Conclusion

Summary

The tree of shapes of an image, as a unified mix of the component trees, was presented and analyzed in the present notes. Its existence was proven for a semicontinuous image. It relies on the simple model according to which shapes have mostly uniform reflectance properties and are distinguishable from the background. This model has no parameters and is well-posed, though it cannot really account for occlusions and subjective contours notably. The classical area openings of mathematical morphology have their counterpart in the grain filter, which has the advantage of being self-dual on continuous images. If small grains, or small shapes, are filtered out, the tree has a structure essentially finite, and a weak Morse theory was developed under these assumptions: several notions of singularity or criticality were defined and shown to be equivalent. To close the theoretical part, the procedure merging the component trees to build the tree of shapes was detailed, providing an algorithm valid in any dimension. In the most frequent two-dimensional case, a more efficient algorithm, the Fast Level Set Transform, was described. Though not as fast as an optimized component tree algorithm, the FLST is quite usable. A variant based on a bilinear interpolation of the image was also presented, which has the advantage of exhibiting more regular level lines. However, it is slightly slower than the FLST and needs as parameters the levels of quantization to represent the image in a finite structure. Finally, a wide range of concrete applications was exposed and briefly explained.

Extensions

Bilevel Sets

As has been discussed, the tree of shapes can be a better explicit representation of the image than the component trees. However, some important

objects are not represented, many of them being bilevel sets, defined by lower *and* upper thresholds. These can be recovered by taking differences of nested shapes, as for example connected components of private pixels of a shape. The resulting sets can be incorporated in the tree. Pushing this idea further, we could also consider differences between a shape and a number of its descendants. As the combinatorial complexity of this can be too expensive, a more structured approach may be needed.

Self-Dual Filters

Whereas some operators, like the median filter, have no natural representation in terms of the tree of shapes, we have seen that the grain filter is a paradigm for *connected* self-dual filters. Keshet [48] brought forth a plurality of other self-dual operators: self-dual erosions and openings. As these operators are only defined in the discrete case, it would be interesting to find a formulation in the continuous case and to study its properties in that framework. In particular, a continuous equivalent to the unfolding transform could be investigated. A good candidate for this would use the minimal total variation along a path originating from the shape and reaching the point at infinity p_∞.

Fast Level Set Transform in Dimension 3

Applying the FLST to 3D images is not very efficient: for each component, we need to compute its Euler characteristic. Whereas it is updated locally in 2D, the locality property is lost in 3D. So the cost of this step in linear in the number of pixels of the component. The merging algorithm of Chap. 5 is better in this regard. It requires the detection of an exterior boundary pixel of components, which is also linear in the number of pixels of the component, but only for upper components of maximal branches in the component tree. A better alternative would be a level surface oriented approach, extending to 3D the algorithm of [109]. Indeed, this does not require the computation of the Euler characteristic. While it also involves following level surfaces, the complexity is only the area of the level surface, not the volume of the component, as a face of a boundary voxel is known right away during the algorithm.

Color Images

An extension of the tree of shapes to color images seems unlikely at first sight. Indeed, the definition relies essentially on level sets, therefore on a total order in the codomain, namely $\mathbb{R}$. As no perceptually relevant order for color codomain $\mathbb{R}^3$ can be imagined, the definition of shapes seems not applicable. Nevertheless, color MSER, or equivalently color component trees, were successfully defined in [33]. This could also be applied to color shapes.

Glossary

$C(D)$	Set of continuous functions on D, 39
G_ε	Grain filter of area ε, 52
$M_\varepsilon, M'_\varepsilon$	Area opening operators of area ε, 40
$NFA(S \leftrightarrow S')$	Number of false alarms of a match, 159
S_x	Smallest limit node containing point x, 21
$[A, B]$	Interval of a tree between nodes A and B, 20
LS_x	The set of level shapes at point x, 24
$\mathrm{cc}(A, x)$	Connected component of A containing point x 11
$\eta_-(x), \eta_+(x)$	end-points of the interval associated to the maximal monotone section containing point x, 80
$\mathcal{L}(u)$	Set of lower components of image u, 18
$\mathcal{M}(S)$	Maximal monotone section containing shape S 24
$\mathcal{P}(X)$	Set of all subsets of X, 18
$\mathcal{S}(u)$	Set of shapes of image u, 18
$\mathcal{T}$	A tree, 18
$\mathcal{U}(u)$	Set of upper components of image u, 18
$\overline{\Omega}$	Domain of image (a set homeomorphic to the closed unit ball of $\mathbb{R}^N$), 11
$\mathrm{Sat}(A, p_\infty)$	Saturation of the set A (w.r.t. point p_∞), 12
$m_{i\,j}, \mu_{i\,j}$	moments and centered moments of a shape, 157
p_∞	Point "at infinity" (a point of $\overline{\Omega}$), 12
$sig(X)$	Signature of set X, 92
$sig(\lambda)$	Signature at level λ, 92
$\mathcal{CC}(A)$	Set of connected components of the point set A 11
$\mathcal{E}$	Set of regional extrema of a function, 91
$\mathcal{USC}(D)$	Set of upper semicontinuous functions on D, 39

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