's, and $\tilde{\Gamma}'$ of lifts of the Γ_i's. (Each lift of Γ_i' is a geodesic line.) $\tilde{\Gamma}$ and $\tilde{\Gamma}'$ decompose $\mathbf{H}^2$ into locally finite cell complexes. $\tilde{g}: \mathbf{H}^2 \to \mathbf{H}^2$ preserves these patterns: $\tilde{g}(\tilde{\Gamma}) = \tilde{\Gamma}'$.

Let $\tilde{\Gamma}^{(0)}$ and $\tilde{\Gamma}'^{(0)}$ be the set of vertices of $\tilde{\Gamma}$ and $\tilde{\Gamma}'$, respectively.

Lemma A.2. *$\tilde{g} \mid \tilde{\Gamma}^{(0)} : \tilde{\Gamma}^{(0)} \to \tilde{\Gamma}'^{(0)}$ is equivariant with respect to $\tilde{f} \mid \tilde{\Gamma}^{(0)}$ and $\tilde{f}' \mid \tilde{\Gamma}'^{(0)}$.*

Proof. Take a vertex $\tilde{v} \in \tilde{\Gamma}^{(0)}$. $\tilde{v}$ is an intersection point of two infinite curves $\tilde{\Gamma}_i$ and $\tilde{\Gamma}_j$, which are lifts of Γ_i and Γ_j. By property (iii) of the Γ_i's, the pair $\{\tilde{\Gamma}_i, \tilde{\Gamma}_j\}$ is uniquely determined by $\tilde{v}$. We have

$$\tilde{g}\,\tilde{f}(\tilde{\Gamma}_i) = \tilde{f}'\,\tilde{g}(\tilde{\Gamma}_i) \quad \text{and} \quad \tilde{g}\,\tilde{f}(\tilde{\Gamma}_j) = \tilde{f}'\,\tilde{g}(\tilde{\Gamma}_j).$$

(*Proof.* Let $\inf(\tilde{\Gamma}_i)$ denote the pair of the two "infinite" points of $\tilde{\Gamma}_i$ in S_∞. Then

$$\inf(\tilde{g}\,\tilde{f}(\tilde{\Gamma}_i)) = (\tilde{g})\hat{}(\tilde{f})\hat{}(\inf(\tilde{\Gamma}_i)) = (\tilde{f}')\hat{}(\tilde{g})\hat{}(\inf(\tilde{\Gamma}_i)) = \inf(\tilde{f}'\,\tilde{g}(\tilde{\Gamma}_i))$$

because $(\tilde{g})\hat{}\,|\,S_\infty = id$ and

$$(\tilde{f})\hat{}\,|\,S_\infty = (\tilde{f}')\hat{}\,|\,S_\infty.$$

$\tilde{g}\,\tilde{f}(\tilde{\Gamma}_i)$ and $\tilde{f}'\,\tilde{g}(\tilde{\Gamma}_i)$ are geodesic lines in $\mathbf{H}^2$ with the same pair of infinite points. Then they coincide. Similarly, $\tilde{g}\,\tilde{f}(\tilde{\Gamma}_j) = \tilde{f}'\,\tilde{g}(\tilde{\Gamma}_j)$.)

Therefore,

$$\{\tilde{g}\,\tilde{f}(\tilde{v})\} = \tilde{g}\,\tilde{f}(\tilde{\Gamma}_i) \cap \tilde{g}\,\tilde{f}(\tilde{\Gamma}_j) = \tilde{f}'\,\tilde{g}(\tilde{\Gamma}_i) \cap \tilde{f}'\,\tilde{g}(\tilde{\Gamma}_j) = \{\tilde{f}'\,\tilde{g}(\tilde{v})\},$$

which proves $\tilde{g}\,\tilde{f}(\tilde{v}) = \tilde{f}'\,\tilde{g}(\tilde{v})$. □

Remember that the $\tilde{\Gamma}_i$'s are geodesic lines with respect to the lifted metric $\tilde{\sigma}$ of σ. Using Lemma A.2, we can find an isotopy

$$\tilde{g}^{(1)}_\tau : \mathbf{H}^2 \to \mathbf{H}^2, \quad 0 \le \tau \le 1,$$

such that $\tilde{g}^{(1)}_0 = \tilde{g}$, $\tilde{g}^{(1)}_\tau(\tilde{\Gamma}) = \tilde{\Gamma}'$, $\tilde{g}^{(1)}_\tau$ is equivariant with respect to the group of covering translations of $\mathbf{H}^2 \to D\Sigma$, and the final stage $\tilde{g}^{(1)}_1$ is "linear" from each edge of $\tilde{\Gamma}$ to an edge of $\tilde{\Gamma}'$ with respect to $\tilde{\sigma}$ and $\tilde{\sigma}'$ (the lifted metric of σ'). Then

$$\tilde{g}^{(1)}_1|\tilde{\Gamma} : \tilde{\Gamma} \to \tilde{\Gamma}'$$

is equivariant with respect to $\tilde{f}|\tilde{\Gamma}$ and $\tilde{f}'|\tilde{\Gamma}'$.

Finally by the Alexander trick, we can deform $\tilde{g}^{(1)}_1$ within each cell and obtain an isotopy

$$\tilde{g}^{(2)}_\tau : \mathbf{H}^2 \to \mathbf{H}^2, \quad 0 \le \tau \le 1,$$

such that $\tilde{g}^{(2)}_0 = \tilde{g}^{(1)}_1$, $\tilde{g}^{(2)}_\tau|\tilde{\Gamma} = \tilde{g}^{(1)}_\tau|\tilde{\Gamma}$, $\tilde{g}^{(2)}_\tau$ is equivariant with respect to the group of covering translations of

$$\mathbf{H}^2 \to D\Sigma,$$

and the final stage

$$\tilde{g}^{(2)}_1 : \mathbf{H}^2 \to \mathbf{H}^2$$

is equivariant with respect to $\tilde{f}$ and $\tilde{f}'$. This homeomorphism $\tilde{g}^{(2)}_1$ projects down to a homeomorphism

$$h : D\Sigma \to D\Sigma.$$

The restricted homeomorphism

$$h|\Sigma : \Sigma \to \Sigma$$

is the one whose existence is asserted by Theorem 2.2. This completes the proof of Case 1.

Case 2. The underlying space $|M|$ of M is S^2.

The proof will be accompanied by several lemmas.

Lemma A.3. *The orders of f and $f' : \Sigma \to \Sigma$ are equal.*

Proof. Let $L(g)$ denote the Lefschetz number of a homeomorphism $g : \Sigma \to \Sigma$. Then the order of f is equal to the smallest positive integer n such that $L(f^n) < 0$ because if $f^n \neq id_\Sigma$, $L(f^n)$ is equal to the number of the fixed points of f^n (see [23, pp. 130, 121]) which is non-negative, while $L(id_\Sigma) = \chi(\Sigma) < 0$. Thus our assumption $f \simeq f'$ implies that their orders coincide. □

Lemma A.4. *There is a bijective correspondence between the set of multiple points of f and the same set of f' which preserves the valencies.*

Proof. We impose a hyperbolic metric on Σ and identify the universal covering $\tilde{\Sigma}$ with $\mathbf{H}^2$, which is compactified to $D = \mathbf{H}^2 \cup S_\infty$ as before. Let $F(f)$ denote the set of fixed points of f. Take a point $p_0 \in F(f)$ and lift it to $\tilde{p}_0 \in \mathbf{H}^2$. Let

$$\tilde{f} : \mathbf{H}^2 \to \mathbf{H}^2$$

be a lift of f which fixes $\tilde{p}_0$. By Nielsen [50, Sect. 2], $\tilde{f}$ extends to a homeomorphism

$$(\tilde{f})\hat{} : D \to D.$$

The lift

$$\tilde{f} : \mathbf{H}^2 \to \mathbf{H}^2$$

is a periodic map because it is a lift of a periodic map and it has a fixed point. Then

$$(\tilde{f})\hat{} : D \to D$$

is also a periodic map. By [31], an orientation-preserving periodic map of a 2-disk D is conjugate to a rotation. In particular, $(\tilde{f})\hat{}|S_\infty$ has no fixed points.

By our assumption

$$f \simeq f' : \Sigma \to \Sigma,$$

$\tilde{f}$ is homotopic, through a lifted homotopy, to a lift $\tilde{f}'$ of f'. Then by the same argument as in Case 1, $(\tilde{f})\hat{}|S_\infty = (\tilde{f}')\hat{}|S_\infty$. By Brouwer's fixed point theorem,

$$(\tilde{f}')\hat{} : D \to D$$

has a fixed point. But $(\tilde{f}')|_{S_\infty}$ $(= (\tilde{f})\hat{}|_{S_\infty})$ has no fixed points, so the fixed point of $\tilde{f}'$ is in $\mathbf{H}^2$. Then the same argument as for $\tilde{f}$ can apply to $\tilde{f}'$, and

$$(\tilde{f}')\hat{} : D \to D$$

is conjugate to a rotation. Thus the fixed point $\tilde{p}_0'$ of $\tilde{f}'$ is uniquely determined in $\mathbf{H}^2$, which projects to a fixed point p_0' of f'. The correspondence

$$\varphi : F(f) \to F(f')$$

is defined by sending p_0 to p_0'.

φ is independent of the choice of the lift $\tilde{p}_0$, but it might depend on the homotopy between f and f'. We will fix the homotopy throughout the argument. Clearly φ is bijective because the roles of f and f' are symmetric.

The valency of p_0 (resp. p_0') with respect to f (resp. f') is the same as the valency of $\tilde{p}_0$ (resp. $\tilde{p}_0'$) with respect to $\tilde{f}$ (resp. $\tilde{f}'$), which can be read off from the action of $(\tilde{f})\hat{}$ (resp. $(\tilde{f}')\hat{}$) on S_∞. But $(\tilde{f})\hat{}|S_\infty = (\tilde{f}')\hat{}|S_\infty$. Thus the valency of p_0 is equal to the valency of $p_0' = \varphi(p_0)$.

Similarly for each factor m of the order of f we can construct a bijective correspondence between $F(f^m)$ and $F((f')^m)$. This correspondence preserves the valencies exactly as above. This completes the proof of Lemma A.4. $\square$

Lemmmas A.3, A.4 and Nielsen's theorem (Theorem 1.2) imply that $f : \Sigma \to \Sigma$ and $f' : \Sigma \to \Sigma$ are conjugate. (Remember that we are considering a closed Σ in Case 2.) In particular, we have

Lemma A.5. *$M = \Sigma/f$ and $M' = \Sigma'/f'$ are homeomorphic as orbifolds.*

Let

$$p_1, \quad p_2, \ldots, \quad p_s, \qquad s \geq 3,$$

be the cone points of M with valencies

$$(m_1, \lambda_1, \sigma_1), \quad (m_2, \lambda_2, \sigma_2), \ldots, \quad (m_s, \lambda_s, \sigma_s),$$

respectively. Consider a polygon P (s-gon) in $\mathbf{H}^2$ whose angles are

$$\pi/\lambda_1, \quad \pi/\lambda_2, \ldots, \quad \pi/\lambda_s.$$

See Fig. A.5. (Such a P exists because $\chi^{\text{orb}}(M) < 0$.) M can be considered as a hyperbolic "bi-hedron" having two faces, each congruent with P.

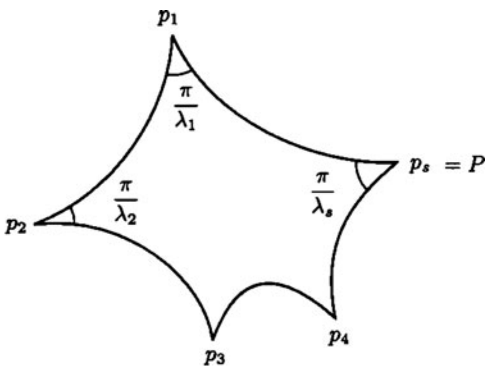

Fig. A.5 A hyperbolic polygon

Let σ be the hyperbolic metric on Σ obtained by lifting the bi-hedron metric on M through the projection $\Sigma \to M$. We lift this metric σ farther to the universal covering $\tilde{\Sigma}$ to make it a hyperbolic plane $\mathbf{H}^2(\sigma)$. The plane $\mathbf{H}^2(\sigma)$ is tessellated by tiles, each congruent with P. The fundamental region for the action of $\pi_1^{\mathrm{orb}}(M)$ ([62, Sect. 13]) is $F = P \cup \overline{P}$, where $\overline{P}$ is a flipped P having an edge in common with it.

Let r_i be the rotation (in $\mathbf{H}^2(\sigma)$) of angle $2\pi/\lambda_i$ centered at vertex $p_i (\in P)$, $i = 1, 2, \ldots, s$. It is known that the rotations r_1, r_2, …, r_s generate the group $\pi_1^{\mathrm{orb}}(M)$, the orientation-preserving automorphism group of the tessellation, (cf. Milnor [45] or [41]). Moreover, from our construction, r_i is a lift of

$$f^{\sigma_i m_i} : \Sigma \to \Sigma, \quad i = 1,\ 2, \ldots,\ s.$$

Let

$$f_\tau : \Sigma \to \Sigma, \quad 0 \leq \tau \leq 1,$$

be a homotopy between f and $f' : f_0 = f,\ f_1 = f'$. Then, through a lifted homotpy

$$(f_\tau^{\sigma_i m_i})^{\tilde{}} : \mathbf{H}^2(\sigma) \to \mathbf{H}^2(\sigma),$$

r_i is homotopic to a homeomorphism

$$r_i' : \mathbf{H}^2(\sigma) \to \mathbf{H}^2(\sigma).$$

This r_i' is a lift of $(f')^{\sigma_i m_i}$, satisfies $(r_i')\hat{}|S_\infty = (r_i)\hat{}|S_\infty$, and is topologically equivalent to a rotation. (See Proof of Lemma A.4.)

Let p_i' be the center of the "rotation" r_i'. We denote the cone point of M' to which p_i' projects by the same notation p_i'. Thus we have obtained the following correspondence:

$$M \ni p_i \longleftrightarrow p_i' \in M', \quad i = 1, 2, \ldots, s.$$

By Lemma A.4, this correspondence preserves the valencies. We impose a structure of a hyperbolic bi-hedron on M' whose faces P', $\overline{P}'$ are congruent with P, $\overline{P}$ preserving the above correspondence of the vertices. Let σ' be the hyperbolic metric on Σ obtained by lifting the bi-hedron metric on M' through the projection $\Sigma \to M'$. We lift σ' to $\tilde{\Sigma}$ to make it a hyperbolic plane $\mathbf{H}^2(\sigma')$, in which the polygon P' is inscribed with vertices

$$p_1',\ p_2', \ldots,\ p_s'.$$

In $\mathbf{H}^2(\sigma)$, the topological rotation r_i' is a genuine rotation with center p_i' of angle $2\pi/\lambda_i$, $i = 1, 2, \ldots, s$. The plane $\mathbf{H}^2(\sigma')$ is tessellated by tiles, each congruent with P'. The rotations r_1', r_2', …, r_s' generate the group $\pi_1^{\mathrm{orb}}(M')$, the orientation-preserving automorphism group of the tessellation. Since P' is congruent with P, the group $\pi_1^{\mathrm{orb}}(M')$ is isomorphic to $\pi_1^{\mathrm{orb}}(M)$ via the correspondence

$$r_i \longleftrightarrow r_i', \quad i = 1, 2, \ldots, s.$$

Note that $\mathbf{H}^2(\sigma)$ and $\mathbf{H}^2(\sigma')$ are merely different "pictures" on the same space $\tilde{\Sigma}$, so $\pi_1^{\text{orb}}(M)$ and $\pi_1^{\text{orb}}(M')$ are considered as subgroups of $Homeo(\tilde{\Sigma})$, the group of all self-homeomorphisms of $\tilde{\Sigma}$. If we fix the action of $\pi_1(\Sigma)$ on $\tilde{\Sigma}$, then $\pi_1(\Sigma)$ is also a subgroup of $Homeo(\tilde{\Sigma})$, contained in $\pi_1^{\text{orb}}(M) \cap \pi_1^{\text{orb}}(M')$. Moreover, the action of $\pi_1(\Sigma)$ is isometric, with respect to $\mathbf{H}^2(\sigma)$ *and* at the same time with respect to $\mathbf{H}^2(\sigma')$.

Lemma A.6. *The isomorphism between $\pi_1^{\text{orb}}(M)$ and $\pi_1^{\text{orb}}(M')$ given by the correspondence $r_i \longleftrightarrow r_i'$ $(i = 1, 2, \ldots, s)$ restricts to the identity on $\pi_1(\Sigma)$.*

Proof. Take an element $g \in \pi_1(\Sigma)$. Since $\pi_1(\Sigma) < \pi_1^{\text{orb}}(M')$, g can be written as a product of $r_1', r_2', \ldots, r_s'$:

$$g = \psi(r_1', r_2', \ldots, r_s').$$

We will show that

$$g = \psi(r_1, r_2, \ldots, r_s).$$

For this, compactify $\mathbf{H}^2(\sigma)$ to $D = \mathbf{H}^2(\sigma) \cup S_\infty$ as before. Then

$$\hat{g}|S_\infty = \psi(r_1', r_2', \ldots, r_s')\hat{\ }|S_\infty = \psi(r_1, r_2, \ldots, r_s)\hat{\ }|S_\infty.$$

Since both g and $\psi(r_1, r_2, \ldots, r_s)$ are isometries of $\mathbf{H}^2(\sigma)$, we have

$$g = \psi(r_1, r_2, \ldots, r_s).$$

$\square$

Let us construct a homeomorphism $\tilde{h} : \tilde{\Sigma} \to \tilde{\Sigma}$ which is equivariant with respect to the actions of $\pi_1^{\text{orb}}(M)$ and $\pi_1^{\text{orb}}(M')$. The construction is obvious. First, map the fundamental region $F = P \cup \overline{P}$ "isometrically" to the fundamental region $F' = P \cup \overline{P}'$. Then extend it equivariantly to the whole space.

By Lemma A.6, we have $\tilde{h}\,g = g\,\tilde{h}$ for all $g \in \pi_1(\Sigma)$. Thus $\tilde{h}$ projects to a homeomorphism $h : \Sigma \to \Sigma$.

Lemma A.7. *$h : \Sigma \to \Sigma$ satisfies $f = h^{-1} f' h$.*

Proof. Let $\tilde{f} : \tilde{\Sigma} \to \Sigma$ be a lift of $f : \Sigma \to \Sigma$. Since $\tilde{f}$ preserves the tessellation, $\tilde{f}$ is written as a product of $r_1, r_2, \ldots, r_s$:

$$\tilde{f} = \varphi(r_1, r_2, \ldots, r_s).$$

Then

$$\varphi(r_1', r_2', \ldots, r_s')$$

is a lift of f'; denote it by $\tilde{f}'$. Since $\tilde{h}$ is equivariant with respect to the actions of $\pi_1^{\text{orb}}(M)$ and $\pi_1^{\text{orb}}(M')$, we have

$$\tilde{h}\,\tilde{f} = \tilde{h}\,\varphi(r_1, r_2, \ldots, r_s) = \varphi(r_1', r_2', \ldots, r_s')\,\tilde{h} = \tilde{f}'\,\tilde{h},$$

so $h\,f = f'\,h$ as asserted. □

Lemma A.8. *$h : \Sigma \to \Sigma$ is isotopic to the identity.*

Proof. It will be sufficient to prove $(\tilde{h})\hat{}|S_\infty = id.$, because then $h : \Sigma \to \Sigma$ preserves the free homotopy class of every simple closed curve.

Let g be any element of $\pi_1(\Sigma)$ different from 1. Then $g : \mathbf{H}^2 \to \mathbf{H}^2$ is a hyperbolic transformation, and for any point $x \in \mathbf{H}^2$, $g^n(x)$ converges (in D) to a definite point $V_g \in S_\infty$ as $n \to +\infty$. Also $g^n(x)$ converges to another definite point $U_g \in S_\infty$ as $n \to -\infty$. Nielsen [50, Sect. 1] calles U_g, V_g the *negative* and the *positive* fundamental points of g.

Then, in D, we have

$$(\tilde{h})\hat{}(V_g) = (\tilde{h})\hat{}(\lim_{n\to\infty} g^n(x)) = \lim_{n\to\infty} (\tilde{h})(g^n(x)) = \lim_{n\to\infty} g^n(\tilde{h})(x) = V_g.$$

Also $(\tilde{h})\hat{}(U_g) = U_g$. But the set of fundamental points

$$\{U_g \mid g \in \pi_1(\Sigma)\} \cup \{V_g \mid g \in \pi_1(\Sigma)\}$$

is *dense* is S_∞ ([53, Sect. 1 Case b)]). This proves $(\tilde{h})\hat{}|S_\infty = id.$ □

The proof is completed in Case 2.

Case 3. The underlying space $|M|$ of M is D^2.

Making the double DM, the proof is reduced to Case 2.

Now we have completed the proof in the case when Σ is *connected*.

In the general case when Σ is not necessarily connected, divide the set of the components of Σ into cycles under the permutation caused by f. Since f' is homotopic to f, f' causes the same permutation. Then in each cycle we can argue just as in the proof of Theorem 2.3(ii). (Beware that we need here the "homotopy implies isotopy" theorem, [10, 21].) This completes the proof of Theorem A.1. □

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