

Appendix A

Clifford Theory and Schur Elements for Generic Hecke Algebras

Let W be a complex reflection group and let us denote by $\mathcal{H}(W)$ its generic Hecke algebra. Suppose that the assumptions 4.2.3 are satisfied. Let W' be another complex reflection group such that, for some specialization of the parameters, $\mathcal{H}(W)$ becomes the twisted symmetric algebra of a finite cyclic group G over the symmetric subalgebra $\mathcal{H}(W')$. Then, if we know the Schur elements and the blocks of $\mathcal{H}(W)$, we can use Propositions 2.3.15 and 2.3.18 in order to calculate the Schur elements and the blocks of $\mathcal{H}(W')$.

In particular, in all the cases of exceptional irreducible complex reflection groups that will be studied below, if we denote by χ' the (irreducible) restriction to $\mathcal{H}(W')$ of an irreducible character $\chi \in \text{Irr}(\mathcal{H}(W))$, then the corresponding Schur elements verify

$$s_\chi = |W : W'| s_{\chi'}.$$

Throughout the Appendix, we denote by Φ_n the n^{th} \mathbb{Q} -cyclotomic polynomial, *i.e.*, the minimal polynomial of ζ_n over \mathbb{Q} . The notation for the irreducible characters of the exceptional irreducible complex reflection groups is the one used by the GAP package CHEVIE and is explained in Subsection 5.2.3.

A.1 The Groups G_4 , G_5 , G_6 , G_7

The following table gives the specializations of the parameters of the generic Hecke algebra $\mathcal{H}(G_7)$, $(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2)$, which give the generic Hecke algebras of the groups G_4 , G_5 and G_6 ([49], Table 4.6).

Group	Index	S	T	U
G_7	1	x_0, x_1	y_0, y_1, y_2	z_0, z_1, z_2
G_5	2	$1, -1$	y_0, y_1, y_2	z_0, z_1, z_2
G_6	3	x_0, x_1	$1, \zeta_3, \zeta_3^2$	z_0, z_1, z_2
G_4	6	$1, -1$	$1, \zeta_3, \zeta_3^2$	z_0, z_1, z_2

Specializations of the parameters for $\mathcal{H}(G_7)$.

Lemma A.1.1.

(1) The algebra $\mathcal{H}(G_7)$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2) \mapsto (1, -1; y_0, y_1, y_2; z_0, z_1, z_2)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_5)$ with parameters $(y_0, y_1, y_2; z_0, z_1, z_2)$.

(2) The algebra $\mathcal{H}(G_7)$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2) \mapsto (x_0, x_1; 1, \zeta_3, \zeta_3^2; z_0, z_1, z_2)$$

is the twisted symmetric algebra of the cyclic group C_3 over the symmetric subalgebra $\mathcal{H}(G_6)$ with parameters $(x_0, x_1; z_0, z_1, z_2)$.

(3) The algebra $\mathcal{H}(G_6)$ specialized via

$$(x_0, x_1; z_0, z_1, z_2) \mapsto (1, -1; z_0, z_1, z_2)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_4)$ with parameters (z_0, z_1, z_2) .

Proof. We have

$$\mathcal{H}(G_7) = \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - z_0)(U - z_1)(U - z_2) = 0 \end{array} \right. \right\rangle.$$

(1) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, S^2 = 1, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - z_0)(U - z_1)(U - z_2) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle T, U \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_5).$$

(2) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, T^3 = 1, \\ (S - x_0)(S - x_1) = 0, \\ (U - z_0)(U - z_1)(U - z_2) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, U \rangle.$$

Then

$$A = \bigoplus_{i=0}^2 T^i \bar{A} = \bigoplus_{i=0}^2 \bar{A} T^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_6).$$

(3) Let

$$A := \left\langle S, U \left| \begin{array}{l} SUSUSU = USUSUS, S^2 = 1, \\ (U - z_0)(U - z_1)(U - z_2) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle U, SUS \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_4). \quad \blacksquare$$

The Schur elements of all irreducible characters of $\mathcal{H}(G_7)$ are calculated in [49]. They are obtained via Galois transformations (permutation of indeterminates, permutation of roots of unity or combination of the two) from the following ones:

$$\begin{aligned} s_{\phi_{1,0}} &= \Phi_1(x_0/x_1) \cdot \Phi_1(x_0 y_0^2 z_0^2 / x_1 y_1 y_2 z_1 z_2) \cdot \Phi_1(y_0/y_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_1(z_0/z_1) \\ &\cdot \Phi_1(z_0/z_2) \cdot \Phi_1(x_0 y_0 z_0 / x_1 y_1 z_1) \cdot \Phi_1(x_0 y_0 z_0 / x_1 y_1 z_2) \cdot \Phi_1(x_0 y_0 z_0 / x_1 y_2 z_1) \\ &\cdot \Phi_1(x_0 y_0 z_0 / x_1 y_2 z_2), \end{aligned}$$

$$\begin{aligned} s_{\phi_{2,9'}} &= 2y_2/y_0 \Phi_1(y_0/y_1) \cdot \Phi_1(y_2/y_0) \cdot \Phi_1(z_1/z_0) \cdot \Phi_1(z_2/z_0) \cdot \Phi_1(r/x_0 y_0 z_0) \\ &\cdot \Phi_1(r/x_0 y_2 z_1) \cdot \Phi_1(r/x_0 y_2 z_2) \cdot \Phi_1(r/x_1 y_0 z_0) \cdot \Phi_1(r/x_1 y_2 z_1) \cdot \Phi_1(r/x_1 y_2 z_2) \end{aligned}$$

$$\text{where } r = \sqrt[3]{x_0 x_1 y_1 y_2 z_1 z_2},$$

$$\begin{aligned} s_{\phi_{3,6}} &= 3\Phi_1(x_1/x_0) \cdot \Phi_1(x_0 y_0 z_0/r) \cdot \Phi_1(x_0 y_0 z_1/r) \cdot \Phi_1(x_0 y_0 z_2/r) \cdot \Phi_1(x_0 y_1 z_0/r) \\ &\cdot \Phi_1(x_0 y_1 z_1/r) \cdot \Phi_1(x_0 y_1 z_2/r) \cdot \Phi_1(x_0 y_2 z_0/r) \cdot \Phi_1(x_0 y_2 z_1/r) \cdot \Phi_1(x_0 y_2 z_2/r) \end{aligned}$$

$$\text{where } r = \sqrt[3]{x_0^2 x_1 y_0 y_1 y_2 z_0 z_1 z_2}.$$

Following Theorem 4.2.4 and [51], Table 8.1, if we set

$$\begin{aligned} X_i^{12} &:= (\zeta_2)^{-i} x_i \quad (i = 0, 1), \\ Y_j^{12} &:= (\zeta_3)^{-j} y_j \quad (j = 0, 1, 2), \\ Z_k^{12} &:= (\zeta_3)^{-k} z_k \quad (k = 0, 1, 2), \end{aligned}$$

then $\mathbb{Q}(\zeta_{12})(X_0, X_1, Y_0, Y_1, Y_2, Z_0, Z_1, Z_2)$ is a splitting field for $\mathcal{H}(G_7)$. Hence, the factorization of the Schur elements over that field is as described by Theorem 4.2.5.

A.2 The Groups $G_8, G_9, G_{10}, G_{11}, G_{12}, G_{13}, G_{14}, G_{15}$

The following table gives the specializations of the parameters of the generic Hecke algebra $\mathcal{H}(G_{11})$, $(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3)$, which give the generic Hecke algebras of the groups G_8, \dots, G_{15} ([49], Table 4.9).

Group	Index	S	T	U
G_{11}	1	x_0, x_1	y_0, y_1, y_2	z_0, z_1, z_2, z_3
G_{10}	2	$1, -1$	y_0, y_1, y_2	z_1, z_1, z_2, z_3
G_{15}	2	x_0, x_1	y_0, y_1, y_2	$\sqrt{u_0}, \sqrt{u_1}, -\sqrt{u_0}, -\sqrt{u_1}$
G_9	3	x_0, x_1	$1, \zeta_3, \zeta_3^2$	z_0, z_1, z_2, z_3
G_{14}	4	x_0, x_1	y_0, y_1, y_2	$1, i, -1, -i$
G_8	6	$1, -1$	$1, \zeta_3, \zeta_3^2$	z_0, z_1, z_2, z_3
G_{13}	6	x_0, x_1	$1, \zeta_3, \zeta_3^2$	$\sqrt{u_0}, \sqrt{u_1}, -\sqrt{u_0}, -\sqrt{u_1}$
G_{12}	12	x_0, x_1	$1, \zeta_3, \zeta_3^2$	$1, i, -1, -i$

Specializations of the parameters for $\mathcal{H}(G_{11})$.

Lemma A.2.1.

(1) The algebra $\mathcal{H}(G_{11})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3) \mapsto (1, -1; y_0, y_1, y_2; z_0, z_1, z_2, z_3)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_{10})$ with parameters $(y_0, y_1, y_2; z_0, z_1, z_2, z_3)$.

(2) The algebra $\mathcal{H}(G_{11})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3) \mapsto (x_0, x_1; 1, \zeta_3, \zeta_3^2; z_0, z_1, z_2, z_3)$$

is the twisted symmetric algebra of the cyclic group C_3 over the symmetric subalgebra $\mathcal{H}(G_9)$ with parameters $(x_0, x_1; z_0, z_1, z_2, z_3)$.

(3) The algebra $\mathcal{H}(G_9)$ specialized via

$$(x_0, x_1; z_0, z_1, z_2, z_3) \mapsto (1, -1; z_0, z_1, z_2, z_3)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_8)$ with parameters (z_0, z_1, z_2, z_3) .

(4) The algebra $\mathcal{H}(G_{11})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3) \mapsto (x_0, x_1; y_0, y_1, y_2; 1, i, -1, -i)$$

is the twisted symmetric algebra of the cyclic group C_4 over the symmetric subalgebra $\mathcal{H}(G_{14})$ with parameters $(x_0, x_1; y_0, y_1, y_2)$.

(5) The algebra $\mathcal{H}(G_{14})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2) \mapsto (x_0, x_1; 1, \zeta_3, \zeta_3^2)$$

is the twisted symmetric algebra of the cyclic group C_3 over the symmetric subalgebra $\mathcal{H}(G_{12})$ with parameters (x_0, x_1) .

(6) The algebra $\mathcal{H}(G_{11})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3) \mapsto (x_0, x_1; y_0, y_1, y_2; \sqrt{u_0}, \sqrt{u_1}, -\sqrt{u_0}, -\sqrt{u_1})$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_{15})$ with parameters $(x_0, x_1; y_0, y_1, y_2; u_0, u_1)$.

(7) The algebra $\mathcal{H}(G_{15})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; u_0, u_1) \mapsto (x_0, x_1; 1, \zeta_3, \zeta_3^2; u_0, u_1)$$

is the twisted symmetric algebra of the cyclic group C_3 over the symmetric subalgebra $\mathcal{H}(G_{13})$ with parameters $(x_0, x_1; u_0, u_1)$.

Proof. We have

$$\mathcal{H}(G_{11}) = \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3) = 0 \end{array} \right. \right\rangle.$$

(1) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, S^2 = 1, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle T, U \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{10}).$$

(2) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, T^3 = 1, \\ (S - x_0)(S - x_1) = 0, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, U \rangle.$$

Then

$$A = \bigoplus_{i=0}^2 T^i \bar{A} = \bigoplus_{i=0}^2 \bar{A} T^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_9).$$

(3) Let

$$A := \left\langle S, U \left| \begin{array}{l} SUSUSU = USUSUS, S^2 = 1, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle U, SUS \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_8).$$

(4) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, U^4 = 1, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1)(T - y_2) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, T \rangle.$$

Then

$$A = \bigoplus_{i=0}^3 U^i \bar{A} = \bigoplus_{i=0}^3 \bar{A} U^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{14}).$$

(5) Let

$$A := \left\langle S, T \left| \begin{array}{l} STSTSTST = TSTSTSTS, T^3 = 1, \\ (S - x_0)(S - x_1) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, TST^2, T^2ST \rangle.$$

Then

$$A = \bigoplus_{i=0}^2 T^i \bar{A} = \bigoplus_{i=0}^2 \bar{A} T^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{12}).$$

(6) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U^2 - u_0)(U^2 - u_1) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, T, U^2 \rangle.$$

Then

$$A = \bar{A} \oplus U\bar{A} = \bar{A} \oplus \bar{A}U \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{15}).$$

(7) Let

$$A := \left\langle U^2, S, T \left| \begin{array}{l} STU^2 = U^2ST, U^2STST = TU^2STS, T^3 = 1, \\ (S - x_0)(S - x_1) = 0, \\ (U^2 - u_0)(U^2 - u_1) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle U^2, S, T^2ST \rangle.$$

Then

$$A = \bigoplus_{i=0}^2 T^i \bar{A} = \bigoplus_{i=0}^2 \bar{A} T^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{13}). \quad \blacksquare$$

The Schur elements of all irreducible characters of $\mathcal{H}(G_{11})$ are calculated in [49]. They are obtained via Galois transformations from the following ones:

$$\begin{aligned} s_{\phi_{1,0}} &= \Phi_1(x_0/x_1) \cdot \Phi_1(y_0/y_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_1(z_0/z_1) \cdot \Phi_1(z_0/z_2) \cdot \Phi_1(z_0/z_3) \cdot \\ &\Phi_1(x_0y_0z_0/x_1y_1z_1) \cdot \Phi_1(x_0y_0z_0/x_1y_1z_2) \cdot \Phi_1(x_0y_0z_0/x_1y_1z_3) \cdot \Phi_1(x_0y_0z_0/x_1y_2z_1) \\ &\cdot \Phi_1(x_0y_0z_0/x_1y_2z_2) \cdot \Phi_1(x_0y_0z_0/x_1y_2z_3) \cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_1z_2) \\ &\cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_1z_3) \cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_2z_3) \cdot \Phi_1(x_0^2y_0^2z_0^3/x_1^2y_1y_2z_1z_2z_3), \end{aligned}$$

$$\begin{aligned} s_{\phi_{2,1}} &= -2z_1/z_0 \Phi_1(y_0/y_2) \cdot \Phi_1(y_1/y_2) \cdot \Phi_1(z_0/z_2) \cdot \Phi_1(z_0/z_3) \cdot \Phi_1(z_1/z_2) \cdot \Phi_1(z_1/z_3) \\ &\cdot \Phi_1(y_0z_0z_1/y_2z_2z_3) \cdot \Phi_1(y_1z_0z_1/y_2z_2z_3) \cdot \Phi_1(r/x_0y_2z_2) \cdot \Phi_1(r/x_0y_2z_3) \cdot \Phi_1(r/x_1y_2z_2) \\ &\cdot \Phi_1(r/x_1y_2z_3) \cdot \Phi_1(r/x_0y_0z_1) \cdot \Phi_1(r/x_0y_1z_1) \cdot \Phi_1(r/x_1y_0z_1) \cdot \Phi_1(r/x_1y_1z_1) \end{aligned}$$

$$\text{where } r = \sqrt[3]{x_0x_1y_0y_1z_0z_1},$$

$$\begin{aligned} s_{\phi_{3,2}} &= 3\Phi_1(x_1/x_0) \cdot \Phi_1(z_1/z_3) \cdot \Phi_1(z_2/z_3) \cdot \Phi_1(z_0/z_3) \cdot \Phi_1(r/x_1y_0z_3) \cdot \Phi_1(r/x_1y_1z_3) \cdot \\ &\Phi_1(r/x_1y_2z_3) \cdot \Phi_1(x_0y_0z_0/r) \cdot \Phi_1(x_0y_0z_1/r) \cdot \Phi_1(x_0y_0z_2/r) \cdot \Phi_1(x_0y_1z_0/r) \cdot \Phi_1(x_0y_1z_1/r) \\ &\cdot \Phi_1(x_0y_1z_2/r) \cdot \Phi_1(x_0y_2z_0/r) \cdot \Phi_1(x_0y_2z_1/r) \cdot \Phi_1(x_0y_2z_2/r) \end{aligned}$$

$$\text{where } r = \sqrt[3]{x_0^2x_1y_0y_1y_2z_0z_1z_2},$$

$$\begin{aligned} s_{\phi_{4,21}} &= -4\Phi_1(y_0/y_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_1(r/x_0y_0z_0) \cdot \Phi_1(r/x_1y_0z_0) \cdot \Phi_1(x_0y_0z_1/r) \\ &\cdot \Phi_1(x_0y_0z_2/r) \cdot \Phi_1(x_0y_0z_3/r) \cdot \Phi_1(x_1y_0z_1/r) \cdot \Phi_1(x_1y_0z_2/r) \cdot \Phi_1(x_1y_0z_3/r) \end{aligned}$$

$$\begin{aligned}
& \cdot \Phi_1(x_0x_1y_0y_1z_0z_1/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_0z_2/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_0z_3/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_2z_0z_1/r^2) \cdot \Phi_1(x_0x_1y_0y_2z_0z_2/r^2) \cdot \Phi_1(x_0x_1y_0y_2z_0z_3/r^2)
\end{aligned}$$

where $r = \sqrt[4]{x_0^2x_1^2y_0^2y_1y_2z_0z_1z_2z_3}$.

Following Theorem 4.2.4 and [51], Table 8.1, if we set

$$\begin{aligned}
X_i^{24} &:= (\zeta_2)^{-i} x_i \quad (i = 0, 1), \\
Y_j^{24} &:= (\zeta_3)^{-j} y_j \quad (j = 0, 1, 2), \\
Z_k^{24} &:= (\zeta_4)^{-k} z_k \quad (k = 0, 1, 2, 3),
\end{aligned}$$

then $\mathbb{Q}(\zeta_{24})(X_0, X_1, Y_0, Y_1, Y_2, Z_0, Z_1, Z_2, Z_3)$ is a splitting field for $\mathcal{H}(G_{11})$. Hence, the factorization of the Schur elements over that field is as described by Theorem 4.2.5.

A.3 The Groups G_{16} , G_{17} , G_{18} , G_{19} , G_{20} , G_{21} , G_{22}

The following table gives the specializations of the parameters of the generic Hecke algebra $\mathcal{H}(G_{19})$, $(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3, z_4)$, which give the generic Hecke algebras of the groups G_{16}, \dots, G_{22} ([49], Table 4.12).

Group	Index	S	T	U
G_{19}	1	x_0, x_1	y_0, y_1, y_2	z_0, z_1, z_2, z_3, z_4
G_{18}	2	$1, -1$	y_0, y_1, y_2	z_0, z_1, z_2, z_3, z_4
G_{17}	3	x_0, x_1	$1, \zeta_3, \zeta_3^2$	z_0, z_1, z_2, z_3, z_4
G_{21}	5	x_0, x_1	y_0, y_1, y_2	$1, \zeta_5, \zeta_5^2, \zeta_5^3, \zeta_5^4$
G_{16}	6	$1, -1$	$1, \zeta_3, \zeta_3^2$	z_0, z_1, z_2, z_3, z_4
G_{20}	10	$1, -1$	y_0, y_1, y_2	$1, \zeta_5, \zeta_5^2, \zeta_5^3, \zeta_5^4$
G_{22}	15	x_0, x_1	$1, \zeta_3, \zeta_3^2$	$1, \zeta_5, \zeta_5^2, \zeta_5^3, \zeta_5^4$

Specializations of the parameters for $\mathcal{H}(G_{19})$.

Lemma A.3.1.

(1) The algebra $\mathcal{H}(G_{19})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3, z_4) \mapsto (1, -1; y_0, y_1, y_2; z_0, z_1, z_2, z_3, z_4)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_{18})$ with parameters $(y_0, y_1, y_2; z_0, z_1, z_2, z_3, z_4)$.

(2) The algebra $\mathcal{H}(G_{19})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3, z_4) \mapsto (x_0, x_1; 1, \zeta_3, \zeta_3^2; z_0, z_1, z_2, z_3, z_4)$$

is the twisted symmetric algebra of the cyclic group C_3 over the symmetric subalgebra $\mathcal{H}(G_{17})$ with parameters $(x_0, x_1; z_0, z_1, z_2, z_3, z_4)$.

(3) The algebra $\mathcal{H}(G_{17})$ specialized via

$$(x_0, x_1; z_0, z_1, z_2, z_3, z_4) \mapsto (1, -1; z_0, z_1, z_2, z_3, z_4)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_{16})$ with parameters $(z_0, z_1, z_2, z_3, z_4)$.

(4) The algebra $\mathcal{H}(G_{19})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2; z_0, z_1, z_2, z_3, z_4) \mapsto (x_0, x_1; y_0, y_1, y_2; 1, \zeta_5, \zeta_5^2, \zeta_5^3, \zeta_5^4)$$

is the twisted symmetric algebra of the cyclic group C_5 over the symmetric subalgebra $\mathcal{H}(G_{21})$ with parameters $(x_0, x_1; y_0, y_1, y_2)$.

(5) The algebra $\mathcal{H}(G_{21})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2) \mapsto (1, -1; y_0, y_1, y_2)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_{20})$ with parameters (y_0, y_1, y_2) .

(6) The algebra $\mathcal{H}(G_{21})$ specialized via

$$(x_0, x_1; y_0, y_1, y_2) \mapsto (x_0, x_1; 1, \zeta_3, \zeta_3^2)$$

is the twisted symmetric algebra of the cyclic group C_3 over the symmetric subalgebra $\mathcal{H}(G_{22})$ with parameters (x_0, x_1) .

Proof. We have

$$\mathcal{H}(G_{19}) = \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3)(U - z_4) = 0 \end{array} \right. \right\rangle.$$

(1) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, S^2 = 1, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3)(U - z_4) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle T, U \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{18}).$$

(2) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, T^3 = 1, \\ (S - x_0)(S - x_1) = 0, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3)(U - z_4) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, U \rangle.$$

Then

$$A = \bigoplus_{i=0}^2 T^i \bar{A} = \bigoplus_{i=0}^2 \bar{A} T^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{17}).$$

(3) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} SUSUSU = USUSUS, S^2 = 1, \\ (U - z_0)(U - z_1)(U - z_2)(U - z_3)(U - z_4) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle U, SUS \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{16}).$$

(4) Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, U^5 = 1, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1)(T - y_2) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, T \rangle.$$

Then

$$A = \bigoplus_{i=0}^4 U^i \bar{A} = \bigoplus_{i=0}^4 \bar{A} U^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{21}).$$

(5) Let

$$A := \left\langle S, T \left| \begin{array}{l} STSTSTSTST = TSTSTSTSTS, S^2 = 1, \\ (T - y_0)(T - y_1)(T - y_2) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle T, STS \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{20}).$$

(6) Let

$$A := \left\langle S, T \left| \begin{array}{l} STSTSTSTST = TSTSTSTSTS, T^3 = 1, \\ (S - x_0)(S - x_1) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, TST^2, T^2ST \rangle.$$

Then

$$A = \bigoplus_{i=0}^2 T^i \bar{A} = \bigoplus_{i=0}^2 \bar{A} T^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{22}). \quad \blacksquare$$

The Schur elements of all irreducible characters of $\mathcal{H}(G_{19})$ are calculated in [49]. They are obtained via Galois transformations from the following ones:

$$\begin{aligned} s_{\phi_{1,0}} &= \Phi_1(x_0/x_1) \cdot \Phi_1(y_0/y_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_1(z_0/z_1) \cdot \Phi_1(z_0/z_2) \cdot \Phi_1(z_0/z_3) \cdot \Phi_1(z_0/z_4) \\ &\cdot \Phi_1(x_0y_0z_0/x_1y_1z_1) \cdot \Phi_1(x_0y_0z_0/x_1y_1z_2) \cdot \Phi_1(x_0y_0z_0/x_1y_1z_3) \cdot \Phi_1(x_0y_0z_0/x_1y_1z_4) \\ &\cdot \Phi_1(x_0y_0z_0/x_1y_2z_1) \cdot \Phi_1(x_0y_0z_0/x_1y_2z_2) \cdot \Phi_1(x_0y_0z_0/x_1y_2z_3) \cdot \Phi_1(x_0y_0z_0/x_1y_2z_4) \\ &\cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_1z_2) \cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_1z_3) \cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_1z_4) \\ &\cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_2z_3) \cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_2z_4) \cdot \Phi_1(x_0y_0^2z_0^2/x_1y_1y_2z_3z_4) \\ &\cdot \Phi_1(x_0^2y_0^2z_0^3/x_1^2y_1y_2z_1z_2z_3) \cdot \Phi_1(x_0^2y_0^2z_0^3/x_1^2y_1y_2z_1z_2z_4) \cdot \Phi_1(x_0^2y_0^2z_0^3/x_1^2y_1y_2z_1z_3z_4) \\ &\cdot \Phi_1(x_0^2y_0^2z_0^3/x_1^2y_1y_2z_2z_3z_4) \cdot \Phi_1(x_0^2y_0^3z_0^4/x_1^2y_1^2y_2z_1z_2z_3z_4) \cdot \Phi_1(x_0^2y_0^3z_0^4/x_1^2y_1y_2^2z_1z_2z_3z_4) \\ &\cdot \Phi_1(x_0^3y_0^4z_0^4/x_1^3y_1^2y_2^2z_1z_2z_3z_4), \end{aligned}$$

$$\begin{aligned} s_{\phi_{2,31'}} &= -2\Phi_1(y_0/y_2) \cdot \Phi_1(y_1/y_2) \cdot \Phi_1(z_0/z_2) \cdot \Phi_1(z_0/z_3) \cdot \Phi_1(z_0/z_4) \cdot \Phi_1(z_1/z_2) \\ &\cdot \Phi_1(z_1/z_3) \cdot \Phi_1(z_1/z_4) \cdot \Phi_1(y_0z_0z_1/y_2z_2z_3) \cdot \Phi_1(y_0z_0z_1/y_2z_2z_4) \cdot \Phi_1(y_0z_0z_1/y_2z_3z_4) \\ &\cdot \Phi_1(y_1z_0z_1/y_2z_2z_3) \cdot \Phi_1(y_1z_0z_1/y_2z_2z_4) \cdot \Phi_1(y_1z_0z_1/y_2z_3z_4) \cdot \Phi_1(y_0y_1z_0z_1^2/y_2^2z_2z_3z_4) \\ &\cdot \Phi_1(y_0y_1z_0^2z_1/y_2^2z_2z_3z_4) \cdot \Phi_1(r/x_0y_0z_0) \cdot \Phi_1(x_0y_0z_1/r) \cdot \Phi_1(x_1y_0z_0/r) \cdot \Phi_1(r/x_1y_0z_1) \\ &\cdot \Phi_1(r/x_1y_2z_2) \cdot \Phi_1(r/x_1y_2z_3) \cdot \Phi_1(r/x_1y_2z_4) \cdot \Phi_1(r/x_0y_2z_2) \cdot \Phi_1(r/x_0y_2z_3) \cdot \Phi_1(r/x_0y_2z_4) \\ &\cdot \Phi_1(rz_0z_1/x_0y_2z_2z_3z_4) \cdot \Phi_1(rz_0z_1/x_1y_2z_2z_3z_4) \end{aligned}$$

$$\text{where } r = \sqrt[3]{x_0x_1y_0y_1z_0z_1},$$

$$\begin{aligned} s_{\phi_{3,22'}} &= 3\Phi_1(x_1/x_0) \cdot \Phi_1(z_0/z_3) \cdot \Phi_1(z_0/z_4) \cdot \Phi_1(z_1/z_3) \cdot \Phi_1(z_1/z_4) \cdot \Phi_1(z_2/z_3) \\ &\cdot \Phi_1(z_2/z_4) \cdot \Phi_1(x_0z_0z_1/x_1z_3z_4) \cdot \Phi_1(x_0z_0z_2/x_1z_3z_4) \cdot \Phi_1(x_0z_1z_2/x_1z_3z_4) \cdot \Phi_1(r/x_1y_0z_3) \\ &\cdot \Phi_1(r/x_1y_0z_4) \cdot \Phi_1(r/x_1y_1z_3) \cdot \Phi_1(r/x_1y_1z_4) \cdot \Phi_1(r/x_1y_2z_3) \cdot \Phi_1(r/x_1y_2z_4) \\ &\cdot \Phi_1(x_0y_0z_0/r) \cdot \Phi_1(x_0y_0z_1/r) \cdot \Phi_1(x_0y_0z_2/r) \cdot \Phi_1(x_0y_1z_0/r) \cdot \Phi_1(x_0y_1z_1/r) \\ &\cdot \Phi_1(x_0y_1z_2/r) \cdot \Phi_1(x_0y_2z_0/r) \cdot \Phi_1(x_0y_2z_1/r) \cdot \Phi_1(x_0y_2z_2/r) \cdot \Phi_1(r^2/x_0x_1y_0y_1z_3z_4) \\ &\cdot \Phi_1(r^2/x_0x_1y_0y_2z_3z_4) \cdot \Phi_1(r^2/x_0x_1y_1y_2z_3z_4) \end{aligned}$$

$$\text{where } r = \sqrt[3]{x_0^2x_1y_0y_1y_2z_0z_1z_2},$$

$$\begin{aligned} s_{\phi_{4,18}} &= -4\Phi_1(y_1/y_0) \cdot \Phi_1(y_0/y_2) \cdot \Phi_1(z_0/z_4) \cdot \Phi_1(z_1/z_4) \cdot \Phi_1(z_2/z_4) \cdot \Phi_1(z_3/z_4) \\ &\cdot \Phi_1(x_0y_0z_0/r) \cdot \Phi_1(x_0y_0z_1/r) \cdot \Phi_1(x_0y_0z_2/r) \cdot \Phi_1(x_0y_0z_3/r) \cdot \Phi_1(x_1y_0z_0/r) \cdot \Phi_1(x_1y_0z_1/r) \\ &\cdot \Phi_1(x_1y_0z_2/r) \cdot \Phi_1(x_1y_0z_3/r) \cdot \Phi_1(r/x_0y_1z_4) \cdot \Phi_1(r/x_1y_1z_4) \cdot \Phi_1(r/x_0y_2z_4) \cdot \Phi_1(r/x_1y_2z_4) \end{aligned}$$

$$\begin{aligned}
& \cdot \Phi_1(r^2/x_0x_1y_0y_1z_0z_1) \cdot \Phi_1(r^2/x_0x_1y_0y_1z_0z_2) \cdot \Phi_1(x_0x_1y_0y_1z_0z_3/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_1z_1z_2/r^2) \cdot \Phi_1(r^2/x_0x_1y_0y_1z_1z_3) \cdot \Phi_1(r^2/x_0x_1y_0y_1z_2z_3) \\
& \cdot \Phi_1(r^2/x_0x_1y_1y_2z_0z_4) \cdot \Phi_1(r^2/x_0x_1y_1y_2z_1z_4) \cdot \Phi_1(r^2/x_0x_1y_1y_2z_2z_4) \\
& \cdot \Phi_1(r^2/x_0x_1y_1y_2z_3z_4)
\end{aligned}$$

where $r = \sqrt[4]{x_0^2x_1^2y_0^2y_1y_2z_0z_1z_2z_3}$,

$$\begin{aligned}
s_{\phi_{5,16}} &= 5\Phi_1(x_0/x_1) \cdot \Phi_1(y_2/y_0) \cdot \Phi_1(y_2/y_1) \cdot \Phi_1(x_0y_0z_0/r) \cdot \Phi_1(x_0y_0z_1/r) \cdot \Phi_1(x_0y_0z_2/r) \\
& \cdot \Phi_1(x_0y_0z_3/r) \cdot \Phi_1(x_0y_0z_4/r) \cdot \Phi_1(x_0y_1z_0/r) \cdot \Phi_1(x_0y_1z_1/r) \cdot \Phi_1(x_0y_1z_2/r) \cdot \Phi_1(x_0y_1z_3/r) \\
& \cdot \Phi_1(x_0y_1z_4/r) \cdot \Phi_1(r/x_1y_2z_0) \cdot \Phi_1(r/x_1y_2z_1) \cdot \Phi_1(r/x_1y_2z_2) \cdot \Phi_1(r/x_1y_2z_3) \cdot \Phi_1(r/x_1y_2z_4) \\
& \cdot \Phi_1(x_0x_1y_0y_1z_0z_1/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_0z_2/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_0z_3/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_1z_0z_4/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_1z_2/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_1z_3/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_1z_1z_4/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_2z_3/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_2z_4/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_1z_3z_4/r^2)
\end{aligned}$$

where $r = \sqrt[5]{x_0^3x_1^2y_0^2y_1^2y_2z_0z_1z_2z_3z_4}$,

$$\begin{aligned}
s_{\phi_{6,15}} &= -6\Phi_1(z_0/z_1) \cdot \Phi_1(z_0/z_2) \cdot \Phi_1(z_0/z_3) \cdot \Phi_1(z_0/z_4) \cdot \Phi_1(r/x_0y_0z_0) \cdot \Phi_1(r/x_0y_1z_0) \\
& \cdot \Phi_1(r/x_0y_2z_0) \cdot \Phi_1(x_1y_0z_0/r) \cdot \Phi_1(x_1y_1z_0/r) \cdot \Phi_1(x_1y_2z_0/r) \cdot \Phi_1(x_0x_1y_0y_1z_0z_1/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_1z_0z_2/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_0z_3/r^2) \cdot \Phi_1(x_0x_1y_0y_1z_0z_4/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_2z_0z_1/r^2) \cdot \Phi_1(x_0x_1y_0y_2z_0z_2/r^2) \cdot \Phi_1(x_0x_1y_0y_2z_0z_3/r^2) \\
& \cdot \Phi_1(x_0x_1y_0y_2z_0z_4/r^2) \cdot \Phi_1(x_0x_1y_1y_2z_0z_1/r^2) \cdot \Phi_1(x_0x_1y_1y_2z_0z_2/r^2) \\
& \cdot \Phi_1(x_0x_1y_1y_2z_0z_3/r^2) \cdot \Phi_1(x_0x_1y_1y_2z_0z_4/r^2) \cdot \Phi_1(x_0^2x_1y_0y_1y_2z_0z_1z_2/r^3) \\
& \cdot \Phi_1(x_0^2x_1y_0y_1y_2z_0z_1z_3/r^3) \cdot \Phi_1(x_0^2x_1y_0y_1y_2z_0z_1z_4/r^3) \cdot \Phi_1(x_0^2x_1y_0y_1y_2z_0z_2z_3/r^3) \\
& \cdot \Phi_1(x_0^2x_1y_0y_1y_2z_0z_2z_4/r^3) \cdot \Phi_1(x_0^2x_1y_0y_1y_2z_0z_3z_4/r^3)
\end{aligned}$$

where $r = \sqrt[6]{x_0^3x_1^3y_0^2y_1^2y_2^2z_0^2z_1z_2z_3z_4}$.

Following Theorem 4.2.4 and [51], Table 8.1, if we set

$$\begin{aligned}
X_i^{60} &:= (\zeta_2)^{-i}x_i \quad (i = 0, 1), \\
Y_j^{60} &:= (\zeta_3)^{-j}y_j \quad (j = 0, 1, 2), \\
Z_k^{60} &:= (\zeta_5)^{-k}z_k \quad (k = 0, 1, 2, 3, 4),
\end{aligned}$$

then $\mathbb{Q}(\zeta_{60})(X_0, X_1, Y_0, Y_1, Y_2, Z_0, Z_1, Z_2, Z_3, Z_4)$ is a splitting field for $\mathcal{H}(G_{19})$. Hence, the factorization of the Schur elements over that field is as described by Theorem 4.2.5.

A.4 The Groups G_{25} , G_{26}

The following table gives the specialization of the parameters of the generic Hecke algebra $\mathcal{H}(G_{26})$, $(x_0, x_1; y_0, y_1, y_2)$, which gives the generic Hecke algebra of the group G_{25} ([50], Theorem 6.3).

Group	Index	S	T
G_{26}	1	x_0, x_1	y_0, y_1, y_2
G_{25}	2	$1, -1$	y_0, y_1, y_2

Specializations of the parameters for $\mathcal{H}(G_{26})$.

Lemma A.4.1. *The algebra $\mathcal{H}(G_{26})$ specialized via*

$$(x_0, x_1; y_0, y_1, y_2) \mapsto (1, -1; y_0, y_1, y_2)$$

is the twisted symmetric algebra of the cyclic group C_2 over the symmetric subalgebra $\mathcal{H}(G_{25})$ with parameters (y_0, y_1, y_2) .

Proof. We have

$$\mathcal{H}(G_{26}) = \left\langle S, T, U \left| \begin{array}{l} STST = TSTS, UTU = TUT, SU = US, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - y_0)(U - y_1)(U - y_2) = 0 \end{array} \right. \right\rangle.$$

Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STST = TSTS, UTU = TUT, SU = US, S^2 = 1, \\ (T - y_0)(T - y_1)(T - y_2) = 0, \\ (U - y_0)(U - y_1)(U - y_2) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle SUS, T, U \rangle.$$

Then

$$A = \bar{A} \oplus S\bar{A} = \bar{A} \oplus \bar{A}S \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G_{25}). \quad \blacksquare$$

The Schur elements of all irreducible characters of $\mathcal{H}(G_{26})$ are calculated in [50]. They are obtained via Galois transformations from the following ones:

$$\begin{aligned} s_{\phi_{1,0}} &= -\Phi_1(x_0/x_1) \cdot \Phi_1(y_0/y_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_0/x_1y_2) \\ &\cdot \Phi_1(x_0y_0^2/x_1y_1^2) \cdot \Phi_1(x_0y_0^2/x_1y_2^2) \cdot \Phi_2(x_0y_0^3/x_1y_1^2y_2) \cdot \Phi_2(x_0y_0^3/x_1y_1y_2^2) \\ &\cdot \Phi_6(x_0y_0^2/x_1y_1y_2) \cdot \Phi_2(y_0^2/y_1y_2) \cdot \Phi_6(y_0/y_1) \cdot \Phi_6(y_0/y_2), \end{aligned}$$

$$\begin{aligned} s_{\phi_{2,3}} &= y_1/y_0 \Phi_1(x_0/x_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_1(y_1/y_2) \cdot \Phi_1(x_0y_0/x_1y_2) \cdot \Phi_1(x_0y_1/x_1y_2) \\ &\cdot \Phi_2(x_0y_0/x_1y_2) \cdot \Phi_2(x_0y_1/x_1y_2) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_6(x_0y_0y_1/x_1y_2^2) \\ &\cdot \Phi_2(y_0y_1/y_2^2) \cdot \Phi_6(y_0/y_1), \end{aligned}$$

$$\begin{aligned} s_{\phi_{3,6}} &= -\Phi_1(x_0/x_1) \cdot \Phi_3(x_0/x_1) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_0/x_1y_2) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \\ &\Phi_2(x_0y_1/x_1y_2) \cdot \Phi_2(x_0y_2/x_1y_0) \cdot \Phi_2(x_0y_2/x_1y_1) \cdot \Phi_2(y_0y_1/y_2^2) \cdot \Phi_2(y_0y_2/y_1^2) \cdot \Phi_2(y_1y_2/y_0^2), \end{aligned}$$

$$s_{\phi_{3,1}} = -\Phi_1(x_1/x_0) \cdot \Phi_1(y_0/y_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_2(y_0/y_2) \cdot \Phi_1(y_1/y_2) \cdot \Phi_2(y_0y_1/y_2^2) \\ \cdot \Phi_2(y_0^2/y_1y_2) \cdot \Phi_6(y_0/y_2) \cdot \Phi_2(x_0y_0/x_1y_2) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_1(x_0y_0^2/x_1y_1^2) \\ \cdot \Phi_2(x_0y_0^2y_1/x_1y_2^2),$$

$$s_{\phi_{6,2}} = \Phi_1(x_0/x_1) \cdot \Phi_1(y_1/y_0) \cdot \Phi_1(y_0/y_2) \cdot \Phi_1(y_1/y_2) \cdot \Phi_2(y_2/y_0) \cdot \Phi_6(y_0/y_2) \\ \cdot \Phi_2(y_0y_2/y_1^2) \cdot \Phi_1(x_0y_1/x_1y_2) \cdot \Phi_2(x_1y_0/x_0y_2) \cdot \Phi_2(x_0y_1/x_1y_2) \cdot \Phi_2(x_0y_0^3/x_1y_1^2y_2),$$

$$s_{\phi_{8,3}} = 2\Phi_1(y_0/y_1) \cdot \Phi_1(y_0/y_2) \cdot \Phi_2(x_1y_2/x_0y_1) \cdot \Phi_2(x_1y_1/x_0y_2) \cdot \Phi_2(ry_0/x_1y_2^2) \\ \cdot \Phi_2(ry_0/x_1y_1^2) \cdot \Phi_1(ry_2/x_1y_0y_1) \cdot \Phi_1(ry_1/x_1y_0y_2) \cdot \Phi_3(ry_0/x_1y_1y_2) \cdot \Phi_3(ry_0/x_0y_1y_2) \\ \text{where } r = \sqrt[3]{-x_0x_1y_1y_2},$$

$$s_{\phi_{9,7}} = \Phi_1(\zeta_3^2) \cdot \Phi_6(y_0/y_1) \cdot \Phi_6(y_2/y_0) \cdot \Phi_6(y_1/y_2) \cdot \Phi_2(\zeta_3x_0y_1y_2/x_1y_0^2) \\ \cdot \Phi_2(\zeta_3x_0y_0y_2/x_1y_1^2) \cdot \Phi_2(\zeta_3x_0y_0y_1/x_1y_2^2) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(\zeta_3x_0/x_1).$$

Following Theorem 4.2.4 and [51], Table 8.2, if we set

$$X_i^6 := (\zeta_2)^{-i}x_i \quad (i = 0, 1), \\ Y_j^6 := (\zeta_3)^{-j}y_j \quad (j = 0, 1, 2),$$

then $\mathbb{Q}(\zeta_3)(X_1, X_2, Y_1, Y_2, Y_2)$ is a splitting field for $\mathcal{H}(G_{26})$. Hence, the factorization of the Schur elements over that field is as described by Theorem 4.2.5.

A.5 The Group G_{28} (“ F_4 ”)

Let $\mathcal{H}(G_{28})$ be the generic Hecke algebra of the real reflection group G_{28} over the ring $\mathbb{Z}[x_0^{\pm 1}, x_1^{\pm 1}, y_0^{\pm 1}, y_1^{\pm 1}]$. We have

$$\mathcal{H}(G_{28}) = \left\langle S_1, S_2, T_1, T_2 \left| \begin{array}{l} S_1S_2S_1 = S_2S_1S_2, \quad T_1T_2T_1 = T_2T_1T_2, \\ S_1T_1 = T_1S_1, \quad S_1T_2 = T_2S_1, \quad S_2T_2 = T_2S_2, \\ S_2T_1S_2T_1 = T_1S_2T_1S_2, \\ (S_i - x_0)(S_i - x_1) = (T_i - y_0)(T_i - y_1) = 0 \end{array} \right. \right\rangle.$$

The Schur elements of all irreducible characters of $\mathcal{H}(G_{28})$ have been calculated in [47]. They are obtained via Galois transformations from the following ones:

$$s_{\phi_{1,0}} = \Phi_1(y_0/y_1) \cdot \Phi_6(y_0/y_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_6(x_0/x_1) \cdot \Phi_1(x_0y_0^2/x_1y_1^2) \cdot \Phi_6(x_0y_0/x_1y_1) \\ \cdot \Phi_1(x_0^2y_0/x_1^2y_1) \cdot \Phi_4(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_0/x_1y_1),$$

$$s_{\phi_{2,4''}} = -y_1/y_0\Phi_6(y_0/y_1) \cdot \Phi_3(x_0/x_1) \cdot \Phi_6(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \\ \cdot \Phi_1(x_0^2y_0/x_1^2y_1) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_1(x_0^2y_1/x_1^2y_0),$$

$$s_{\phi_{4,8}} = 2\Phi_6(y_0/y_1) \cdot \Phi_6(x_1/x_0) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_2(x_1y_1/x_0y_0) \cdot \Phi_2(x_0y_0/x_1y_1),$$

$$s_{\phi_{4,1}} = \Phi_1(y_0/y_1) \cdot \Phi_6(y_0/y_1) \cdot \Phi_1(x_1/x_0) \cdot \Phi_6(x_0/x_1) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_6(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_0/x_1y_1),$$

$$s_{\phi_{6,6''}} = 3\Phi_1(y_1/y_0) \cdot \Phi_1(y_1/y_0) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_0) \cdot \Phi_6(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_2(x_1y_0/x_0y_1),$$

$$s_{\phi_{8,3''}} = -y_1/y_0 \Phi_6(y_0/y_1) \cdot \Phi_6(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_1/x_0) \cdot \Phi_3(x_0/x_1) \cdot \Phi_1(x_0y_1^2/x_1y_0^2) \cdot \Phi_1(x_0y_0^2/x_1y_1^2),$$

$$s_{\phi_{9,2}} = \Phi_1(y_0/y_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0y_1^2/x_1y_0^2) \cdot \Phi_4(x_0y_0/x_1y_1) \cdot \Phi_1(x_1^2y_0/x_0^2y_1) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_0y_0/x_1y_1),$$

$$s_{\phi_{12,4}} = 6\Phi_3(y_0/y_1) \cdot \Phi_3(x_1/x_0) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_2(x_0y_1/x_1y_0) \cdot \Phi_2(x_0y_0/x_1y_1) \cdot \Phi_2(x_1y_1/x_0y_0),$$

$$s_{\phi_{16,5}} = 2x_1y_1/x_0y_0 \Phi_6(y_0/y_1) \cdot \Phi_6(x_1/x_0) \cdot \Phi_4(x_0y_1/x_1y_0) \cdot \Phi_4(x_0y_0/x_1y_1).$$

Following Theorem 4.2.4, if we set

$$X_i^2 := (\zeta_2)^{-i} x_i \quad (i = 0, 1),$$

$$Y_j^2 := (\zeta_2)^{-j} y_j \quad (j = 0, 1),$$

then $\mathbb{Q}(X_0, X_1, Y_0, Y_1)$ is a splitting field for $\mathcal{H}(G_{28})$. Hence, the factorization of the Schur elements over that field is as described by Theorem 4.2.5.

A.6 The Group G_{32}

Let $\mathcal{H}(G_{32})$ be the generic Hecke algebra of the complex reflection group G_{32} over the ring $\mathbb{Z}[x_0^{\pm 1}, x_1^{\pm 1}, x_2^{\pm 1}]$. We have

$$\mathcal{H}(G_{32}) = \left\langle S_1, S_2, S_3, S_4 \left| \begin{array}{l} S_i S_{i+1} S_i = S_{i+1} S_i S_{i+1}, \\ S_i S_j = S_j S_i \text{ when } |i - j| > 1, \\ (S_i - x_0)(S_i - x_1)(S_i - x_2) = 0 \end{array} \right. \right\rangle.$$

The Schur elements of all irreducible characters of $\mathcal{H}(G_{32})$ have been calculated in [50]. They are obtained via Galois transformations from the following ones:

$$s_{\phi_{1,0}} = \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0^3/x_1x_2^2) \cdot \Phi_1(x_0^3/x_1^2x_2) \cdot \Phi_1(x_0^5/x_1^3x_2^2) \cdot \Phi_1(x_0^5/x_1^2x_2^3) \cdot \Phi_2(x_0^4/x_1x_2^3) \cdot \Phi_2(x_0^4/x_1^3x_2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_2(x_0^2/x_1x_2)$$

$$\cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_0/x_1) \cdot \Phi_6(x_0^3/x_1x_2^2) \cdot \Phi_6(x_0^3/x_1^2x_2) \cdot \Phi_6(x_0^2/x_1x_2) \cdot \Phi_4(x_0^2/x_1x_2) \\ \cdot \Phi_4(x_0/x_2) \cdot \Phi_4(x_0/x_1) \cdot \Phi_3(x_0^2/x_1x_2) \cdot \Phi_{10}(x_0/x_2) \cdot \Phi_{10}(x_0/x_1) \cdot \Phi_5(x_0^2/x_1x_2),$$

$$s_{\phi_{4,1}} = \Phi_1(x_0^4/x_1x_2^3) \cdot \Phi_1(x_0^3/x_1x_2^2) \cdot \Phi_1(x_0^3/x_1^2x_2) \cdot \Phi_1(x_0^2x_1/x_2^3) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_2) \\ \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_2(x_0^5/x_1x_2^4) \cdot \Phi_2(x_0^3x_1/x_2^4) \cdot \Phi_2(x_0^3/x_1x_2^2) \cdot \Phi_2(x_0^2/x_1x_2) \\ \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_0/x_1) \cdot \Phi_4(x_0/x_2) \cdot \Phi_3(x_0^2/x_1x_2) \cdot \Phi_{10}(x_0/x_1) \\ \cdot \Phi_{15}(x_0/x_2),$$

$$s_{\phi_{5,4}} = \Phi_1(x_0^3x_1^2/x_2^5) \cdot \Phi_1(x_0^2x_1/x_2^3) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_0) \\ \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_2(x_0^3/x_1x_2^2) \cdot \Phi_2(x_0x_1^2/x_2^3) \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \\ \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0^4x_1/x_2^5) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0/x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_0/x_1) \\ \cdot \Phi_4(x_0/x_1) \cdot \Phi_3(x_0x_1/x_2^2) \cdot \Phi_{12}(x_0/x_2),$$

$$s_{\phi_{6,8}} = x_1^2/x_0^2 \Phi_1(x_0/x_1) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_1/x_2) \\ \cdot \Phi_1(x_0x_1^2/x_2^3) \cdot \Phi_1(x_0^2x_1/x_2^3) \cdot \Phi_2(x_0x_1^2/x_2^3) \cdot \Phi_2(x_0^2x_1/x_2^3) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_2(x_1^2/x_0x_2) \\ \cdot \\ \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_0/x_2) \cdot \Phi_6(x_0x_1/x_2^2) \cdot \Phi_6(x_0/x_2) \cdot \\ \Phi_6(x_1/x_2) \cdot \Phi_{10}(x_0/x_1) \cdot \Phi_5(x_0x_1/x_2^2),$$

$$s_{\phi_{10,2}} = \Phi_1(x_0^2x_1/x_2^3) \cdot \Phi_1(x_0^3/x_1x_2^2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_2/x_0) \\ \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_2(x_0^4/x_1^3x_2) \cdot \Phi_2(x_0^3/x_1x_2^2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0x_2/x_1^2) \\ \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_6(x_0^2x_1/x_2^3) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_0/x_1) \\ \cdot \Phi_4(x_0/x_2) \cdot \Phi_3(x_0^2/x_1x_2),$$

$$s_{\phi_{15,6}} = \Phi_1(x_0^3/x_1x_2^2) \cdot \Phi_1(x_0^3/x_1^2x_2) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_2/x_1) \cdot \Phi_1(x_2/x_1) \\ \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_0/x_2) \cdot \Phi_2(x_0^3x_2/x_1^4) \cdot \Phi_2(x_0^3x_1/x_2^4) \cdot \Phi_2(x_1^2/x_0x_2) \cdot \Phi_2(x_0x_1/x_2^2) \\ \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_6(x_0^2/x_1x_2) \cdot \Phi_6(x_0/x_1) \\ \cdot \Phi_6(x_0/x_2) \cdot \Phi_4(x_0^2/x_1x_2),$$

$$s_{\phi_{15,8}} = \Phi_1(x_1^2x_2/x_0^3) \cdot \Phi_1(x_0^2x_2/x_1^3) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \\ \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_0/x_1) \cdot \Phi_2(x_1x_2/x_0^2) \cdot \Phi_2(x_0x_2/x_1^2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0x_1/x_2^2) \\ \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0/x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_1/x_2) \cdot \Phi_4(x_0x_1/x_2^2),$$

$$s_{\phi_{20,3}} = \Phi_1(x_0^2x_2/x_1^3) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0^4/x_1x_2^3) \\ \cdot \Phi_1(x_0^3/x_1x_2^2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_2(x_0x_1^2/x_2^3) \cdot \Phi_2(x_1^2/x_0x_2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_2(x_0x_1/x_2^2) \\ \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_2/x_0) \cdot \Phi_6(x_0^3/x_1^2x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_4(x_0/x_2) \cdot \Phi_3(x_0x_1/x_2^2),$$

$$s_{\phi_{20,5}} = -\Phi_1(x_1^3/x_0^2x_2) \cdot \Phi_1(x_0^3/x_1^2x_2) \cdot \Phi_1(x_0x_1^2/x_2^3) \cdot \Phi_1(x_0^2x_1/x_2^3) \cdot \Phi_1(x_2/x_1) \\ \cdot \Phi_1(x_2/x_1) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_2(x_1^3/x_0x_2^2) \cdot \Phi_2(x_0^3/x_1x_2^2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \\ \Phi_2(x_0x_1/x_2^2) \cdot \Phi_6(x_0x_1/x_2^2) \cdot \Phi_6(x_1/x_0) \cdot \Phi_6(x_1/x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_3(x_0x_1/x_2^2),$$

$$s_{\phi_{20,7}} = \Phi_1(x_0^3x_1/x_2^4) \cdot \Phi_1(x_0x_1^2/x_2^3) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_0/x_2) \cdot \\ \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_2/x_0) \cdot \Phi_2(x_0^3x_2/x_1^4) \cdot \Phi_2(x_0x_1^2/x_2^3) \cdot \Phi_2(x_0x_1/x_2^2) \\ \cdot \Phi_2(x_1^2/x_0x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_1/x_2) \cdot \Phi_3(x_0^2/x_1x_2),$$

$$s_{\phi_{20,12}} = 2\Phi_1(x_2/x_1) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_1/x_0) \\ \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_1) \cdot \Phi_2(x_0x_1^2/x_2^3) \cdot \Phi_2(x_0x_2^2/x_1^3) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_2(x_1x_2/x_0^2) \\ \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0/x_2) \cdot \Phi_6(x_2/x_1) \cdot \Phi_6(x_1/x_2) \cdot \Phi_3(x_0^2/x_1x_2),$$

$$s_{\phi_{24,6}} = \Phi_1(x_1^3/x_0^2x_2) \cdot \Phi_1(x_2^3/x_0^2x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_2) \\ \cdot \Phi_1(x_2/x_1) \cdot \Phi_1(x_1/x_2) \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0x_2/x_1^2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_6(x_0/x_1) \\ \cdot \Phi_6(x_0/x_2) \cdot \Phi_4(x_0/x_1) \cdot \Phi_4(x_0/x_2) \cdot \Phi_5(x_0^2/x_1x_2),$$

$$s_{\phi_{30,4}} = \Phi_1(x_0^5/x_1^3x_2^2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \\ \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \cdot \Phi_2(x_1/x_0) \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_2/x_0) \cdot \Phi_2(x_0^5/x_1x_2^4) \cdot \Phi_2(x_0x_2^2/x_1^3) \\ \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0x_2/x_1^2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_6(x_0/x_1) \cdot \Phi_6(x_1/x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_4(x_0/x_2),$$

$$s_{\phi_{30,12'}} = \Phi_1(x_1^5/x_0^3x_2^2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_0/x_2) \\ \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_2/x_1) \cdot \Phi_2(x_1^5/x_0x_2^4) \cdot \Phi_2(x_1x_2^2/x_0^3) \\ \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_1x_2/x_0^2) \cdot \Phi_2(x_1^2/x_0x_2) \cdot \Phi_6(x_1/x_0) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_1/x_2) \cdot \Phi_4(x_1/x_2),$$

$$s_{\phi_{36,5}} = \Phi_1(1/\zeta_3) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(\zeta_3x_0^2/x_1x_2) \cdot \Phi_1(\zeta_3^2x_0x_2/x_1^2) \\ \cdot \Phi_1(x_2^2/\zeta_3^2x_0x_1) \cdot \Phi_2(x_1x_2/x_0^2) \cdot \Phi_2(x_0^2x_2/\zeta_3x_1^3) \cdot \Phi_2(\zeta_3^2x_0^2x_1/x_2^3) \cdot \Phi_6(x_0^2/x_1x_2) \cdot \\ \Phi_6(x_0/x_1) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_1/x_2) \cdot \Phi_5(\zeta_3x_0/x_2) \cdot \Phi_5(\zeta_3x_0/x_1),$$

$$s_{\phi_{40,8}} = \Phi_1(x_0^3x_1^2/x_2^5) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_0/x_2) \\ \cdot \Phi_1(x_1^2x_2/x_0^3) \cdot \Phi_1(x_2/x_1) \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_2(x_0/x_2) \\ \cdot \Phi_6(x_0/x_1) \cdot \Phi_6(x_1/x_2) \cdot \Phi_4(x_0/x_1) \cdot \Phi_4(x_0/x_2) \cdot \Phi_3(x_0x_2/x_1^2),$$

$$s_{\phi_{45,6}} = \Phi_1(\zeta_3) \cdot \Phi_1(\zeta_3^2x_0^2/x_1x_2) \cdot \Phi_1(\zeta_3x_0x_2/x_1^2) \cdot \Phi_1(\zeta_3x_0x_1/x_2^2) \cdot \Phi_1(x_2/x_0) \cdot \\ \Phi_1(x_1/x_2) \cdot \Phi_2(\zeta_3^2x_0^2x_2/x_1^3) \cdot \Phi_2(\zeta_3^2x_1^2x_2/x_0^3) \cdot \Phi_2(x_0/\zeta_3^2x_2) \cdot \Phi_2(\zeta_3x_1/x_2) \cdot \Phi_2(x_0x_1/x_2^2) \\ \cdot \Phi_6(x_1/x_0) \cdot \Phi_6(x_1/x_2) \cdot \Phi_6(x_0/x_2) \cdot \Phi_6(x_0x_1/x_2^2) \cdot \Phi_4(\zeta_3x_0/x_2) \cdot \Phi_4(\zeta_3x_1/x_2),$$

$$s_{\phi_{60,7}} = \Phi_1(x_0x_1^2/x_2^3) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_0/x_2) \\ \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_1x_2^2/x_0^3) \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0^4x_1/x_2^5) \cdot \Phi_2(x_1^2/x_0x_2) \cdot \Phi_2(x_1x_2/x_0^2) \\ \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_1/x_2) \cdot \Phi_6(x_0/x_1) \cdot \Phi_6(x_2/x_1) \cdot \Phi_4(x_0/x_1),$$

$$s_{\phi_{60,11''}} = \Phi_1(x_1x_2^3/x_0^4) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_1/x_2) \cdot \Phi_1(x_2/x_0) \\ \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_0x_2^2/x_1^4) \cdot \Phi_2(x_1^2/x_0x_2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0^2/x_1x_2) \\ \cdot \Phi_2(x_0/x_1) \cdot \Phi_2(x_0/x_1) \cdot \Phi_6(x_0x_1/x_2^2) \cdot \Phi_6(x_1/x_0),$$

$$s_{\phi_{60,12}} = 2\Phi_1(x_0x_2^2/x_1^3) \cdot \Phi_1(x_0x_1^2/x_2^3) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_1/x_0) \cdot \Phi_1(x_2/x_0) \cdot \\ \Phi_1(x_2/x_0) \cdot \Phi_1(x_2/x_0) \cdot \Phi_2(x_2/x_1) \cdot \Phi_2(x_1/x_2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_2(x_0^2/x_1x_2) \cdot \Phi_6(x_0^2/x_1x_2) \\ \cdot \Phi_6(x_1/x_2) \cdot \Phi_6(x_2/x_1) \cdot \Phi_4(x_0/x_1) \cdot \Phi_4(x_0/x_2),$$

$$s_{\phi_{64,8}} = 2\Phi_1(rx_1/x_2^2) \cdot \Phi_1(x_2^2/rx_0) \cdot \Phi_1(x_0/x_2) \cdot \Phi_1(x_2/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_1/x_0) \\ \cdot \Phi_1(x_0^3/x_1x_2^2) \cdot \Phi_1(x_0x_2^2/x_1^3) \cdot \Phi_2(rx_0^2/x_1^2x_2) \cdot \Phi_2(rx_1^2/x_0^2x_2) \cdot \Phi_3(x_0x_1/x_2^2) \cdot \Phi_{10}(r/x_2) \\ \cdot \Phi_{15}(r/x_0)$$

$$\text{where } r = \sqrt[3]{x_0x_1},$$

$$s_{\phi_{80,9}} = 2\Phi_1(x_0x_2^2/x_1^3) \cdot \Phi_1(x_0x_1^2/x_2^3) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_0/x_1) \cdot \Phi_1(x_2/x_0) \cdot \Phi_1(x_2/x_0) \\ \cdot \Phi_1(x_2/x_1) \cdot \Phi_1(x_2/x_1) \cdot \Phi_2(x_0/x_2) \cdot \Phi_2(x_0/x_1) \cdot \Phi_4(x_1x_2/x_0^2) \cdot \Phi_4(x_0/x_1) \cdot \Phi_4(x_0/x_2) \\ \cdot \Phi_3(x_0^2/x_1x_2) \cdot \Phi_{12}(x_1/x_2),$$

$$s_{\phi_{81,10}} = 3\Phi_2(rx_2/x_0^2) \cdot \Phi_2(rx_2/x_1^2) \cdot \Phi_2(rx_0/x_2^2) \cdot \Phi_2(rx_0/x_1^2) \cdot \Phi_2(rx_1/x_0^2) \cdot \Phi_2(rx_1/x_2^2) \\ \cdot \Phi_2(x_0x_1/x_2^2) \cdot \Phi_2(x_0x_2/x_1^2) \cdot \Phi_2(x_1x_2/x_0^2) \cdot \Phi_2(r/x_2) \cdot \Phi_2(r/x_0) \cdot \Phi_2(r/x_1) \cdot \Phi_4(r^2/x_0x_1) \\ \cdot \Phi_4(r^2/x_0x_2) \cdot \Phi_4(r^2/x_1x_2) \cdot \Phi_5(r/x_0) \cdot \Phi_5(r/x_2) \cdot \Phi_5(r/x_1)$$

where $r = \sqrt[3]{x_0x_1x_2}$.

Following Theorem 4.2.4 and [51], Table 8.2, if we set

$$X_i^6 := (\zeta_3)^{-i} x_i \quad (i = 0, 1, 2),$$

then $\mathbb{Q}(\zeta_3)(X_0, X_1, X_2)$ is a splitting field for $\mathcal{H}(G_{32})$. Hence, the factorization of the Schur elements over that field is as described by Theorem 4.2.5.

A.7 The Groups $G(de, e, r)$

The generic Hecke algebras of the groups $G(de, e, r)$ are presented in Chapter 5. Here we will only give some applications of Clifford theory and a description of their Schur elements.

A.7.1 The Groups $G(de, e, r)$, $r > 2$

Proposition 1.6 of [2] yields the specialization of the parameters of the generic Hecke algebra $\mathcal{H}(G(de, 1, r))$, $(x_0, x_1; u_0, u_1, \dots, u_{de-1})$, which gives the generic Hecke algebra of the group $G(de, e, r)$.

Lemma A.7.1. *The algebra $\mathcal{H}(G(de, 1, r))$ specialized via*

$$\begin{cases} x_i \mapsto x_i & (0 \leq i \leq 1), \\ u_k \mapsto \zeta_e^{[k/d]} v_{k \bmod d}^{1/e} & (0 \leq k \leq de - 1) \end{cases}$$

is the twisted symmetric algebra of the cyclic group C_e over the symmetric subalgebra $\mathcal{H}(G(de, e, r))$ with parameters $(x_0, x_1; v_0, v_1, \dots, v_{d-1})$.

Proof. The algebra $\mathcal{H}(G(de, 1, r))$ is generated by the elements $\mathbf{s}, \mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_{r-1}$ satisfying the relations:

- $\mathbf{s}\mathbf{t}_1\mathbf{s} = \mathbf{t}_1\mathbf{s}\mathbf{t}_1\mathbf{s}$, $\mathbf{s}\mathbf{t}_j = \mathbf{t}_j\mathbf{s}$, for $j \neq 1$,
- $\mathbf{t}_j\mathbf{t}_{j+1}\mathbf{t}_j = \mathbf{t}_{j+1}\mathbf{t}_j\mathbf{t}_{j+1}$, $\mathbf{t}_i\mathbf{t}_j = \mathbf{t}_j\mathbf{t}_i$, for $|i - j| > 1$,
- $(\mathbf{s} - u_0)(\mathbf{s} - u_1) \cdots (\mathbf{s} - u_{de-1}) = (\mathbf{t}_j - x_0)(\mathbf{t}_j - x_1) = 0$.

Let A be the algebra obtained from $\mathcal{H}(G(de, 1, r))$ via the given specialization, i.e., the algebra generated by the elements $\mathbf{s}, \mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_{r-1}$ satisfying the same braid relations as above, as well as:

$$(\mathbf{s}^e - v_0)(\mathbf{s}^e - v_1) \cdots (\mathbf{s}^e - v_{d-1}) = (\mathbf{t}_j - x_0)(\mathbf{t}_j - x_1) = 0.$$

If $\bar{A} := \langle \mathbf{s}^e, \tilde{\mathbf{t}}_1 := \mathbf{s}^{-1} \mathbf{t}_1 \mathbf{s}, \mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_{r-1} \rangle$, then

$$A = \bigoplus_{i=0}^{e-1} \mathbf{s}^i \bar{A} = \bigoplus_{i=0}^{e-1} \bar{A} \mathbf{s}^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G(de, e, r)). \quad \blacksquare$$

For $x_1 = -1$, the algebra $\mathcal{H}(G(de, 1, r))$ becomes the generic Ariki-Koike algebra $\mathcal{H}_{de,r}$ associated to $G(de, 1, r)$. Set $n := de$ and $x := x_0$. The following result, which has been obtained independently by Geck, Iancu and Malle [36] and by Mathas [54], gives a description of the Schur elements of $\mathcal{H}_{n,r}$. Recall that the irreducible characters of $G(n, 1, r)$ are parametrized by the n -partitions of r .

Theorem A.7.2. *Let λ be an n -partition of r with ordinary standard symbol $B_\lambda = (B_\lambda^{(0)}, B_\lambda^{(1)}, \dots, B_\lambda^{(n-1)})$. Fix $L \geq h_\lambda$, where h_λ is the height of λ . We set $B_{\lambda,L} := (B_\lambda^{(0)}[L - h_\lambda], B_\lambda^{(1)}[L - h_\lambda], \dots, B_\lambda^{(n-1)}[L - h_\lambda]) = (B_{\lambda,L}^{(0)}, B_{\lambda,L}^{(1)}, \dots, B_{\lambda,L}^{(n-1)})$ and $B_{\lambda,L}^{(s)} = (b_1^{(s)}, b_2^{(s)}, \dots, b_L^{(s)})$. Let $a_L := r(n-1) + \binom{n}{2} \binom{L}{2}$ and $b_L := nL(L-1)(2nL - n - 3)/12$. Then the Schur element of the irreducible character χ_λ is given by the formulae $s_\lambda = (-1)^{a_L} x^{b_L} (x-1)^{-r} (u_0 u_1 \dots u_{n-1})^{-r} \nu_\lambda / \delta_\lambda$, where*

$$\nu_\lambda = \prod_{0 \leq s < t < n} (u_s - u_t)^L \prod_{0 \leq s, t < n} \prod_{b_s \in B_{\lambda,L}^{(s)}} \prod_{1 \leq k \leq b_s} (x^k u_s - u_t)$$

and

$$\delta_\lambda = \prod_{0 \leq s < t < n} \prod_{(b_s, b_t) \in B_{\lambda,L}^{(s)} \times B_{\lambda,L}^{(t)}} (x^{b_s} u_s - x^{b_t} u_t) \prod_{0 \leq s < n} \prod_{1 \leq i < j \leq L} (x^{b_i^{(s)}} u_s - x^{b_j^{(s)}} u_s).$$

Following [17], Table 1, the field of definition of $G(n, 1, r)$ is $K := \mathbb{Q}(\zeta_n)$. By Theorem 4.2.4, if we set

$$\begin{aligned} X^{|\mu(K)|} &:= x, \\ U_k^{|\mu(K)|} &:= (\zeta_n)^{-k} u_k \quad (k = 0, 1, \dots, n-1), \end{aligned}$$

then the algebra $K(X, U_0, U_1, \dots, U_{n-1}) \mathcal{H}_{n,r}$ is split semisimple. We easily deduce that the factorization of the Schur elements of this algebra is as described by Theorem 4.2.5.

A.7.2 The Groups $G(de, e, 2)$, e Odd

Lemma A.7.1 holds when $r = 2$ and e is odd.

A.7.3 The Groups $G(de, e, 2)$, e Even

Suppose that $e = 2f$ for some $f \geq 1$. Proposition 1.6 of [2] yields the specialization of the parameters of the generic Hecke algebra $\mathcal{H}(G(2fd, 2, 2))$, $(x_0, x_1; y_0, y_1; z_0, z_1, \dots, z_{fd-1})$, which gives the generic Hecke algebra of the group $G(2fd, 2f, 2)$.

Lemma A.7.3. *The algebra $\mathcal{H}(G(2fd, 2, 2))$ specialized via*

$$\begin{cases} x_i \mapsto x_i & (0 \leq i \leq 1), \\ y_j \mapsto y_j & (0 \leq j \leq 1), \\ z_k \mapsto \zeta_f^{[k/d]} u_{k \bmod d}^{1/f} & (0 \leq k \leq fd - 1) \end{cases}$$

is the twisted symmetric algebra of the cyclic group C_f over the symmetric subalgebra $\mathcal{H}(G(2fd, 2f, 2))$ with parameters $(x_0, x_1; y_0, y_1; u_0, u_1, \dots, u_{d-1})$.

Proof. We have

$$\mathcal{H}(G(2fd, 2, 2)) = \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1) = 0, \\ (U - z_0)(U - z_1) \cdots (U - z_{fd-1}) = 0 \end{array} \right. \right\rangle.$$

Let

$$A := \left\langle S, T, U \left| \begin{array}{l} STU = TUS = UST, \\ (S - x_0)(S - x_1) = 0, \\ (T - y_0)(T - y_1) = 0, \\ (U^f - u_0)(U^f - u_1) \cdots (U^f - u_{d-1}) = 0 \end{array} \right. \right\rangle$$

and

$$\bar{A} := \langle S, T, U^f \rangle.$$

Then

$$A = \bigoplus_{i=0}^{f-1} U^i \bar{A} = \bigoplus_{i=0}^{f-1} \bar{A} U^i \quad \text{and} \quad \bar{A} \cong \mathcal{H}(G(2fd, 2f, 2)). \quad \blacksquare$$

Set $n := fd = de/2$. The group $G(2n, 2, 2)$ has $4n$ irreducible characters of degree 1,

$$\chi_{ijk} \ (0 \leq i, j \leq 1) \ (0 \leq k < n),$$

and $n^2 - n$ irreducible characters of degree 2,

$$\chi_{kl}^1, \chi_{kl}^2 \ (0 \leq k < l < n).$$

Following [49], Theorem 3.11, the Schur elements of the irreducible characters of $\mathcal{H}(G(2n, 2, 2))$ are:

$$s_{\chi_{ijk}} = \Phi_1(x_i x_{1-i}^{-1}) \cdot \Phi_1(y_j y_{1-j}^{-1}) \cdot \prod_{l=0, l \neq k}^{n-1} \left(\Phi_1(z_k z_l^{-1}) \cdot \Phi_1(x_i x_{1-i}^{-1} y_j y_{1-j}^{-1} z_k z_l^{-1}) \right),$$

$$s_{\chi_{kl}^{1,2}} = -2 \prod_{m=0, m \neq k, l}^{n-1} \left(\Phi_1(z_k z_m^{-1}) \cdot \Phi_1(z_l z_m^{-1}) \right) \cdot \prod_{i=0}^1 \left(\Phi_1(X_i X_{1-i}^{-1} Y_i Y_{1-i}^{-1} Z_k Z_l^{-1}) \right. \\ \left. \cdot \Phi_1(X_i X_{1-i}^{-1} Y_{1-i} Y_i^{-1} Z_l Z_k^{-1}) \right)$$

where $X_i^2 := x_i$, $Y_j^2 := y_j$, $Z_k^2 := z_k$.

Following [17], Table 1, the field of definition of $G(2n, 2, 2)$ is $K := \mathbb{Q}(\zeta_{2n})$. By Theorem 4.2.4, if we set

$$\begin{aligned} \mathcal{X}_i^{|\mu(K)|} &:= (\zeta_2)^{-i} x_i \quad (i = 0, 1), \\ \mathcal{Y}_j^{|\mu(K)|} &:= (\zeta_2)^{-j} y_j \quad (j = 0, 1), \\ \mathcal{Z}_k^{|\mu(K)|} &:= (\zeta_n)^{-k} z_k \quad (k = 0, 1, \dots, n-1), \end{aligned}$$

then the algebra $K(\mathcal{X}_0, \mathcal{X}_1, \mathcal{Y}_0, \mathcal{Y}_1, \mathcal{Z}_0, \mathcal{Z}_1, \dots, \mathcal{Z}_{n-1})\mathcal{H}(G(2n, 2, 2))$ is split semisimple. Hence, the factorization of the Schur elements of this algebra is as described by Theorem 4.2.5.

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Index

- β -number, 105
- β -number shifted, 105
- ϕ -bad
 - prime ideal, 86
 - prime number, 86
- α -function, 87

- adapted
 - family, 14
 - morphism, 14
- Ariki-Koike algebra, 107
- Artin-like presentation, 74

- block, 22, 58
- block-idempotent, 22
- braid group, 73
- Brauer
 - graph, 58
 - reciprocity, 57
- Brauer-Nesbitt lemma, 49

- Casimir element, 31
- central character, 30
- charged
 - content, 107
 - height of a multipartition, 106
 - standard symbol, 107
 - symbol, 107
- cyclotomic specialization, 80
- cyclotomic specialization associated with
 - an essential hyperplane, 92
 - no essential hyperplane, 92

- decomposition
 - map, 53
 - matrix, 53

- defect
 - of a block, 58
 - of a character, 58
- diagram of a multipartition, 109
- discrete valuation, 8
- discrete valuation ring, 8
- distinguished
 - braid reflection, 74
 - reflection, 73

- essential
 - algebra, 62
 - hyperplane, 82

- field of definition, 72

- Grothendieck group, 49

- Hecke algebra
 - cyclotomic, 81
 - generic, 75
 - spetsial, 81
- height
 - of a multipartition, 105
 - of a partition, 105

- ℓ -adic topology, 9
- integral
 - closure, 3
 - element, 3
- integrally closed, 4

- Jacobson radical, 9

- Krull ring, 8

- length function, 74
- localization
 - of a module, 3
 - of a ring, 2
- modular constituent, 57
- monodromy generator, 74
- morphism associated with a monomial, 12
- multipartition, 105
- node, 109
- ordinary
 - content, 106
 - standard symbol, 105
 - symbol, 106
- p-block associated with
 - an essential hyperplane, 83, 92
 - no essential hyperplane, 83, 92
- p-essential
 - hyperplane, 82
 - monomial, 63, 79
- p-residue, 109
- p-residue equivalent, 109
- partition, 104
- primitive
 - element, 10
 - monomial, 62
- pseudo-reflection, 72
- pure braid group, 73
- realized over, 51
- reflecting hyperplane, 72
- reflection group, 72
- regular variety, 73
- residue equivalent, 109
- Rouquier block, 84
- Rouquier block associated with
 - an essential hyperplane, 87, 92
 - no essential hyperplane, 87, 92
- Rouquier ring, 84
- Schur element, 33
- semi-continuity, 83
- semi-local ring, 9
- semisimple algebra, 24
- Shephard-Todd classification, 72
- size
 - of a multipartition, 105
 - of a partition, 105
- Specht module, 108
- specialization, 52
- split algebra, 25
- stuttering multipartition, 123
- symmetric
 - algebra, 30
 - subalgebra, 35
- symmetrizing form, 30
- Tits' deformation theorem, 55
- trace function, 30
- twisted symmetric algebra of a finite group, 37
- valuation, 5
- valuation ring, 5
- weight system, 106

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Addresses:

Professor J.-M. Morel, CMLA,
École Normale Supérieure de Cachan,
61 Avenue du Président Wilson, 94235 Cachan Cedex, France
E-mail: Jean-Michel.Morel@cmla.ens-cachan.fr

Professor F. Takens, Mathematisch Instituut,
Rijksuniversiteit Groningen, Postbus 800,
9700 AV Groningen, The Netherlands
E-mail: F.Takens@rug.nl

Professor B. Teissier, Institut Mathématique de Jussieu,
UMR 7586 du CNRS, Équipe “Géométrie et Dynamique”,
175 rue du Chevaleret,
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