Appendix

In this Appendix we collect some useful facts related to the almost-analytic extension of a smooth function and give, following Dimassi-Sjöstrand [7], the proof of the Dyn'kin-Helffer-Sjostrand formula (12.48).

A.1 Almost-Analytic Extension and the Dyn'kin–Helffer–Sjöstrand Formula

Let $x, y \in \mathbb{R}$, and let $z = x + iy \in \mathbb{C}$. We denote by $L(dz) = dxdy$ the Lebesgue measure of $\mathbb{C}$, so that $dz \wedge d\bar{z} = -2idx \wedge dy = -2iL(dz)$, and write $\partial/\partial\bar{z} = (\partial/\partial x + i\partial/\partial y)/2$.

In the first place we recall the *complex Gauss-Green formula*, whose proof is easily deduced from the classical Gauss-Green formula when applied to the real and imaginary part of a complex-valued function.

Lemma A.1.1 (Complex Gauss-Green formula). *Let $D \subset \Omega \subset \mathbb{C}$ be open sets such that D has a C^1 boundary ∂D in Ω. Let $f \in C_0^1(\Omega)$. Then with the proper orientations (that is, with ∂D oriented in such a way that D is kept to the left) one has*

$$\int_{\partial D} f(z)dz = -\iint_D \frac{\partial f}{\partial \bar{z}}(z)dz \wedge d\bar{z} = 2i \iint_D \frac{\partial f}{\partial \bar{z}}(z)L(dz). \tag{A.1}$$

Notice that (A.1) may also be rewritten as

$$\langle \frac{\partial \mathbf{1}_D}{\partial \bar{z}} | f \rangle = \frac{i}{2} \int_{\partial D} f(z)dz, \;\; f \in C_0^1(\Omega), \tag{A.2}$$

where the left-hand side denotes the distribution duality, and $\mathbf{1}_D$ is the characteristic function of the set D.

Next, following Hörmander [28], we recall the following result about the fundamental solution of the $\partial/\partial\bar{z}$-operator.

Lemma A.1.2. *With the notation of Lemma A.1.1, let $\zeta \in D$. Then, with the proper orientations,*

$$f(\zeta) = -\frac{1}{\pi}\iint_D \frac{\partial f}{\partial \bar{z}}(z)(z-\zeta)^{-1}L(dz) + \frac{1}{2\pi i}\int_{\partial D} f(z)(z-\zeta)^{-1}dz, \forall f \in C_0^1(\Omega). \tag{A.3}$$

In particular, when $D = \Omega$ there is no curvilinear integral in (A.3), that is,

$$f(\zeta) = -\frac{1}{\pi}\iint_\Omega \frac{\partial f}{\partial \bar{z}}(z)(z-\zeta)^{-1}L(dz), \ \ \forall f \in C_0^1(\Omega), \tag{A.4}$$

and, furthermore, considering $(x,y) \longmapsto E_\zeta(x,y) = \pi^{-1}(z-\zeta)^{-1}$, which is a locally integrable function, gives

$$\frac{\partial E_\zeta}{\partial \bar{z}} = \delta_\zeta, \tag{A.5}$$

that is, E_ζ is a fundamental solution of $\partial/\partial\bar{z}$.

Proof. For $\zeta \in D$, we apply the complex Gauss-Green formula (A.1) to the function $f(z)/(z-\zeta)$, with D replaced by $D \setminus B_\varepsilon$, where B_ε is a disc of radius ε with center at ζ, and ε is picked small. Hence

$$2i\iint_{D\setminus B_\varepsilon} \frac{\partial f}{\partial \bar{z}}(z)(z-\zeta)^{-1}L(dz) = \int_{\partial D} f(z)(z-\zeta)^{-1}dz - \int_{\partial B_\varepsilon} f(z)(z-\zeta)^{-1}dz,$$

with the proper orientations (i.e. ∂B_ε is oriented in the formula as the boundary of B_ε). Since

$$\int_{\partial B_\varepsilon} f(z)(z-\zeta)^{-1}dz = f(\zeta)\int_{\partial B_\varepsilon}(z-\zeta)^{-1}dz + O(\varepsilon) \longrightarrow 2\pi i f(\zeta), \text{ as } \varepsilon \to 0+,$$

letting $\varepsilon \to 0+$ gives (A.3). □

We now pass to the proof of Theorem 12.2.1. In the first place we show that given a function $f \in C_0^2(\mathbb{R})$, it is always possible to find an extension $\tilde{f} \in C_0^1(\mathbb{C})$ such that $\partial\tilde{f}/\partial\bar{z} = O(|\mathrm{Im}\, z|)$, as in the statement of Theorem 12.2.1, and for which formula (A.4) holds true. We have in fact the following lemma.

Lemma A.1.3. *Given any $f \in C_0^2(\mathbb{R})$ there exists $\tilde{f} \in C_0^1(\mathbb{C})$ such that*

$$\tilde{f}\big|_{\mathbb{R}} = f, \quad \textit{and} \quad \frac{\partial \tilde{f}}{\partial \bar{z}}(z) = O(|\mathrm{Im}\, z|). \tag{A.6}$$

Moreover, one has

$$f(t) = -\frac{1}{\pi}\iint_{\mathbb{C}} \frac{\partial f}{\partial \bar{z}}(z)(z-t)^{-1}L(dz), \ \ \forall t \in \mathbb{R}. \tag{A.7}$$

Proof. We denote by $g^{(k)}$ the kth-derivative with respect to x or y of a function g. Take $\chi \in C_0^\infty(\mathbb{R})$ with $0 \le \chi \le 1$, $\chi\big|_{|y|\le 1} = 1$, $\chi\big|_{|y|\ge 2} = 0$. Then define

$$\tilde{f}(x+iy) := \Big(f(x) + iyf^{(1)}(x)\Big)\chi(y).$$

It is clear that $\tilde{f} \in C_0^1(\mathbb{C})$. One then computes

$$\frac{\partial \tilde{f}}{\partial x}(x+iy) = \Big(f^{(1)}(x) + iyf^{(2)}(x)\Big)\chi(y),$$
$$\frac{\partial \tilde{f}}{\partial y}(x+iy) = if^{(1)}(x)\chi(y) + \Big(f(x) + iyf^{(1)}(x)\Big)\chi^{(1)}(y),$$

so that

$$\frac{\partial \tilde{f}}{\partial \bar{z}}(z) = \frac{y}{2}\Big(if^{(2)}(x)\chi(y) - f^{(1)}(x)\chi^{(1)}(y)\Big) + if(x)\chi^{(1)}(y). \tag{A.8}$$

Since

$$\operatorname{supp}\chi^{(1)} \subset \{y;\ 1 \le |y| \le 2\},$$

we notice that

$$\Big|\frac{\chi^{(1)}(y)}{y}\Big| \le |\chi^{(1)}(y)| \le C,$$

whence we may write

$$\frac{\partial \tilde{f}}{\partial \bar{z}}(z) = \frac{y}{2}\Big(if^{(2)}(x)\chi(y) - f^{(1)}(x)\chi^{(1)}(y)\Big) + iyf(x)\frac{\chi^{(1)}(y)}{y}.$$

which proves (A.6).

The fact that formula (A.7) holds, follows immediately from Lemma A.1.2 by taking $\Omega = \mathbb{C}$ and $\zeta = x \in \mathbb{R}$. □

We next prove Theorem 12.2.1.

Proof (of Theorem 12.2.1). Denote by $Q \in \mathscr{L}(H,H)$ the right-hand side of (12.48). For $u, v \in H$ consider

$$((z-P)^{-1}u, v) = \int (z-t)^{-1}(dE(t)u, v),$$

where $t \mapsto E(t)$ is the spectral family associated with P. It follows that

$$(Qu, v) = -\frac{1}{\pi}\iint_{\mathbb{C}} \frac{\partial \tilde{f}}{\partial \bar{z}}(z)\Big(\int (z-t)^{-1}(dE(t)u, v)\Big)L(dz) =$$

(by Fubini's theorem, using the fact that $\tilde{f}$ is compactly supported and that $\int dE(t) = \mathrm{Id}_H$)

$$= \int \Big(-\frac{1}{\pi} \iint_{\mathbb{C}} \frac{\partial \tilde{f}}{\partial \bar{z}}(z)(z-t)^{-1} L(dz) \Big) (dE(t)u, v) = \int f(t)(dE(t)u, v) = f(P),$$

which concludes the proof. □

However, when P is a pseudodifferential operator and when dealing with the pseudodifferential nature of $f(P)$, as in the proof of Theorem 12.2.2, one does need to consider almost-analytic extensions of a given $f \in C_0^\infty(\mathbb{R})$. The next lemma grants the existence of such almost-analytic extension. (This result also holds, by using a locally-finite partition of unity of $\mathbb{R}$, for functions belonging to $C^\infty(\mathbb{R})$. See Treves [70, Chapter X, Section 2], for more on this.)

Lemma A.1.4. *Let $f \in C_0^\infty(\mathbb{R})$. There exists $\tilde{f} \in C_0^\infty(\mathbb{C})$, called an* **almost-analytic extension** *of f, such that*

$$\tilde{f}|_{\mathbb{R}} = f, \text{ and } \forall N \geq 1 \ \exists C_N > 0 \text{ such that } \Big| \frac{\partial \tilde{f}}{\partial \bar{z}}(z) \Big| \leq C_N |\mathrm{Im}\, z|^N, \forall z \in \mathbb{C}, \quad \text{(A.9)}$$

$$\mathrm{supp}\, \tilde{f} \subset \{ z = x + iy \in \mathbb{C};\ x \in \mathrm{supp}\, f,\ |y| \leq C \},$$

where, recall, $\partial/\partial\bar{z} = (\partial/\partial x + i\partial/\partial y)/2$. In addition one has

$$f(t) = -\frac{1}{\pi} \iint_{\mathbb{C}} \frac{\partial \tilde{f}}{\partial \bar{z}}(z) \big(z-t\big)^{-1} L(dz),\ \ t \in \mathbb{R}. \quad \text{(A.10)}$$

Proof. Take $\chi \in C_0^\infty(\mathbb{R})$ as in the proof of Lemma A.1.3. One can then choose a monotone increasing sequence $R_k \nearrow +\infty$ growing sufficiently fast so as to have that the series

$$\tilde{f}(x+iy) := \sum_{k \geq 0} \frac{f^{(k)}(x)}{k!} (iy)^k \chi(R_k y)$$

is uniformly convergent, along with the series of the derivatives to all orders. One now computes

$$\frac{\partial \tilde{f}}{\partial x}(z) = \sum_{k \geq 0} \frac{f^{(k+1)}(x)}{k!} (iy)^k \chi(R_k y),$$

and

$$i \frac{\partial \tilde{f}}{\partial y}(z) = \sum_{k \geq 0} \frac{f^{(k+1)}(x)}{k!} i^{k+2} y^k \chi(R_{k+1} y) + \sum_{k \geq 0} \frac{f^{(k)}(x)}{k!} (iy)^k R_k \chi^{(1)}(R_k y),$$

obtaining

$$\frac{\partial \tilde{f}}{\partial \bar{z}}(z) = \frac{1}{2}\sum_{k\geq 0}\frac{f^{(k+1)}(x)}{k!}(iy)^k\Big(\chi(R_k y)-\chi(R_{k+1}y)\Big) + \frac{i}{2}\sum_{k\geq 0}\frac{f^{(k)}(x)}{k!}(iy)^k R_k \chi^{(1)}(R_k y).$$

Since

$$\operatorname{supp}\chi^{(1)}(R_k\cdot) \subset \{y;\, R_k^{-1} \leq |y| \leq 2R_k^{-1}\},$$

and

$$\operatorname{supp}(\chi(R_k\cdot)-\chi(R_{k+1}\cdot)) \subset \{y;\, R_{k+1}^{-1} \leq |y| \leq R_k^{-1}\},$$

one sees that (A.9) holds.

Formula (A.10) follows as before from Lemma A.1.2. This completes the proof. □

Main Notation

B.1 General Notation

- $\mathbb{N} = \{1,2,\ldots\}$, $\mathbb{Z}_+ = \{0,1,2,\ldots\}$, $\mathbb{R}_+ = (0,+\infty)$, and $\overline{\mathbb{R}}_+ = [0,+\infty)$.
- We denote by $\sharp A$ or by $\mathrm{card} A$ the cardinality of the set A.
- As usual, given functions f and g, we write $f \sim g$ *as* $x \to x_0$ when $\lim_{x\to x_0} \dfrac{f(x)}{g(x)} = 1$.
- Let $A, B > 0$. We write $A \lesssim B$ (or equivalently $B \gtrsim A$) when there is a universal constant $C > 0$ such that $A \le CB$. We therefore write $A \approx B$ whenever $A \lesssim B$ and $B \lesssim A$.
- Given sequences $\{A_j\}_j$, $\{B_j\}_j \subset \mathbb{R}$, we write

  $$A_j \lesssim B_j, \ \ as \ \ j \to +\infty$$

  (or equivalently $B_j \gtrsim A_j$ *as* $j \to +\infty$) if there are constants $C > 0$ and $j_0 \in \mathbb{N}$ such that

  $$A_j \le CB_j, \ \ \forall j \ge j_0.$$

  We hence write

  $$A_j \approx B_j, \ \ as \ \ j \to +\infty,$$

  when $A_j \lesssim B_j$ and $B_j \lesssim A_j$ as $j \to +\infty$.
- The $N \times N$ identity matrix is denoted by I, regardless N, or by I_N, or by $I_{\mathbb{C}^N}$.
- The norm of a vector v belonging to $\mathbb{R}^N$, resp. $\mathbb{C}^N$, is denoted by $|v|$, resp. $|v|_{\mathbb{C}^N}$. The inner product in $\mathbb{R}^N$, resp. the Hermitian product in $\mathbb{C}^N$, of vectors v, v', is denoted by $\langle v, v' \rangle$, resp. $\langle v, v' \rangle_{\mathbb{C}^N}$.
- The canonical inner product in L^2 is denoted by $(\cdot,\cdot)$, the induced norm by $\|\cdot\|_0$.
- The ring of $N \times N$ complex matrices is always denoted by M_N.
- An **excision** function is a C^∞ function χ with $0 \le \chi \le 1$, supported away from the origin and $\equiv 1$ outside a large compact set. The standard one used in these notes is such that $\chi \equiv 0$ for $|X| \le 1/2$ and $\chi \equiv 1$ for $|X| \ge 1$.
- The duality between $\mathscr{S}$ and $\mathscr{S}'$ is denoted by $\langle u | \varphi \rangle_{\mathscr{S}',\mathscr{S}}$, where $u \in \mathscr{S}'$ and $\varphi \in \mathscr{S}$.

- The set of linear continuous maps between Hilbert (or Banach, or Fréchet) spaces H_1 and H_2 is denoted by $\mathscr{L}(H_1, H_2)$.
- We write Tr for the operator-trace, and Tr for the $N \times N$ matrix-trace.
- The symbol of the standard harmonic oscillator $p_0(x,\xi) = (|\xi|^2 + |x|^2)/2$, also written $p_0(X) = |X|^2/2$, with $X = (x,\xi)$.
- The matrices J and K:

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad K = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

- We say that $f(h) = O(h^{N_0})$ for some $N_0 \in \mathbb{Z}_+$ if there exists $C_{N_0} > 0$ such that $|f(h)| \le C_{N_0} h^{N_0}$. We say that $f(h) = O(h^\infty)$ if for any given $N_0 \in \mathbb{Z}_+$ one has $f(h) = O(h^{N_0})$.
- The characteristic function of a set V is denoted by $\mathbf{1}_V$.
- $\hookrightarrow\hookrightarrow$ denotes compact embedding.

B.2 Symbol, Function and Operator Spaces

- The symbol space $S(m,g)$ in the Weyl-Hörmander calculus, see Definition 3.1.6.
- The global weight-function $m(X) = (1+|X|^2)^{1/2}$ and the global metric $g_X = |dX|^2/m(X)^2$, $X \in \mathbb{R}^{2n}$, see (3.13).
- The symbol space $S_{\mathrm{cl}}(m^\mu, g)$, see Definition 3.2.3.
- The global polynomial differential (GPD) symbols, see Definition 3.2.6 and also Definition 3.2.9.
- The smoothing symbol class $S(m^{-\infty}, g)$, see Definition 3.2.14.
- The function space B^s, see Definition 3.2.25 and Proposition 3.2.26.
- The symbol space $S(\mu, r)$, see Definition 6.1.1.
- The set $\mathrm{OP}S_{\mathrm{cl}}(\mu, r)$ of pseudodifferential operators, see Definition 6.1.2.
- The semiclassical symbol space $S^k_\delta(m^\mu, g)$, see (9.1) of Definition 9.1.1.
- The semiclassical symbol space $S^k_\delta(m^\mu)$, see (9.2) of Definition 9.1.1.
- The smoothing semiclassical symbol class $S^{-\infty}(m^{-\infty}, g)$, see (9.7).
- The smoothing semiclassical symbol class $S^{-\infty}(m^\mu)$, see (9.8).
- The class of classical semiclassical symbols $S^k_{\mathrm{cl}}(m^\mu)$, see Point 1. of Definition 9.1.9.
- The class of classical semiclassical symbols $S^k_{0,\mathrm{cl}}(m^\mu, g)$, see Point 2. of Definition 9.1.9.
- The class of semiclassical GPD symbols, see Definition 9.4.1.
- The isometry of L^2, also automorphism of $\mathscr{S}'$ and $\mathscr{S}$ ($E > 0$)

$$U_E \colon u(x) \longmapsto E^{-n/4} u(x/\sqrt{E}),$$

see (9.4).

B.3 The Spectral Counting Function and the Spectral ζ-Function

- The spectral counting functions $N(\lambda)$ and $N_0(\lambda)$, see (4.4).
- The spectral ζ-function $\zeta_A(\lambda)$ associated with a positive operator A with a discrete spectrum, see (4.10).

B.4 Dynamical Quantities and Assumptions

- Hypotheses $(H1)$-$(H3)$, see Assumption 11.1.1.
- Hypotheses $(H4)$ and $(H5)$, see Theorem 12.2.4.
- Hypothesis $(H5')$, see Proposition 12.2.17 (and just a few lines above it).
- Hypothesis $(H6)$, see Theorem 12.2.16.
- The functions $J_p(E)$ and $T_p(E)$ associated with a periodic Hamiltonian trajectory of a symbol p at energy E, see Lemma 11.1.2.
- The averaged action-integral $A_p(E)$, see Definition 11.1.5.
- The Leray-Liouville measure $L_{f,\lambda}(dX)$ associated with the hypersurface $f^{-1}(\lambda)$, see Definition 12.2.11, Proposition 12.2.14 and Corollary 12.2.15.

B.5 Classes of Systems

- Non-commutative harmonic oscillator (for short NCHO), see Definition 3.2.11.
- The class $\mathcal{Q}_2$, see Definition 12.1.1.
- The class $\mathcal{Q}_2^s$, see Definition 12.1.2.

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LECTURE NOTES IN MATHEMATICS

Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

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