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Index of Notation

\mathbb{R} : the real numbers	$N_C(\bar{x})$: normal cone
$\overline{\mathbb{R}}$: the extended real numbers	$\widehat{N}_C(\bar{x})$: regular normal cone
\mathbb{N} : the natural numbers	$R_C(\bar{x})$: local recession cone
\mathbb{Q} : the rational numbers	K^* : polar cone
\mathcal{B} : closed unit ball	C° : polar set
$ x $: Euclidean norm	f^*, f^{**} : conjugate, biconjugate
$\langle x, y \rangle$: canonical inner product	δ_C : indicator function
C^{k+} : strictly continuous derivatives	σ_C : support function
$C \setminus D$: relative complement	γ_C : gauge function
bdry C : boundary	$d_C(x), d(x, C)$: distance from C
cl C : closure	logexp: log-exponential function
int C : interior	vecmax: vector-max function
rint C : relative interior	$DS(\bar{x} \bar{v}), DS(\bar{x})$: graphical derivative
cosm C : cosmic closure	$\widehat{D}S(\bar{x} \bar{v}), \widehat{D}S(\bar{x})$: regular derivative
hzn C : horizon	$D^*S(\bar{x} \bar{v}), D^*S(\bar{x})$: coderivative
con C : convex hull of set C	$\widehat{D}^*S(\bar{x} \bar{v}), \widehat{D}^*S(\bar{x})$: regular coderivative
con f : convex hull of function f	$D_*S(\bar{x} \bar{v}), D_*S(\bar{x})$: strict derivative
pos C , pos f : positive hull	$f \# g$: epi-addition
dom f : effective domain	$\lambda * f$: epi-multiplication
epi f : epigraph	$\mathcal{N}(\bar{x})$: neighborhood collection
hypo f : hypograph	\mathcal{N}_∞ : subsets of \mathbb{N} containing all ν sufficiently large
dir w : the direction of w	$\mathcal{N}_\infty^\#$: infinite subsets of \mathbb{N}
gph S : graph of S	g-lim: graphical limits
cl f : lsc regularization	p-lim: pointwise limits
cl S : graphical closure	e-lim: epigraphical limits
lip f : Lipschitz modulus of function f	h-lim: hypographical limits
lip $F, \text{lip } S$: Lipschitz modulus of mappings	$\lim^\infty, \lim \inf^\infty, \lim \sup^\infty$: horizon limits
C^∞ : horizon cone	\xrightarrow{p} : pointwise convergence
f^∞ : horizon function	\xrightarrow{e} : epigraphical convergence
S^∞ : horizon mapping	\xrightarrow{h} : hypographical convergence
lev $_{\leq \alpha} f$: lower level set	\xrightarrow{c} : cosmic convergence
$df(\bar{x})$: subderivative function	\xrightarrow{t} : total convergence
$\widehat{d}f(\bar{x})$: regular subderivative function	\xrightarrow{g} : graphical convergence
$\partial f(\bar{x})$: subgradient set	$\overline{\cdot}$: convergence within C
$\widehat{\partial}f(\bar{x})$: regular subgradient set	$\overrightarrow{\cdot}$: f -attentive convergence
$\partial^\infty f(\bar{x})$: horizon subgradient set	$\overleftarrow{\cdot}$: convergence indexed by \mathbb{N}
$\widetilde{\partial}f(\bar{x})$: convexified subgradient set	sets, cl-sets: spaces of sets
$d^2f(\bar{x}), d^2f(\bar{x} v)$: second subderivatives	fens, lsc-fens: spaces of functions
$e_\lambda f$: Moreau envelope	maps, osc-maps: spaces of mappings
$P_\lambda f$: proximal mapping	\widehat{d}_ρ : estimates of set and epi-distances
P_C : projection mapping	d, d_ρ : set distances, epi-distances
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