A

Appendix

We present some elementary mathematical material that is used throughout this book. Most of these basics in mathematics can be found in Feller (1968, 1971) and Motwani and Raghavan (1995).

A.1 Probability Distributions

Definition A.1 (Binomial distribution). *A random variable X follows the binomial distribution with parameters n and p if*

$$\mathrm{Prob}(X = k) \;=\; \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

for $k \in \{0, \ldots, n\}$. Its expectation is $E(X) = np$.

Illustratively, the random variable X counts the number of successes in n independent Bernoulli trials with probability p for a success.

Definition A.2 (Geometric distribution). *A random variable X follows the geometric distribution with parameter p if*

$$\mathrm{Prob}(X = k) \;=\; p^k \cdot (1-p)$$

for $k \in \mathbb{N}_0$. Its expectation is $E(X) = 1/p$.

Illustratively, X counts the number of consecutive successes before the first failure in independent Bernoulli trials with success probability p.

Definition A.3 (Poisson distribution). *Let λ be a positive real number. A random variable X follows the Poisson distribution with parameter λ if*

$$\mathrm{Prob}(X = k) \;=\; \frac{\lambda^k e^{-\lambda}}{k!}$$

for $k \in \mathbb{N}_0$. Its expectation is $E(X) = \lambda$.

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A.2 Deviation Inequalities

Proposition A.4 (Markov's inequality). *Let X be a random variable assuming only non-negative values. Then for all $t \in \mathbb{R}^+$,*

$$Prob(X \geq k \cdot E(X)) \leq 1/k.$$

Proposition A.5 (Chernoff bounds). *Let $X_1, X_2, \ldots, X_n$ be independent Poisson trials such that for $1 \leq i \leq n$ $Prob(X_i = 1) = p_i$, where $0 < p_i < 1$. Let $X = \sum_{i=1}^{n} X_i$, $\mu = E(X) = \sum_{i=1}^{n} p_i$. Then the following inequalities hold.*

$$\begin{aligned}
Prob(X \geq (1+\delta)\mu) &\leq \left(\frac{e^\delta}{(1+\delta)^{(1+\delta)}}\right)^\mu && \delta > 0 \\
Prob(X \geq (1+\delta)\mu) &\leq e^{-\mu\delta^2/3} && 0 < \delta \leq 1 \\
Prob(X \leq (1-\delta)\mu) &\leq e^{-\mu\delta^2/2} && 0 < \delta \leq 1
\end{aligned}$$

Proposition A.6 (Chernoff-Hoeffding bound). *Let $X_1, \ldots, X_n$ be independent random variables such $a_i \leq X_i \leq b_i$ for $1 \leq i \leq n$. Denote $X = \sum_{i=1}^{n} X_i$. Then for any $\delta \geq 0$ the following inequalities hold.*

$$\begin{aligned}
Prob(X \geq \mu + \delta) &\leq e^{-2\delta^2/\sum_{i=1}^{n}(b_i - a_i)^2} \\
Prob(X \leq \mu - \delta) &\leq e^{-2\delta^2/\sum_{i=1}^{n}(b_i - a_i)^2}
\end{aligned}$$

A.3 Other Useful Formulas

Proposition A.7 (Union bound). *For a finite or countably infinite sequence $A_1, A_2, A_3, \ldots,$ of events*

$$\text{Prob}\left(\cup_{i \geq 1} A_i\right) \leq \sum_{i \geq 1} \text{Prob}(A_i).$$

Proposition A.8 (Law of total probability). *For an event A and a partition of the sample space Ω into mutually disjoint events $B_1, \ldots, B_k$, i.e., $\cup_{i=1}^{k} B_i = \Omega$, it holds*

$$\text{Prob}(A) = \sum_{i=1}^{k} \text{Prob}(A \mid B_i) \cdot \text{Prob}(B_i),$$

where $\text{Prob}(A \mid B_i)$ denotes the conditional probability of A, given B_i.

Proposition A.9 (Stirling's formula). *For any* $n \in \mathbb{N}$

$$\sqrt{2\pi n}n^n e^{-n} \; < \; n! \; < \; \sqrt{3\pi n}n^n e^{-n}$$

holds.

Proposition A.10 (Inequalities with e**).**

$$\begin{aligned} e^x &\geq 1 + x \text{ for } x \in \mathbb{R} \\ e^{-x} &\leq 1 - \frac{x}{2} \text{ for } 0 \leq x \leq 1 \\ e^x &\leq \frac{1}{1-x} \text{ for } x < 1 \\ \left(1 - \frac{1}{n}\right)^n \leq e^{-1} &\leq \left(1 - \frac{1}{n}\right)^{n-1} \text{ for } n \in \mathbb{N} \end{aligned}$$

Proposition A.11 (Binomial coefficients). *Let* $n \geq k \geq 0$*. The binomial coefficients are defined as*

$$\binom{n}{k} = \binom{n}{n-k} = \frac{n!}{k!(n-k)!},$$

and it holds

$$\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \frac{n^k}{k!} \leq \left(\frac{ne}{k}\right)^k.$$

Proposition A.12 (Harmonic sum). *Let* $H_n = \sum_{i=1}^{n} 1/i$ *be the* n*th Harmonic sum. Then for any* $n \in \mathbb{N}$

$$H_n = \ln n + \Theta(1).$$

Proposition A.13 (Coupon collector's theorem). *In the coupon collector's problem,* n *different coupons are given and at each trial a coupon is chosen uniformly at random. Let* X *be a random variable describing the number of trials required to choose each coupon at least once. Then*

$$E(X) = nH_n$$

holds, where H_n *denotes the* n*th Harmonic number, and*

$$\lim_{n \to \infty} \text{Prob}(X \leq n(\ln n - c)) = e^{-e^c}$$

holds for each constant $c \in \mathbb{R}$*.*

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