

Appendix A

A.1 Conjectures

1. (Á. Baricz [45]) The inequality (3.23) holds for all $a, b > 0$ and for all $r, s \in (0, 1)$.
2. (Á. Baricz and E. Neuman [59]) The function $x \in \mathbb{R} \mapsto \mathcal{J}_p(x) \in [1, \infty)$ is strictly log-convex for all $p > -1$, while the function $x \in (0, \infty) \mapsto I_p(x) \in (0, \infty)$ is strictly log-concave for all $p > 0$.
3. (Á. Baricz [54]) The function $x \in \mathbb{R} \mapsto \mathcal{J}_p^{(2k)}(x) \in (0, \infty)$ is strictly log-convex for all $k \in \mathbb{N}$ and $p > -1$, while the function $x \in (0, \infty) \mapsto \mathcal{J}_p^{(2k+1)}(x) \in (0, \infty)$ is strictly log-concave for all $k \in \mathbb{N}$ and $p > -1$.
4. (Á. Baricz [54]) The inequality (3.97) holds for all $a, b \geq 0$ and $p > -1$.
5. (Á. Baricz and S. Wu [63]) The inequality (3.169) holds for all $p \in (-1, p_0)$ and $|x| < j_{p,1}$, while (3.170) and (3.171) hold for all $p > -1$ and $|x| < j_{p,1}$.

A.2 Open Problems

1. (Á. Baricz [43]) Find the explicit range of b, c, p for which u_p is univalent in \mathbb{D} .
2. (S. András and Á. Baricz [24]) Is it true that if $p > -1$ increases, then the image region $\mathcal{J}_p(\mathbb{D})$ decreases, more precisely if $p > q > -1$, then $\mathcal{J}_p(\mathbb{D}) \subset \mathcal{J}_q(\mathbb{D})$? This problem is related to Corollary 2.14 which provides a partial affirmative answer. Namely, it is true that if $p > q \geq -1/4$, then $\mathcal{J}_p(\mathbb{D}) \subset \mathcal{J}_q(\mathbb{D})$.
3. (Á. Baricz [41]) For which $b, p, c \in \mathbb{R}$ does $\sigma(r_1) + \sigma(r_2) \leq 2\sigma(\sqrt{r_1 r_2})$ hold for all $r_1, r_2 \in (0, 1)$, where $\sigma(r) = u_p(1 - r^2)/u_p(r^2)$?
4. (Á. Baricz [41]) Suppose that the power series $f(r) = \sum_{n \geq 0} A_n r^n$ is convergent for all $r \in (0, 1)$, where $A_n > 0$ for all $n \geq 0$. Let us consider the function $m_f : (0, 1) \rightarrow (0, \infty)$, defined by $m_f(r) = f(1 - r^2)/f(r^2)$. Find conditions concerning the coefficients A_n , which guarantee that $m_f(r_1) + m_f(r_2) \leq 2m_f(\sqrt{r_1 r_2})$ for all $r_1, r_2 \in (0, 1)$.

A.3 Matlab Programs for Graphs

1. Fig. 1.1

```
x=0:0.1:10;
y1=besselj(-1/2,x);
y2=besselj(0,x);
y3=besselj(1/2,x);
plot(x,y1,'b. ');hold on;
plot(x,y2,'g ');
plot(x,y3,'r--');
```

2. Fig. 1.2

```
x=0:0.1:4;
y1=besseli(-1/2,x);
y2=besseli(0,x);
y3=besseli(1/2,x);
plot(x,y1,'b. ');hold on;
plot(x,y2,'g ');
plot(x,y3,'r--');
```

3. Fig. 2.1

```
clear;
fi=0:0.001:2*pi;
z=cos(fi)+i*sin(fi);
gz=cos(z);
plot(z,'g '); hold on;
plot(gz,'b ');
axis equal;
axis([-1.1 1.6 -1.1 1.1]);
```

4. Fig. 2.2

```
clear;
fi=0:0.001:2*pi;
z=cos(fi)+i*sin(fi);
fz=sin(z)./z;
plot(fz,'r ');hold on;
plot(z,'g '); hold on;
axis equal;
axis([-1.1 1.6 -1.1 1.1]);
```

5. Fig. 2.3

```
clear;
fi=0:0.001:2*pi;
z=cos(fi)+i*sin(fi);
fz=sin(z)./z;
gz=cos(z);
plot(fz,'r ');hold on;
plot(z,'g '); hold on;
```

```

plot(gz,'b');
axis equal;
axis([-1.1 1.6 -1.1 1.1]);

```

6. Fig. 2.4

```

clear;
fi=0:0.001:2*pi;
z=cos(fi)+i*sin(fi);
fz=sin(z)./z;
gz=cos(z);
hz=3*((sin(z))./(z.^3)-(cos(z))./(z.^2));
plot(fz,'r');hold on;
plot(z,'g'); hold on;
plot(gz,'b');
plot(hz,'y');
axis equal;
axis([-1.1 1.6 -1.1 1.1]);

```

7. Fig. 3.1

```

clear;
plot(0,1,'o');hold on; %point A
plot(pi/2,2/pi,'o'); %point B
x=0:0.05:2;
y1=2/pi-4/(pi^2)*(x-pi/2);
y2=sin(x)./x;
y3=1+2/pi*(2/pi-1)*x;
plot(x,y1,'r-'); hold on;
plot(x,y2,'g');
plot(x,y3,'b-');

```

8. Fig. 3.2

```

clear;
x=-1.5:0.1:1.5;
y1=2/9*(2+5/2*cos(sqrt(3/5)*x));
y2=sin(x)./x;
y3=cos(x/sqrt(3));
plot(x,y1,'b-'); hold on;
plot(x,y2,'r-');
plot(x,y3,'g');

```

9. Fig. 3.3

```

clear;
x=0:0.05:pi/2;
y1=cos(x);
y2=(pi^2-4*x.^2)./(pi^2+4*x.^2);
y3=((pi^2-4*x.^2)./(pi^2+4*x.^2)).^(pi^2/16);
plot(x,y1,'g'); hold on;
plot(x,y2,'b-');
plot(x,y3,'r-');

```

10. Fig. 3.4

```

clear;
x=0:0.01:pi;
y1=sin(x)./x;
y2=(pi^2-x.^2)./(pi^2+x.^2);
y3=((pi^2-x.^2)./(pi^2+x.^2)).^(pi^2/12);
p1=plot(x,y1,'g'); hold on;
p2=plot(x,y2,'b. ');
p3=plot(x,y3,'r-. ');
set(p2,'Markersize',1);
set(p3,'Markersize',1);

```

11. Fig. 3.5

```

clear;
x=0:0.01:1.5;
y1=tan(x)./x;
y2=(pi^2+4*x.^2)./(pi^2-4*x.^2);
y3=((pi^2+4*x.^2)./(pi^2-4*x.^2)).^(pi^2/24);
p1=plot(x,y1,'g'); hold on;
p2=plot(x,y2,'r.- ');
p3=plot(x,y3,'b. ');
set(p2,'Markersize',4);
set(p3,'Markersize',4);

```

12. Fig. 3.6

```

clear;
x=0:0.01:1.5;
y1=cosh(x);
y2=ones(1,length(x));
y3=((pi^2+4*x.^2)./(pi^2-4*x.^2)).^(pi^2/16);
p1=plot(x,y1,'g'); hold on;
p2=plot(x,y2,'b. ');
p3=plot(x,y3,'r.- ');
set(p2,'Markersize',4);
set(p3,'Markersize',4);

```

13. Fig. 3.7

```

clear;
x=0:0.01:3;
y1=sinh(x)./x;
y2=ones(1,length(x));
y3=((pi^2+x.^2)./(pi^2-x.^2)).^(pi^2/12);
p1=plot(x,y1,'g'); hold on;
p2=plot(x,y2,'b. ');
p3=plot(x,y3,'r.- ');
set(p2,'Markersize',4);
set(p3,'Markersize',4);

```

14. Fig. 3.8

```

clear;
x=0:0.01:pi/2;
y1=tanh(x)./x;
y2=ones(1,length(x));
y3=((pi^2-4*x.^2)/(pi^2+4*x.^2)).^(pi^2/24);
p1=plot(x,y1,'g'); hold on;
p2=plot(x,y2,'b. ');
p3=plot(x,y3,'r-. ');
set(p2,'Markersize',4);
set(p3,'Markersize',4);
ylim([0 1.1]);

```

15. Fig. 3.9

```

clear;
x=0:0.01:1;
g=0.577215665;
y1=gamma(x)-(1-x)/(1+x).*(exp((1-g)*x)/(x));
p1=plot(x,y1,'b');
set(p1,'Markersize',4);

```

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Index

- Abramowitz, M., 2, 8, 9, 19, 64, 117, 162, 175, 185
 Acharya, A.P., 4
 Aczél, J., 73, 104
 admissible function method, 28
 Ahlfors, L.V., xiii
 Aksentev, L.A., 26
 Alexander duality theorem, 25
 Alexander transform, 32
 Alexander, J.W., 5, 25
 Ali, R.M., 62
 Almkvist, G., 3, 99
 Alzer, H., 75, 161
 Amos, D.E., 123
 Anderson, G.D., 3, 4, 6, 71, 73–76, 85, 88, 89, 94, 97–99, 104, 107, 141, 143
 András, S., 19, 63, 66, 68, 76, 138, 158, 189
 Andrews, G.E., 2, 3
 Apostol, T.M., 153, 158
 arithmetic mean, 86
 Askey inequality, 114
 Askey, R., 2, 3, 5, 15, 114
 Atkinson, K.E., 117
 Aumann, G., 73
 Avhadiev, F.G., 26
 Balasubramanian, R., 3, 4, 7, 62, 75, 81, 83, 88, 96, 104, 106, 112, 121
 Baricz, Á., xiv, 5–7, 10, 11, 14, 16, 19, 23, 27–30, 33, 35, 36, 38, 40, 41, 43, 45, 46, 48, 51, 53–55, 57, 59–61, 63, 66, 68, 71–73, 76, 77, 79–83, 85, 86, 88, 90–95, 99–108, 110, 112, 114–116, 118, 120–122, 125–128, 130, 133, 135, 138, 139, 143, 145–147, 149, 153, 158, 161–163, 167, 168, 172, 177, 178, 180, 181, 183, 189
 Barnard, R.W., 4, 62, 74, 88, 89
 Becker criterion for univalence, 26
 Becker, J., 26
 Bencze, M., 136
 Berndt, B.C., 3, 74–76, 99
 Bernoulli numbers, 175
 Bessel differential equation, 8
 Bessel function of the first kind, 8
 beta function, 17
 Bieberbach conjecture, 3
 Biernacki, M., 76, 120
 Bissu, S.K., 127
 Borwein, D., 105
 Borwein, J., 105
 Boucekkine, R., 98
 Brown, R.K., 44, 48
 Cao, J., 145
 Carlson, B.C., 4, 24
 Carlson-Shaffer operator, 62
 Carver, W.B., 137
 Cauchy mean value theorem, 148
 Cauchy–Bunyakovsky–Schwarz inequality, 21
 Chebyshev integral inequality, 22
 Chen, C.P., 7, 134, 162, 166, 184
 Choi, J.H., xiv, 4, 31, 61, 62
 Chu, Y., 98, 113, 141, 144
 close-to-convex function, 25
 close-to-convex function of order alpha, 25
 concave function, 72
 confluent hypergeometric function, 93
 convex function, 24, 72
 convex function of order alpha, 24
 convex function with respect to means, 105
 convex sequence, 32
 Cusa inequality, 136

- Daróczy, Z., 104
 de Branges, L., 3
 Deaño, A., 170
 Debnath, L., 141, 145
 digamma function, 74
 Duren, P.L., 24–26, 33, 58, 60, 69
 Dutka, J., 1
- Eenigenburg, P.J., 51, 58
 Elbert, Á., 167, 169
 Elliott formula, 75
 Elliott, E.B., 75
 elliptic integrals, 3, 99, 106
 Euler-Mascheroni constant, 74
 Evans, R.J., 2, 74
- Fee, G., 105
 Fejér, L., 5, 32
 Fournier, R., 62
 Frobenius method, 9
- gamma function, 2
 Gauss asymptotic formula, 74
 Gauss-Gegenbauer quadrature formula, 117
 Gaussian hypergeometric function, 1, 73
 Gegenbauer polynomials, 117
 generalized and normalized Bessel function of the first kind, 11
 generalized Bessel differential equation, 9
 generalized Bessel function of the first kind, 10
 generalized hypergeometric function, 3
 geometric mean, 86
 Gil, A., 170
 Giordano, C., 16, 126
 Girgensohn, R., 105
 Goodman, A.W., 25, 26
 Grünbaum inequality, 113
 Grünbaum, F.A., 113
 Gronau, D., 113
 Guo, B.N., 76, 142, 145, 159
- Hölder mean, 105
 Hölder-Rogers inequality, 22
 Hadamard product, 33
 Hammarwall, D., 125
 Hansen, P.A., 7
 Hao, Q.D., 141, 144
 Hardy space, 58
 Hardy, G.H., 96
 harmonic mean, 86
- Heikkala, V., 99, 120
 Hermite-Hadamard inequality, 22
 Hille, E., 26
 Huo, Z.H., 145
 Huygens, C., 136
 hyperbolic sine integral, 161
 hypergeometric differential equation, 1
- Ifantis, E.K., 167, 170, 176, 177
 integral representation of Bessel functions, 16, 19
 Ismail, M.E.H., 16, 126, 167, 172
 Iuskevici, A.P., 136
 Ivády, P., 187
- Jack's lemma, 26
 Jack, I.S., 5, 24–26, 49
 Jensen inequality, 21
 Jordan inequality, 140
 Joshi, C.M., 127
- Küstner, R., 4
 Kampé de Fériet, J., 1
 Kanas, S., 30, 48
 Kaplan, W., 5, 26
 Karatsuba, E.A., 3, 75
 Kazi, H., 76
 Keogh, F.R., 58
 Kim, Y.C., xiv, 4, 31, 61, 62
 Kishore formula, 171
 Kishore, N., 171, 175
 Kober inequality, 144
 Koumandos, S., 130, 161
 Kreyszig, E., 44
 Krzyż, J., 76, 120
- Laforgia, A., 16, 126, 169
 Landen identity, 99
 Legendre identity, 75
 Lindén, H., 99
 Littlewood, J.E., 96
 Log, 2
 log, 72
 log-concave function, 72
 log-convex function, 72
 Lorch, L., 129–131
- MacGregor, T.H., 25
 Mahajan inequality, 130

- Mahajan, A., 130
 Matkowski, J., 73, 104, 105, 113
 Mercer, A.Mcd., 114
 Merkes, E.P., 4, 51
 Miller, S.S., 4, 5, 28, 29, 46, 69
 Mitrinović inequality, 130
 Mitrinović, D.S., 21, 22, 64, 115, 128, 130, 137, 140
 Mocanu, P.T., 4, 5, 28, 29, 46, 69
 modified Bessel differential equation, 8
 modified Bessel function of the first kind, 8
 modified spherical Bessel differential equation, 9
 modified spherical Bessel function of the first kind, 9
 Muldoon, M.E., 129–131, 167, 172
 multiplicatively concave function, 103
 multiplicatively convex function, 103
 Murugusundaramoorthy, G., 4
- Naik, S., 3, 62, 75
 Nanjundiah, T.S., 127
 Nehari criterion for univalence, 26
 Nehari, Z., 26
 Neuman, E., 5–7, 15, 16, 71, 76, 97, 114, 116, 118, 120–122, 125, 126, 133, 139, 142, 189
 Nezhmetdinov, I.R., 4, 32
 Niculescu, C.P., 73, 96, 103, 105, 113
 Niu, D.W., 142, 145, 146, 149, 158, 159
 normalized function, 25
 Noshiro, K., 28
- Oppenheim, A., 137
 Owa, S., 4, 5, 26, 28, 39, 40, 69
 Ozaki, S., 5, 26, 32, 55
- Pólya, G., 96
 Pascu, M.N., 24
 Pascu, N.N., 24
 Pascu, R.N., 24
 Pečarić, J., 16, 126
 Petrović inequality, 115
 Petrović, M., 115
 Piessens, R., 167
 Pinchuk, B., 25
 Pochhammer (or Appell) symbol, 1
 Pommerenke, C., 25, 26
 Ponnusamy, S., xiv, 3, 4, 6, 7, 31, 32, 55, 57, 58, 61, 62, 74–76, 81, 83, 88, 96, 104, 106, 112, 120, 121
- Prabhakaran, D.J., 62
 pre-Schwarzian derivative, 26
 probability measure, 17, 19, 21
- Qi, F., 7, 76, 134, 141, 142, 144, 145, 149, 159, 162, 166, 184
 Qiu, S.L., 4, 7, 75, 76, 83, 98, 99, 112
 Qui, L.H., 141
- Rätz, J., 105
 Rønning, F., 4, 62
 Răducanu, D., 24
 Rainville, E.D., 2, 74
 Ramanujan differentiation formula, 75
 Ramanujan refined asymptotic formula, 74
 Rayleigh formula, 171
 Rayleigh function, 171
 Rayleigh inequalities, 170
 recursive relations for Bessel functions, 13
 recursive relations for generalized Bessel functions, 11
 recursive relations for modified Bessel functions, 13
- Redheffer inequality, 133
 Redheffer, R., 133
 Richards, K.C., 4, 74, 88, 89
 Riemann mapping theorem, 24
 Roberts, A.W., 72, 79
 Robertson, M.S., 24, 25, 51
 Roy, R., 2, 3
 Ruiz-Tamarit, J.R., 98
 Ruscheweyh, S., 4, 62
- Sándor, J., 136, 140, 142, 145, 186
 Sabapathy, S., 4
 Saigo, M., 62
 Schild, A., 25
 Schlömilch, O.X., 7
 Schwarz lemma, 69
 Schwarzian derivative, 26
 Scott, W.T., 4, 51
 second-order power mean, 86
 Segura, J., 170
 Selinger, V., 5, 15, 43
 Shaffer, D.B., 4, 24
 Shanmugam, T.N., 4
 Siafarikas, P.D., 167, 170, 176, 177
 Silverman, H., 4, 5, 51, 54
 Simpson, H.C., 120
 sine integral, 161
 Singh, V., 4, 62

- Sivasubramanian, S., 4
 Sonine integral formula, 133
 Spector, S.J., 120
 spherical Bessel differential equation, 8
 spherical Bessel function of the first kind, 9
 Srivastava, H.M., 4, 5, 26, 28, 31, 39, 40, 61, 69, 145
 Stankiewicz, J., 30, 48
 starlike function, 24
 starlike function of order α , 24
 Stegun, I.A., 2, 8, 9, 19, 64, 117, 162, 175, 185
 Stromberg, K.R., 148, 156
 Sugawa, T., xiv, 98
 Sun, J., 7, 166, 173–175, 180, 181
 Sun, Y., xiv
 Swaminathan, A., 4
 Szegő, G., 113, 118

 Temme, N.M., 2
 Thiruvengkatachar, V.K., 127
 Todd, J., 44
 Trif, T., 113
 Turán-type inequality, 127

 Vamanamurthy, M.K., 3, 4, 6, 71, 73–76, 85, 88, 89, 94, 97–99, 104, 107, 120, 141, 143

 van der Corput inequality, 128
 Varberg, D.E., 72, 79
 Vuorinen, M., 3, 4, 6, 7, 31, 32, 55, 57, 62, 71, 73–76, 81, 83, 85, 88, 89, 94, 96–99, 104, 106, 107, 112, 120, 121, 141, 143, 184

 Wang, G., 98, 113, 141, 144
 Warschawski, S.E., 28
 Watson, G.N., xiii, 2, 5, 8, 13, 16, 19, 63, 74, 127, 133, 148, 163, 164, 169–171, 179
 Whittaker, E.T., 2, 74
 Wilf, H.S., 44
 Wu, S., 7, 72, 145, 146, 153, 158, 162, 167, 177, 181, 189

 Xu, S.L., 141

 Zhang, X., 98, 113, 141, 144
 Zhao, C.J., 141
 Zhao, J.W., 7, 134, 162, 166, 184
 Zhu, L., 7, 141, 145, 146, 149–152, 160, 166, 173–175, 180, 181
 Zygmund, A., 161

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