Appendix A
Galois Connections

Let $\mathcal{M}$ be a partially ordered set, with order relation $\subseteq$. A *closure* on $\mathcal{M}$ is a map $X \mapsto \overline{X}$ from $\mathcal{M}$ to $\mathcal{M}$ such that: (i) $X \subseteq \overline{X}$; (ii) $\overline{\overline{X}} = \overline{X}$; (iii) $X \subseteq Y$ implies $\overline{X} \subseteq \overline{Y}$. An element $X \in \mathcal{M}$ is said to be *closed* whenever $X = \overline{X}$. Let $\mathcal{C}(\mathcal{M})$ be the set of all closed elements of $\mathcal{M}$, with the induced order. Then if $\mathcal{M}$ is a complete lattice (see Section 1.1.2), so is $\mathcal{C}(\mathcal{M})$ with respect to the lattice operations ($\mathcal{N} \subseteq \mathcal{C}(\mathcal{M})$)

$$\left.\bigwedge_{X \in \mathcal{N}} X\right|_{\mathcal{C}(\mathcal{M})} = \left.\bigwedge_{X \in \mathcal{N}} X\right|_{\mathcal{M}},$$

$$\left.\bigvee_{X \in \mathcal{N}} X\right|_{\mathcal{C}(\mathcal{M})} = \overline{\left(\left.\bigvee_{X \in \mathcal{N}} X\right|_{\mathcal{M}}\right)}.$$

Let now $\mathcal{L}$ and $\mathcal{M}$ be two partially ordered sets. A *Galois connection* [160, Bir66] between $\mathcal{L}$ and $\mathcal{M}$ is a pair of maps $\alpha : \mathcal{L} \to \mathcal{M}$ and $\beta : \mathcal{M} \to \mathcal{L}$, such that (we write $X^\alpha := \alpha(X)$ and $Y^\beta := \beta(Y)$:

(i) both α and β reverse order,
(ii) $S \subseteq S^{\alpha\beta}$ for each $S \in \mathcal{L}$ and $T \subseteq T^{\beta\alpha}$ for each $T \in \mathcal{M}$.

It follows from the definition that $S \mapsto S^{\alpha\beta}$ (resp. $T \mapsto T^{\beta\alpha}$) is a closure on $\mathcal{L}$ (resp. $\mathcal{M}$).

From now on, we will assume that both $\mathcal{L}$ and $\mathcal{M}$ are complete lattices. So are then the two sets of closed elements $\mathcal{C}(\mathcal{L})$ and $\mathcal{C}(\mathcal{M})$. Furthermore, α (resp. β) is a lattice anti-isomorphism of $\mathcal{C}(\mathcal{L})$ onto $\mathcal{C}(\mathcal{M})$ [resp. $\mathcal{C}(\mathcal{M})$ onto $\mathcal{C}(\mathcal{L})$], that is, for every subset $\mathcal{N} \subseteq \mathcal{C}(\mathcal{L})$, one has

$$\Big(\bigvee_{X \in \mathcal{N}} X\Big)^\alpha = \bigwedge_{X \in \mathcal{N}} X^\alpha, \tag{A.1}$$

$$\Big(\bigwedge_{X \in \mathcal{N}} X\Big)^\alpha = \bigvee_{X \in \mathcal{N}} X^\alpha, \tag{A.2}$$

and similarly for β. Actually [165], in the case where both $\mathcal{L}$ and $\mathcal{M}$ are complete lattices, the two maps α and β are not independent: α generates a Galois connection if, and only if, it satisfies the single condition (A.1); β is then uniquely determined and given by

$$T^\beta = \bigvee_{X^\alpha \geqslant T} X.$$

Further insight into the structure of Galois connections can be found in the paper of Shmuely [178]. Two points are of interest for us.

(1) Galois correspondences between the complete lattices $\mathcal{L}$ and $\mathcal{M}$ are in 1-1 correspondence with certain subsets of $\mathcal{L} \times \mathcal{M}$, called G-ideals. Since the latter form a complete lattice with the natural order inherited $\mathcal{L} \times \mathcal{M}$, it follows that the set of all Galois maps $\alpha : \mathcal{L} \to \mathcal{M}$ that generate a Galois connection also form a complete lattice.

(2) Every Galois connection between $\mathcal{L}$ and $\mathcal{M}$ can be generated by a *binary relation*, that is a subset $\Upsilon \subseteq \mathcal{L} \times \mathcal{M}$; for instance $\Upsilon = \{(S,T) : T \leqslant S^\alpha\}$. Conversely, every binary relation $\Upsilon \subseteq \mathcal{L} \times \mathcal{M}$ generates a Galois connection, namely the one that corresponds to the minimal G-ideal generated (by lattice operations) by $\Upsilon, (0_\mathcal{L}, 1_\mathcal{M})$ and $(1_\mathcal{L}, 0_\mathcal{M})$ where 0, resp. 1, denotes the smallest, resp. largest, element of the lattice indicated.

Next we specialize the discussion to the case $\mathcal{L} = \mathcal{M} = \mathcal{P}(S)$, the complete lattice of all subsets of a given set S. If we assume furthermore that $\alpha = \beta$ (such an α is called an *involution*), the resulting self-dual Galois connection on $\mathcal{P}(S)$ is exactly what was called *compatibility on* S in Definition 1.1.4. Indeed $\alpha = \beta$ is equivalent to the corresponding binary relation Υ being symmetric: $(X, Y) \in \Upsilon$ if and only if $(Y, X) \in \Upsilon$, which we can write, as usual, $X \# Y$ (with $\# \equiv \alpha = \beta$). The closed elements of $\mathcal{P}(S)$ are precisely the *assaying subspaces*, which constitute the complete lattice $\mathcal{F}(S, \#)$. The map $\#$ of $\mathcal{F}(S, \#)$ onto itself is an involution and a lattice anti-isomorphism. Property (1) above means that the set $\mathrm{Comp}(S)$ of all compatibilities on S is in a 1-1 correspondence with the set of all symmetric G-ideals of $\mathcal{P}(S) \times \mathcal{P}(S)$ and the latter is a complete lattice with respect to the order inherited from $\mathcal{P}(S) \times \mathcal{P}(S)$. That order is exactly the notion of the weak comparability ("weakly finer", etc.) introduced in Section 1.5. Property (2) yields the notion of generating subset for a Galois connection. These are exactly our *generating subsets*, discussed in Section 1.4.

Finally, we come back to the linear case. Let V be a vector space and $\#$ a linear compatibility on V. By the very definition, the relation $f \# g$ $(f, g \in V)$ is equivalent to $[f] \# [g]$, where $[f]$ is the one-dimensional subspace generated by f. Thus we may take as complete lattice $\mathcal{L}(V)$ the set of all vector subspaces of V. A linear compatibility on V is the same thing as a self-dual (or involutive) Galois map on $\mathcal{L}(V)$. The whole discussion above then goes through.

Note. For lattice theory, we refer to Birkhoff [Bir66]. For Galois connections, see also Ore [160], Pickert [165] or Shmuely [178]. The consideration of Galois connections in the PIP-space context was first made in Antoine–Gustafson [20].

Appendix B
Some Facts About Locally Convex Spaces

In this appendix, we will recall some basic definitions and facts concerning locally convex topological vector spaces (LCS), i.e., topological vector spaces (TVS) which have a base of neighborhoods of zero consisting of convex sets, or equivalently, spaces with a topology that can be defined in terms of a family of seminorms. Our reference is the textbook of Köthe [Köt69], except for the notation of the different topologies, where we follow Schaefer [Sch71].

B.1 Completeness

A LCS $E[\mathsf{t}]$ is *complete* if every Cauchy net has a limit in E; it is *quasi-complete* if every closed bounded set in $E[\mathsf{t}]$ is complete; it is sequentially complete if every Cauchy sequence has a limit in E. Of course, completeness $\Rightarrow$ quasi-completeness $\Rightarrow$ sequential completeness, and the three notions coincide for metrizable spaces, i.e., Banach or Fréchet spaces (a Fréchet space is a complete metrizable LCS).

B.2 Dual Pairs and Canonical Topologies

Two vector spaces E, F form a dual pair $\langle E, F\rangle$ if there is a bilinear form $\langle\cdot|\cdot\rangle$ on $E \times F$, separating in both arguments: $\langle e|f\rangle = 0, \forall\, f \in F$, implies $e = 0$, $\langle e|f\rangle = 0, \forall\, e \in E$, implies $f = 0$. For any LCS E, with dual E', that is, the space of continuous linear functionals on E, $\langle E, E'\rangle$ is a dual pair. So is $\langle E, E^{\times}\rangle$, where $E^{\times}$ denotes the space of continuous conjugate linear functionals on E.

Given the dual pair $\langle E, F\rangle$, the weak topology $\sigma(E, F)$ is the coarsest topology on E for which the linear forms $e \mapsto \langle e|f\rangle,\ f \in F$, are continuous, and in fact, for which the dual of E is F. It is locally convex and Hausdorff. A basis of neighborhoods of zero consists of the sets S^o, where S runs over all finite subsets of F, and $S^o := \{e \in E : |\langle e|f\rangle| \leqslant 1, \forall\, f \in F\}$ is the (absolute) polar of $S \subset F$. The weak topology on F, $\sigma(F, E)$, is defined similarly.

The Mackey topology $\tau(E,F)$ can be defined as the finest topology on E such that the dual is F (its existence is the content of the Mackey-Arens theorem); a basis of neighborhoods of zero is given by the sets T^o, where T runs over all absolutely convex, $\sigma(F,E)$-compact subsets of F.

The strong topology $\beta(F,E)$ is defined by the basis of neighborhoods of zero $\{U^o\}$ where U runs over all absolutely convex $\sigma(F,E)$-closed and bounded subsets of F. We recall that a subset B of a LCS E is *bounded* if, for every neighborhood of zero U, there is a $\rho > 0$ such that $B \subset \rho U$.

A topology $\mathsf{t}(E)$ on E is called a *topology of the dual pair* $\langle E, F\rangle$ if the dual of $E[\mathsf{t}(E)]$ is F. Then one has, for any topology $\mathsf{t}(E)$ of the dual pair:

$$\sigma(E,F) \prec \mathsf{t}(E) \prec \tau(E,F) \prec \beta(E,F).$$

Thus, by definition, the Mackey topology is a topology of the dual pair, while the strong one is not, in general. If we start with a given topology $\mathsf{t}(E)$ on E we have the same inclusions with $F = E'$ (equivalently, with $F = E^{\times}$).[1]

In a dual pair $\langle E, F\rangle$, several classes of subsets of E depend only on the dual pair and *not* on the topology of E, i.e., they coincide for all topologies of the dual pair. Such are: closed subspaces, convex closed subsets, dense and total subsets, bounded subsets.

A LCS $E[\mathsf{t}]$ is *barreled* if $\mathsf{t}(E) = \tau(E,E') = \beta(E,E')$. A metrizable LCS always carries its Mackey topology, $\mathsf{t}(E) = \tau(E,E')$, but need not be barreled. A complete metrizable LCS, i.e., a Banach or a Fréchet space, is always barreled. The strong dual of a metrizable LCS is a complete (DF)-space. This class, whose definition is quite technical, contains also all normed spaces (see [Köt69, Sec. 29.3]).

A LCS E is *bornological* if every seminorm that is bounded on bounded sets is continuous. Every metrizable space is bornological, but a bornological space need not be barreled.

A barreled LCS $E[\mathsf{t}]$ is called a *Montel space* if every bounded subset of E is relatively compact. A Montel space is necessarily quasi-complete and reflexive (see below), and its strong dual is also a Montel space. An infinite dimensional Banach space cannot be Montel. Typical examples are $\omega, \varphi, \mathcal{S}(\mathbb{R}^n), \mathcal{S}^{\times}(\mathbb{R}^n)$.

B.3 Linear Maps

Let $E[\mathsf{t}(E)]$, $F[\mathsf{t}(F)]$ be locally convex spaces with topologies $\mathsf{t}(E)$ and $\mathsf{t}(F)$ and duals E', F', respectively. Let $\alpha : E \to F$ be a linear map. Consider the following statements

[1] In fact, all the statements that follow remain valid if one replaces the dual E' by the conjugate dual $E^{\times}$.

(i) α is continuous from $E[\mathsf{t}(E)]$ into $F[\mathsf{t}(F)]$.
(ii) α is continuous from $E[\tau(E, E')]$ into $F[\tau(F, F')]$.
(iii) α is continuous from $E[\sigma(E, E')]$ into $F[\sigma(F, F')]$.
(iv) There exists a linear map $\alpha' : F' \to E'$ such that

$$\langle \alpha(f)|g\rangle = \langle f|\alpha'(g)\rangle, \quad \forall f \in E, g \in F'.$$

(v) The sesquilinear form $b(f, g) = \langle f|\alpha'(g)\rangle$ is separately continuous in each of its arguments $f \in E, g \in F'$.

Then (ii)-(v) are equivalent and (i) implies all of them. The map α', which is automatically Mackey and weakly continuous, is called the *adjoint* or *transposed* map of α. For Banach or Fréchet spaces, the five statements are obviously equivalent. This also happens if the spaces under consideration are *reflexive* (see below).

B.4 Reflexivity

Given an LCS E, the canonical topologies $\sigma(E', E), \tau(E', E), \beta(E', E)$ are defined in the same way; thus, with the notation of Section 2.3:

$$E'|_\beta \hookrightarrow E'|_\tau \hookrightarrow E'|_\sigma .$$

By definition $\left(E'|_\tau \right)' = E$, but the dual of the strong dual, called the *bidual*, $E'' = (E'|_\beta)'$ may be strictly larger than E. A LCS E is called *semi-reflexive* if E'' coincides with E as a vector space. E is called *reflexive* if, in addition, the strong bidual $E''[\beta(E'', E')] := (E'|_\beta)'|_\beta$ coincides with E as a TVS. The two notions are different in general, but they coincide for Fréchet spaces. In fact, for a Fréchet or a Banach space E, the following properties are equivalent: (i) E is reflexive, (ii) E is semi-reflexive; (iii) E is weakly quasi-complete; and (iv) the strong dual $E'|_\beta$ is reflexive. Notice that an incomplete normed or metrizable space can never be reflexive.

A dual pair $\langle E, F\rangle$ is *reflexive* if each space is the strong dual of the other: $E|'_\beta = F$, $F|'_\beta = E$. Equivalently, if $E|_\tau$ and $F|_\tau$ are both semi-reflexive, or if they are both barreled: $\tau(E, F) = \beta(E, F)$ and $\tau(F, E) = \beta(F, E)$. In a reflexive pair, both spaces are reflexive and quasi-complete for their weak and their Mackey (= strong) topology.

B.5 Projective Limits

Let be given a vector space E, a family $\{E_\alpha\}$ of LCS and maps $i_\alpha : E \to E_\alpha$ such that, for every nonzero $x \in E$, there is some α with $i_\alpha(x) \neq 0$. Then there is a coarsest topology on E that makes all the maps i_α continuous; it is

called the *projective topology* and E with this topology, $E_{\mathrm{proj}} := \varprojlim_\alpha E_\alpha$, is called the *projective limit* of $\{E_\alpha\}$ with respect to the maps i_α. The projective limit is said to be *reduced* if $i_\alpha(E)$ is dense in E_α for each α (this can always be achieved without restriction of generality). The following properties are useful:

- E_{proj} is complete (resp. quasi-complete, sequentially complete) if every E_α is.
- Given any LCS Y, a linear map $t : Y \to E_{\mathrm{proj}}$ is continuous if, and only if, each composed map $t_\alpha = i_\alpha \circ t : Y \to E_\alpha$ is continuous.

The following examples are important:

(i) Let E be a LCS, H a subspace of E. The *subspace topology* on H is the projective topology with respect to the embedding $i : H \to E$.

(ii) Let $\{E_\alpha\}$ be as above and $E = \prod_\alpha E_\alpha$ the product vector space, i.e., the set-theoretic product $\{x = (x_\alpha)\}$, $x_\alpha \in E_\alpha$, with addition and scalar multiplication defined componentwise. The *product topology* on E is the projective topology with respect to the projection maps $p_\alpha : E \to E_\alpha$.

(iii) In a general projective limit, E_{proj} is isomorphic to a closed subspace of the product $\prod_\alpha E_\alpha$. In the case considered here, $\{E_\alpha\}$ is a family of vector subspaces of a given vector space V, each of which is itself a LCS. Then $E_{\mathrm{proj}} = \varprojlim_\alpha E_\alpha$ is the subspace $\cap_\alpha E_\alpha$ with the projective topology. E_{proj} is metrizable if, and only if, the family $\{E_\alpha\}$ contains a cofinal countable subfamily of metrizable spaces (this makes sense since the subspaces are partially ordered by inclusion).

(iv) In particular, if the family $\{E_\alpha\}$ consists of a countable family $\{\mathcal{H}_n, n \in \mathbb{N}\}$ of Hilbert spaces, with mutually consistent norms (see Section 2.2), then $E_{\mathrm{proj}} = \varprojlim_n \mathcal{H}_n$ is called a *countably Hilbert space*. Such spaces and their relation to PIP-spaces have been studied by Antoine–Karwowski [22].

(v) A countably Hilbert space $E_{\mathrm{proj}} = \varprojlim_n \mathcal{H}_n$ is said to be *nuclear* if, for every $\mathcal{H}_n$, there is a larger $\mathcal{H}_m$ such that the embedding $E_{mn} : \mathcal{H}_n \to \mathcal{H}_m$ is a nuclear (i.e., trace class) operator or, equivalently, a Hilbert-Schmidt operator. Such nuclear spaces are precisely those in which the nuclear spectral theorem holds true (see Section 7.1.1).

B.6 Inductive Limits

Let be given a vector space F, a family $\{F_\kappa\}$ of LCS, with $\{\kappa\}$ a directed set, and maps $j_\kappa : F_\kappa \to F$. Then there is a finest topology on F that makes all the maps j_κ continuous; it is called the *inductive topology* and F with this topology, denoted $F_{\mathrm{ind}} := \varinjlim_\kappa F_\kappa$, is called the *inductive limit* of $\{F_\kappa\}$ with

respect to the maps j_κ. The inductive limit is called *strict* if $\nu \leqslant \kappa$ implies that $F_\nu \hookrightarrow F_\kappa$ and F_κ induces on F_ν its original topology. Given any LCS, Y, a linear map $t : F_{\text{ind}} \to Y$ is continuous if, and only if, each composed map $t_\kappa = t \circ j_\kappa : F_\kappa \to Y$ is continuous. Again three cases are worth mentioning.

(i) If E is a LCS and H a closed subspace, the *quotient topology* on E/H is the inductive topology with respect to the canonical surjection $\pi : E \to E/H$.
(ii) For any family $\{F_\kappa\}$, let $F = \sum_\kappa F_\kappa$ be the direct sum, that is, the subspace of $\prod_\kappa F_\kappa$ consisting of elements with finitely many nonzero coordinates;[2] the *direct sum topology* on F is the inductive topology with respect to the embeddings $j_\kappa : F_\kappa \to F$.
(iii) In particular, a LCS is called an *(LF)-space* if it can be represented as a strict inductive limit of an increasing sequence of Fréchet spaces.
(iv) For a general inductive limit, F_{ind} is isomorphic to a quotient of $\sum_\kappa F_\kappa$.

B.7 Duality and Hereditary Properties

Let E be a LCS, H a closed subspace. The orthogonal space of H is $H^\perp := \{f \in E' : \langle f|h\rangle = 0, \forall\, h \in H\}$. $H^\perp$ is a closed subspace of E'. Then the dual of H is $E'/H^\perp$, and the dual of E/H is $H^\perp$.

As for canonical topologies, the hereditary properties are the following:

- The *Mackey topology* is inherited by quotients, but not by closed subspaces in general:
$$\tau(E,E')|_{E/H} = \tau(E/H, H^\perp), \qquad \tau(E,E')|_H \prec \tau(H, E'/H^\perp).$$
We do get equality for subspaces in two cases: if $\tau(E,E')|_H$ is metrizable, or if H is a dense subspace (hence not closed).
- The *weak topology* is inherited both by quotients and closed subspaces, whereas the *strong topology* is inherited by neither of them, in general.

Direct sums and products are dual to each other:

$$\left(\textstyle\prod_\alpha E_\alpha\right)'_{\text{proj}} = \textstyle\sum_\alpha E'_\alpha\,,$$
$$\left(\textstyle\sum_\kappa F_\kappa\right)'_{\text{ind}} = \textstyle\prod_\kappa F'_\kappa\,,$$

and Mackey topologies go through:

$$\left(\textstyle\prod_\alpha E_\alpha\right)\big|_\tau = \left(\textstyle\prod_\alpha E_\alpha|_\tau\right)_{\text{proj}} \quad \text{and} \quad \left(\textstyle\sum_\kappa F_\kappa\right)\big|_\tau = \left(\textstyle\sum_\kappa F_\kappa|_\tau\right)_{\text{ind}}.$$

[2] Some authors denote the direct sum as $\bigoplus_\kappa F_\kappa$

Reduced projective limits and inductive limits are also dual to each other:

$$\left.\begin{aligned} (\varprojlim{}_\alpha E_\alpha)' &= \varinjlim{}_\alpha E'_\alpha \\ (\varinjlim{}_\kappa F_\kappa)' &= \varprojlim{}_\kappa F'_\kappa \end{aligned}\right. \quad \text{(if the left-hand side is reduced).}$$

Combining all the above results, we get finally that the Mackey topology on a projective, resp. inductive, limit is finer than the projective, resp. inductive, limit of the respective Mackey topologies, with equality if the former is metrizable:

$$\left.\begin{aligned} \varprojlim{}_\alpha (E_\alpha|_\tau) &\prec (\varprojlim{}_\alpha E_\alpha)\Big|_\tau \\ \varinjlim{}_\kappa (F_\kappa|_\tau) &\prec (\varinjlim{}_\kappa F_\kappa)\Big|_\tau \end{aligned}\right. \quad \text{(equality if the left-hand side is metrizable)}$$

where, with a slight abuse of language, the symbol $\prec$ means that the topology of the space on the left-hand side is coarser than that of the space on the right-hand side.

Weak topologies go through projective limits only, and there is no general result for strong topologies.

Epilogue

Now it is time to draw some conclusions for this volume. As we explained in the Prologue, PIP-spaces emerged as a common backbone underlying a number of different structures, such as Rigged Hilbert spaces, Nested Hilbert spaces, scales of Banach or Hilbert spaces, etc. The aim of all these constructions is to bypass the inconvenients of the Hilbert space, and notably the L^2 spaces which are the natural environment of quantum mechanics. Indeed, Hilbert space is too "narrow", in the sense that it cannot accommodate many useful objects (δ functions, distributions, plane waves, ...). At the same time, it is too "large", because it contains very singular functions, which result from the requirement of completeness in the Lebesgue norm topology. Of course, the way out of the dilemma is to go beyond Hilbert space, essentially towards distribution theory. But this creates another problem, by creating two different kinds of mathematical objects, namely, nice (test) functions and continuous linear functionals on them. By contrast, the PIP-space point of view stays closer to the original Hilbert space, in the sense that there is a unique vector space, with an inner product, defined for certain pairs only. This is in fact the central point: the existence of an inner product between two vectors is *not* a property of each of them separately, but a property of the pair. Once this fact has been formalized in the form of a compatibility relation on the vector space at hand, all the rest follows naturally.

As we have seen, the development of the subject, while indeed rooted in the mathematical formulation of quantum mechanics, gets its own momentum. Many concrete examples have been discussed, showing the versatility of the concept of PIP-space. Actually the latter brings in a clear mathematical advantage. Indeed, as for any procedure of mathematical abstraction, the theory of PIP-spaces puts at our disposal a series of techniques and mathematical tools which are independent of the specific "example" we are dealing with. We have emphasized the word "example", since the family of spaces that constitute any one of them has been in most cases the subject of an extensive literature in functional analysis. All examples discussed in Chapter 4 and in Chapter 8 enter in this category.

Yet many problems remain open, and we have occasionally stated some of them. More fundamentally, the task ahead is to apply these ideas to concrete

situations, in particular in the realm of signal processing. It is there that ever more sophisticated types of function spaces emerge, generated by specific applications. They always come in families, indexed by one or several parameters controlling smoothness, local or asymptotic behavior, etc. And systematically the relevant objects are not the individual spaces, but the family as a whole. This is precisely the idea of PIP-spaces, and this is why we thought that the time has come to review the formalism.

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Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

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