

Conclusions and Outlook

This monograph has attempted to provide a general framework for the convergence analysis of a variety of numerical methods using layer-adapted meshes for the solution of certain classes of singularly perturbed problems. While for some problems satisfactory answers have been presented, there is still a large number of open issues. We would like to summarise the results here and to point out some directions for future research.

Difference schemes in one dimension have been covered successfully to a large extent, although there are a few minor open questions. For example, the analysis for the fourth-order scheme for reaction-diffusion problems in Sect. 6.1.4 and the analysis for turning-point problems are restricted to Shishkin meshes so far. Similarly, for the two-parameter problem in Sect. 6.3, no analysis for arbitrary meshes and a second-order scheme is available.

For finite element methods—in both one and two dimensions—the situation is different. Here results on arbitrary meshes are restricted to the interpolation error. Convergence and superconvergence estimates are known for special meshes, namely Shishkin-type meshes, only. A general framework for this very important class of methods is still missing.

For systems of strongly coupled convection-diffusion problems, we only have a limited grasp of the situation. Even for one-dimensional problems there are still basic difficulties: when different diffusion parameters are present, can sharp pointwise bounds on derivatives be proved?

For two-dimensional convection-diffusion problems, further work on stability bounds is needed to improve our understanding of these problems. In particular, sharp estimates for the Green's functions and negative-norm stability inequalities are required. In one dimensions these turned out to be the key ingredient for the convergence theory for arbitrary meshes, for the a posteriori error analysis and for dealing with strongly coupled convection-diffusion equations.

Two parameter problems of reaction-convection-diffusion type in two dimensions were considered in a small number of publications, but the presentations are typically very technical, mainly because of complicated solution decompositions involving a large number of different terms. Also the techniques used are very similar to that known from convection-diffusion problems with a single parameter. Further research is required to derive a general comprehensive theory.

Time-dependent problems have not been included in this book because only a few results are available that are insufficient for the development of a general theory. The vast majority of results use first-order backward Euler for discretisation in time. Higher-order time discretisations by A-stable Runge-Kutta methods have been considered in the literature, but their analysis is incomplete because of a lack of resolvent estimates for non-uniform meshes.

This account of open issues in the field is naturally incomplete. Studying some of the material presented in the book, the reader will certainly discover further mathematical problems worthy of investigating.

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