Appendix A
The twistor and Killing spinor equations

In this section we recall basic facts as well as classification results concerning twistor and Killing spinors on Riemannian spin manifolds. The reader is invited to refer to [56, 70, 55, 63] for further statements and references.

A.1 Definitions and examples

Definition A.1.1 *Let (M^n, g) be a Riemannian spin manifold.*

i) A twistor-spinor *on (M^n, g) is a section ψ of ΣM solving*

$$P\psi = 0, \tag{A.1}$$

where $P_X\psi := \nabla_X\psi + \frac{1}{n}X \cdot D\psi$ for every $X \in TM$.

*ii) Given a complex number α, an α-*Killing spinor *on (M^n, g) is a section ψ of ΣM solving*

$$\nabla_X\psi = \alpha X \cdot \psi \tag{A.2}$$

for every $X \in TM$. In case $\alpha \in \mathbb{R}$ (resp. $\alpha \in i\mathbb{R}^*$) an α-Killing spinor is called real Killing (resp. imaginary Killing) spinor.

The operator $P : \Gamma(\Sigma M) \longrightarrow \Gamma(T^*M \otimes \Sigma M)$ is called the *Penrose* or *twistor* operator. It is obtained as the orthogonal projection of the covariant derivative onto the kernel of the Clifford multiplication $\mu : T^*M \otimes \Sigma M \longrightarrow \Sigma M$. Obviously a section of ΣM is a Killing spinor if and only if it is a twistor-spinor which is an eigenvector of D. Beware that our definition of real Killing spinor contains that of parallel spinor, compare [56].

Notes A.1.2

1. The name "Killing spinor" originates from the fact that, if α is real, then the vector field V defined by $g(V, X) := i\langle\psi, X \cdot \psi\rangle$ for all $X \in TM$, is a Killing vector field on (M^n, g).
2. One could give a slightly more general definition of Killing spinor, requiring (A.2) to hold for a given smooth complex-valued function α on M.

If α is real-valued and $n \geq 2$, then O. Hijazi has shown [127] that it must be constant on M, see [134, Prop. 5.12]. On the other hand, H.-B. Rademacher has proved [209] the existence of (and actually completely classified) manifolds carrying non-zero α-Killing spinors with non-constant $\alpha \in C^\infty(M, i\mathbb{R})$, see Theorem A.4.5 below.

On 1-dimensional manifolds M it is a simple exercise to show the following: every section of ΣM is a twistor-spinor, therefore this space is infinite-dimensional. For any $\alpha \in \mathbb{C}$ the space of α-Killing spinors on $(M, g) := (\mathbb{R}, \mathrm{can})$ is $\mathbb{C} \cdot e^{-i\alpha t}$, thus it is 1-dimensional. As for the circle $\mathbb{S}^1(L)$ of length $L > 0$ and carrying the δ-spin structure with $\delta \in \{0, 1\}$ (see Example 1.4.3.1), it admits a non-zero - and hence 1-dimensional - space of α-Killing spinors if and only if $\alpha \in \frac{\pi\delta}{L} + \frac{2\pi}{L}\mathbb{Z}$ (in particular α must be real).

Before we proceed to general properties of twistor-spinors in dimension $n \geq 2$, we discuss a few examples.

Examples A.1.3 We describe the twistor spinors on simply-connected spaceforms.

1. Let $(M^n, g) := (\mathbb{R}^n, \mathrm{can})$, $n \geq 2$, be endowed with its canonical spin structure. It obviously admits a $2^{[\frac{n}{2}]}$-dimensional space of parallel spinors, which are the constant sections of $\Sigma(\mathbb{R}^n) \cong \mathbb{R}^n \times \Sigma_n$. There exists moreover a $2^{[\frac{n}{2}]}$-dimensional space of non-parallel twistor-spinors, which are of the form $\varphi_x := x \cdot \psi$ for every $x \in \mathbb{R}^n$, where ψ is some parallel spinor on $(\mathbb{R}^n, \mathrm{can})$. Since this space stands in direct sum with that of parallel spinors, we deduce that the space of twistor-spinors on $(\mathbb{R}^n, \mathrm{can})$ is at least $2^{[\frac{n}{2}]+1}$-dimensional. We shall show that for $n \geq 3$ the space $\mathrm{Ker}(P)$ is actually at most (hence here exactly) $2^{[\frac{n}{2}]+1}$-dimensional (see Proposition A.2.1.3.b)), however in dimension $n = 2$ there are many more twistor-spinors on $\mathbb{R}^n$ (see Proposition A.2.3).
2. Let $(M^n, g) := (\mathbb{S}^n, \mathrm{can})$, $n \geq 2$, be endowed with its canonical metric (with sectional curvature 1) and spin structure. Since it is a hypersurface of $(\mathbb{R}^{n+1}, \mathrm{can})$ with Weingarten-map $-\mathrm{Id}_{TM}$ w.r.t. the normal vector field $\nu_x := x$ we deduce from the Gauss-type formula (1.21) that the restriction of any parallel spinor (resp. positive half spinor for n odd, see Proposition 1.4.1) onto $\mathbb{S}^n$ is a $\frac{1}{2}$-Killing spinor. Therefore the space of $\frac{1}{2}$-Killing spinors on $(\mathbb{S}^n, \mathrm{can})$ is at least $2^{[\frac{n}{2}]}$-dimensional. Using again Proposition 1.4.1, it is easy to show that the restriction of any spinor of the form $x \mapsto x \cdot \psi$, where ψ is parallel on $(\mathbb{R}^{n+1}, \mathrm{can})$ (and positive if n is odd), gives a $-\frac{1}{2}$-Killing spinor. Therefore the space of $-\frac{1}{2}$-Killing spinors on $(\mathbb{S}^n, \mathrm{can})$ is also at least $2^{[\frac{n}{2}]}$-dimensional. From Propositions A.2.1.4 and A.4.1.2 below we deduce that both spaces have exactly dimension $2^{[\frac{n}{2}]}$ and that the space of twistor-spinors is exactly the direct sum of them.
3. Let $(M^n, g) := (H^n, \mathrm{can})$, $n \geq 2$, be endowed with its canonical metric (with sectional curvature -1) and spin structure. It is a hypersurface of

the Minkowski-space $(\mathbb{R}^{n+1}, \langle\!\langle \cdot\,,\cdot \rangle\!\rangle)$ with Weingarten-map $-\mathrm{Id}_{TM}$ w.r.t. the normal vector field $\nu_x := x$. In the Lorentzian setting Proposition 1.4.1 has an analog: there exists an isomorphism $\Sigma\widetilde{M}_{|_M} \longrightarrow \Sigma M$ (or to a double copy of it if n is odd) for which quite the same Gauss-type-formula as (1.21) holds but for which the relation (1.20) becomes $X \cdot i\nu \cdot \varphi = -X \underset{M}{\cdot} \varphi$ (or $-(X \underset{M}{\cdot} \oplus - X \underset{M}{\cdot})\varphi$ if n is odd) for all $X \in TM$ and $\varphi \in \Sigma\widetilde{M}_{|_M}$. As a consequence the restriction of any parallel spinor (resp. positive half spinor for n odd) onto H^n is a $\frac{i}{2}$-Killing spinor. Analogously the restriction of any spinor of the form $x \mapsto x \cdot \psi$, where ψ is parallel on $(\mathbb{R}^{n+1}, \langle\!\langle \cdot\,,\cdot \rangle\!\rangle)$ (and positive if n is odd), gives a $-\frac{i}{2}$-Killing spinor. We deduce from Proposition A.4.1.2 below that there are no other Killing spinors on (H^n, can) than those constructed and from Proposition A.2.1.3.b) that, if $n \geq 3$, then the space of twistor-spinors is exactly the direct sum of both spaces (for $n = 2$ see Proposition A.2.3).

Non simply-connected spaceforms may also admit non-zero twistor-spinors. This is the case if and only if the (or at least some) twistor-spinor on the corresponding model space is preserved by the π_1-action, see Proposition 1.4.2. This condition is not always fulfilled: for example there does not exist any non-zero imaginary Killing spinor on closed Riemannian spin manifolds (otherwise the Dirac operator would have a purely imaginary eigenvalue, which cannot be on closed manifolds). In dimension 2 flat tori together with their trivial spin structure are the only closed non simply-connected spaceforms admitting twistor-spinors, which are then parallel. In dimension 3 flat tori together with their trivial spin structure are also the only closed flat manifolds admitting twistor-spinors [206], however the quotient of $\mathbb{S}^3$ through any of its finite subgroups carries Killing spinors [13] (for lens spaces it has been proved independently in [84]). The real projective space $\mathbb{R}\mathrm{P}^n$ admits for every $n \equiv 3\ (4)$ non-zero real Killing spinors: in the notations of Corollary 2.1.5, if the spin structure is fixed by $\delta = 0$, then there exists a $2^{\frac{n-1}{2}}$-dimensional space of $-\frac{1}{2}$-Killing spinors if $n \equiv 3\ (8)$ and of $\frac{1}{2}$-ones if $n \equiv 7\ (8)$ respectively (vice-versa for $\delta = 1$). More generally there exists a formula for the dimension of the space of Killing spinors on every (closed) spaceform with positive curvature [38, Thm. 3]. However, up to the knowledge of the author, there does not exist any full classification of complete (even closed) flat manifolds admitting parallel spinors in dimension $n \geq 4$.

Non conformally flat examples are much more involved, see for instance [170] where the authors construct on every $\mathbb{C}^n$ a half conformally flat (non conformally flat) metric carrying a non-zero space of twistor-spinors. For the reader interested in twistor-spinors on singular spaces such as orbifolds we suggest [57].

A.2 Elementary properties of twistor-spinors

The following fundamental results on the twistor-spinor-equation are due to H. Baum, T. Friedrich and A. Lichnerowicz (see [56] for precise references):

Proposition A.2.1 (see [56]) *Let ψ be any twistor-spinor on an $n(\geq 2)$-dimensional Riemannian spin manifold (M^n, g). Then the following holds:*

1. *For any conformal change $\overline{g} := e^{2u} g$ of metric on M^n,*
$$\overline{P} = e^{\frac{u}{2}} \circ P \circ e^{-\frac{u}{2}},$$
where $\overline{P} := P_{\overline{g}}$. In particular $e^{\frac{u}{2}}\overline{\psi}$ is a twistor-spinor on $(M^n, \overline{g})$.
2. *If S denotes the scalar curvature of (M^n, g) then*
$$D^2\psi = \frac{nS}{4(n-1)}\psi. \tag{A.3}$$
3. *If $n \geq 3$ then:*
   a) *for every $X \in TM$,*
$$\nabla_X(D\psi) = \frac{n}{n-2}\Big(-\frac{1}{2}\mathrm{Ric}(X)\cdot\psi + \frac{S}{4(n-1)}X\cdot\psi\Big). \tag{A.4}$$
   b) $\dim(\mathrm{Ker}(P)) \leq 2^{[\frac{n}{2}]+1}$.
   c) *The zero-set of ψ is either discrete in M^n or M^n itself.*
   d) *If (M^n, g) is Einstein with $S \neq 0$ then ψ is the sum of two non-parallel Killing spinors.*
   e) *If $|\psi|$ is a non-zero constant then (M^n, g) is Einstein. Moreover either $S = 0$ and ψ is parallel or $S > 0$ and ψ is the sum of two real non-parallel Killing spinors.*
4. *If M^n is closed then* $\mathrm{Ker}(P)$ *is finite dimensional. In the case $\psi \neq 0$ if furthermore S is constant then either $S = 0$ and ψ is parallel or $S > 0$ and ψ is the sum of two real non-parallel Killing spinors.*

Proof:

1. For any $\varphi \in \Gamma(\Sigma M)$, $f \in C^\infty(M)$ and $X \in TM$ one has
$$\begin{aligned} P_X(f\varphi) &= \nabla_X(f\varphi) + \frac{1}{n}X\cdot D(f\varphi) \\ &\overset{(1.11)}{=} X(f)\varphi + f\nabla_X\varphi \\ &\quad + \frac{1}{n}X\cdot\mathrm{grad}(f)\cdot\varphi + \frac{f}{n}X\cdot D\varphi \\ &= X(f)\varphi + \frac{1}{n}X\cdot\mathrm{grad}(f)\cdot\varphi + fP_X\varphi. \end{aligned} \tag{A.5}$$

We deduce from (1.17) and (1.18) that

$$\begin{aligned}
\overline{P}_X\overline{\varphi} &= \overline{\nabla}_X\overline{\varphi} + \frac{1}{n}X\bar{\cdot}\overline{D}\overline{\varphi} \\
&= \overline{\nabla_X\varphi} - \frac{1}{2}\overline{X\cdot \mathrm{grad}_g(u)\cdot\varphi} - \frac{X(u)}{2}\overline{\varphi} \\
&\quad + \frac{e^{-u}}{n}X\bar{\cdot}(\overline{D\varphi} + \frac{n-1}{2}\overline{\mathrm{grad}_g(u)\cdot\varphi}) \\
&= \overline{P_X\varphi} - \frac{1}{2n}\overline{X\cdot \mathrm{grad}_g(u)\cdot\varphi} - \frac{X(u)}{2}\overline{\varphi}, \qquad \text{(A.6)}
\end{aligned}$$

so that

$$\begin{aligned}
\overline{P}_X(e^{\frac{u}{2}}\overline{\varphi}) &\overset{(A.5)}{=} \frac{e^{\frac{u}{2}}}{2}X(u)\overline{\varphi} + \frac{e^{\frac{u}{2}}}{2n}X\bar{\cdot}\mathrm{grad}_{\overline{g}}(u)\bar{\cdot}\,\overline{\varphi} + e^{\frac{u}{2}}\overline{P}_X\overline{\varphi} \\
&\overset{(A.6)}{=} \frac{e^{\frac{u}{2}}}{2}X(u)\overline{\varphi} + \frac{e^{\frac{u}{2}}}{2n}X\bar{\cdot}\mathrm{grad}_{\overline{g}}(u)\bar{\cdot}\,\overline{\varphi} \\
&\quad + e^{\frac{u}{2}}(\overline{P_X\varphi} - \frac{1}{2n}\overline{X\cdot \mathrm{grad}_g(u)\cdot\varphi} - \frac{X(u)}{2}\overline{\varphi}) \\
&= e^{\frac{u}{2}}\overline{P_X\varphi},
\end{aligned}$$

which shows 1.

2. Let $X, Y \in \Gamma(TM)$, then

$$\nabla_X\nabla_Y\psi = -\frac{1}{n}\nabla_XY\cdot D\psi - \frac{1}{n}Y\cdot\nabla_XD\psi,$$

from which we deduce

$$\begin{aligned}
R^{\nabla}_{X,Y}\psi &= \nabla_{[X,Y]}\psi - [\nabla_X, \nabla_Y]\psi \\
&= \frac{1}{n}(Y\cdot\nabla_XD\psi - X\cdot\nabla_YD\psi). \qquad \text{(A.7)}
\end{aligned}$$

Let $\{e_j\}_{1\le j\le n}$ be a local orthonormal basis of TM. From (1.9) we obtain

$$\begin{aligned}
\frac{1}{2}\mathrm{Ric}(X)\cdot\psi &= \frac{1}{n}\sum_{j=1}^{n}(e_j\cdot e_j\cdot\nabla_XD\psi - e_j\cdot X\cdot\nabla_{e_j}D\psi) \\
&= \frac{1}{n}(-(n-2)\nabla_XD\psi + X\cdot D^2\psi). \qquad \text{(A.8)}
\end{aligned}$$

Hence

$$-\frac{1}{2}S\psi \quad = \quad \frac{1}{2}\sum_{j=1}^{n}e_j\cdot\mathrm{Ric}(e_j)\cdot\psi$$

$$\begin{aligned}
&\overset{(A.8)}{=} \frac{1}{n}\sum_{j=1}^{n}(-(n-2)e_j\cdot\nabla_{e_j}D\psi + e_j\cdot e_j\cdot D^2\psi)\\
&= -\frac{2(n-1)}{n}D^2\psi
\end{aligned}$$

which shows 2.

3. Assume $n \geq 3$.

a) Coming back to (A.8) using (A.3) we obtain

$$\frac{1}{2}\mathrm{Ric}(X)\cdot\psi = -\frac{n-2}{n}\nabla_X D\psi + \frac{S}{4(n-1)}X\cdot\psi$$

which is the result.

b) It follows from (A.4) that ψ is a twistor-spinor if and only if the section $\psi \oplus D\psi$ of $\Sigma M \oplus \Sigma M$ is parallel w.r.t. the covariant derivative

$$\begin{aligned}
\nabla^T_X(\varphi_1\oplus\varphi_2) := &\Big(\nabla_X\varphi_1 + \frac{1}{n}X\cdot\varphi_2\Big)\\
&\oplus\Big(\frac{n}{n-2}(\frac{1}{2}\mathrm{Ric}(X)\cdot\varphi_1 - \frac{S}{4(n-1)}X\cdot\varphi_1) + \nabla_X\varphi_2\Big)
\end{aligned}$$

for all $\varphi_1, \varphi_2 \in \Gamma(\Sigma M)$. From $\mathrm{rk}_{\mathbb{C}}(\Sigma M) = 2^{[\frac{n}{2}]}$ we conclude.

c) We compute the Hessian of $|\psi|^2$. Let $X, Y \in \Gamma(TM)$. From

$$\begin{aligned}
X(|\psi|^2) &= 2\Re e\,(\langle\nabla_X\psi,\psi\rangle)\\
&= -\frac{2}{n}\Re e\,(\langle X\cdot D\psi,\psi\rangle)
\end{aligned}$$

one has

$$\begin{aligned}
\mathrm{Hess}(|\psi|^2)(X,Y) &= -\frac{2}{n}\Re e\,(\langle Y\cdot\nabla_X D\psi,\psi\rangle + \langle Y\cdot D\psi,\nabla_X\psi\rangle)\\
&\overset{(A.4)}{=} \frac{1}{n-2}\Re e(\langle Y\cdot\mathrm{Ric}(X)\cdot\psi,\psi\rangle)\\
&\quad -\frac{S}{2(n-1)(n-2)}\Re e(\langle Y\cdot X\cdot\psi,\psi\rangle)\\
&\quad +\frac{2}{n^2}\Re e(\langle Y\cdot D\psi, X\cdot D\psi\rangle)\\
&= -\frac{|\psi|^2}{n-2}\mathrm{ric}(X,Y)\\
&\quad +\Big(\frac{S|\psi|^2}{2(n-1)(n-2)} + \frac{2|D\psi|^2}{n^2}\Big)g(X,Y). \qquad (A.9)
\end{aligned}$$

If $\psi_p = 0$ then $\mathrm{Hess}(|\psi|^2)_p = \frac{2|D\psi|_p^2}{n^2} g_p$. In the case $\psi \neq 0$ one must have $(D\psi)_p \neq 0$ (otherwise the ∇^T-parallel section $\psi \oplus D\psi$ would vanish at p and hence identically), therefore the Hessian of $|\psi|^2$ is positive definite at p and the result follows.

d) If (M^n, g) is Einstein then (A.4) becomes

$$\begin{aligned}\nabla_X D\psi &= \frac{n}{n-2}\Big(-\frac{S}{2n} + \frac{S}{4(n-1)}\Big)X \cdot \psi \\ &= -\frac{S}{4(n-1)} X \cdot \psi\end{aligned}$$

for every $X \in TM$. In case $S \neq 0$ the spinor ψ can be written as $\psi = \psi_1 + \psi_{-1}$ where

$$\psi_{\pm 1} := \frac{1}{2}\left(\psi \pm \frac{1}{\lambda} D\psi\right)$$

and with $\lambda := \sqrt{\frac{nS}{4(n-1)}}$ (if $S < 0$ the square root may be chosen arbitrarily since changing its sign just exchanges the roles of ψ_1 and ψ_{-1}). We show that $\psi_{\pm 1}$ is a $\mp\frac{\lambda}{n}$-Killing spinor on (M^n, g): for every $X \in TM$,

$$\begin{aligned}\nabla_X \psi_{\pm 1} &= \frac{1}{2}(\nabla_X \psi \pm \frac{1}{\lambda}\nabla_X D\psi) \\ &= \frac{1}{2}\Big(-\frac{1}{n}X \cdot D\psi \pm \frac{1}{\lambda}(-\frac{\lambda^2}{n}X \cdot \psi)\Big) \\ &= \mp\frac{\lambda}{2n} X \cdot (\psi \pm \frac{1}{\lambda} D\psi) \\ &= \mp\frac{\lambda}{n} X \cdot \psi_{\pm 1},\end{aligned}$$

which shows d).

e) If $|\psi|$ is a non-zero constant then (A.9) implies

$$\mathrm{ric} = \Big(\frac{S}{2(n-1)} + \frac{2(n-2)|D\psi|^2}{n^2|\psi|^2}\Big) g,$$

that is, (M^n, g) is Einstein. Moreover from the latter equation the scalar curvature of (M^n, g) is then given by

$$S = \frac{4(n-1)}{n} \cdot \frac{|D\psi|^2}{|\psi|^2} \geq 0. \tag{A.10}$$

If $S > 0$ then using d) we deduce that $\psi = \psi_1 + \psi_{-1}$ where $\psi_{\pm 1}$ is a $\mp\sqrt{\frac{S}{4n(n-1)}}$-Killing spinor with $\sqrt{\frac{S}{4n(n-1)}} \in \mathbb{R}$. In case $S = 0$ the

identity (A.10) requires $D\psi = 0$ and hence $\nabla\psi = 0$, i.e., ψ is parallel on (M^n, g). This proves e).

4. Assume (M^n, g) to be closed and $n \geq 2$. From

$$|\nabla\varphi|^2 = |P\varphi|^2 + \frac{1}{n}|D\varphi|^2 \tag{A.11}$$

and

$$\begin{aligned} P^*P &\overset{(A.11)}{=} \nabla^*\nabla - \frac{1}{n}D^2 \\ &\overset{(1.15)}{=} \frac{n-1}{n}\nabla^*\nabla - \frac{S}{4n}\mathrm{Id} \end{aligned} \tag{A.12}$$

the operator P^*P is elliptic, hence its kernel is finite-dimensional. If furthermore $\psi \neq 0$ and S is constant then integrating the Hermitian product of (A.3) with ψ one obtains

$$\begin{aligned} \frac{nS}{4(n-1)} \int_M |\psi|^2 v_g &= \int_M \langle D^2\psi, \psi\rangle v_g \\ &= \int_M |D\psi|^2 v_g, \end{aligned}$$

which shows $S \geq 0$. On the other hand (A.3) already stands for the limiting-case in T. Friedrich's inequality (3.1), so that ψ must either be parallel (in case $S = 0$) or the sum of two real Killing spinors (in case $S > 0$). This shows 4. and concludes the proof. □

Note A.2.2 Actually Proposition A.2.1.4 implies that Proposition A.2.1.3.b) and c) hold on closed M in dimension $n = 2$ as well: on the one hand we deduce that the only compact orientable surfaces admitting twistor-spinors are $\mathbb{S}^2$ and $\mathbb{T}^2$ carrying any conformal class, the latter one being endowed with its trivial spin structure. For $\mathbb{S}^2$ (resp. $\mathbb{T}^2$) that space is 4-dimensional (resp. 2-dimensional), corresponding to the direct sum of the space of $\frac{1}{2}$-Killing spinors with that of $-\frac{1}{2}$-ones for the canonical metric (resp. to the space of parallel spinors for any flat metric in the conformal class). On the other hand the sum of two Killing spinors on $\mathbb{S}^2$ has at most one zero.

For $\mathbb{R}^2$ and $\mathbb{H}^2$ (see Note A.2.2 for closed M^2) one can again make the space of twistor-spinors explicit, however that space turns out to be infinite-dimensional:

Proposition A.2.3 *Let M be any non-empty connected open subset of $\mathbb{R}^2$ carrying its canonical conformal class and spin structure. Then the space of*

twistor-spinors of M for any metric in this conformal class is isomorphic to the direct sum of the space of holomorphic with that of anti-holomorphic functions on M. In particular the space of twistor-spinors on $\mathbb{R}^2$ and $\mathbb{H}^2$ respectively is infinite-dimensional.

Proof: Since the twistor-spinor-equation is conformally invariant (see Proposition A.2.1.1) we may assume that g is the canonical flat metric on M. Let $\{\varphi_+, \varphi_-\}$ be a basis of parallel spinors on M w.r.t. g such that $ie_1 \cdot e_2 \cdot \varphi_\pm = \pm\varphi_\pm$ where $\{e_1, e_2\}$ denotes the canonical basis of $\mathbb{R}^2$. Then there exist functions $f_+, f_- : M \longrightarrow \mathbb{C}$ such that $\psi = f_+\varphi_+ + f_-\varphi_-$. We compute $P\psi$: for every $X \in TM$,

$$P_X\psi = X(f_+)\varphi_+ + X(f_-)\varphi_- + \frac{1}{2}X \cdot (df_+ \cdot \varphi_+ + df_- \cdot \varphi_-).$$

For the Kähler structure J associated to g and the orientation of M one has however

$$\begin{aligned}
X \cdot Y \cdot \varphi_\pm &= \sum_{j,k=1}^{2} g(X, e_j)g(Y, e_k)e_j \cdot e_k \cdot \varphi \\
&= \sum_{j=1}^{2} g(X, e_j)g(Y, e_j)e_j \cdot e_j \cdot \varphi \\
&\quad +(g(X, e_1)g(Y, e_2) - g(X, e_2)g(Y, e_1))e_1 \cdot e_2 \cdot \varphi_\pm \\
&= -g(X, Y)\varphi - g(X, J(Y))e_1 \cdot e_2 \cdot \varphi_\pm \\
&= (-g(X, Y) \pm ig(X, J(Y)))\varphi_\pm \\
&= -2g(X, p_\pm(Y))\varphi_\pm,
\end{aligned}$$

where $p_\pm(X) := \frac{1}{2}(X \mp iJ(X))$. We deduce that

$$\begin{aligned}
P_X\psi &= X(f_+)\varphi_+ - g(X, p_+(df_+))\varphi_+ \\
&\quad +X(f_-)\varphi_- - g(X, p_-(df_-))\varphi_- \\
&= g(X, p_-(df_+))\varphi_+ + g(X, p_+(df_-))\varphi_-.
\end{aligned}$$

Therefore $P\psi = 0$ if and only if $p_\pm(df_\mp) = 0$, that is, if and only if f_+ is anti-holomorphic and f_- is holomorphic. From $(\mathbb{H}^2, \mathrm{can}_{\mathbb{H}^2}) = (\{z \in \mathbb{C} \text{ s.t. } |z| < 1\}, \frac{4}{(1-|z|^2)^2}\mathrm{can}_{\mathbb{R}^2})$ we conclude the proof. $\square$

Note that Proposition A.2.3 together with Note A.2.2 imply in particular that Proposition A.2.1.3.c) still holds in dimension $n = 2$, since holomorphic and anti-holomorphic functions on a surface vanish either on a discrete subset or identically.

Corollary A.2.4 ([56]) *Let (M^n, g) be an $n(\geq 3)$-dimensional Riemannian spin manifold carrying a non-zero twistor-spinor ψ. Then $Z_\psi := \{x \in M \,|\, \psi(x) = 0\}$ is discrete in M and $(\overline{M}^n := M^n \setminus Z_\psi, \overline{g} := \frac{1}{|\psi|^4} g)$ admits a real Killing spinor, which is parallel if $Z_\psi \neq \varnothing$.*

Proof: The statement on the zero set of ψ has been proved in Proposition A.2.1.3.c). From Proposition A.2.1.1 the spinor $\overline{\phi} := \frac{\overline{\psi}}{|\psi|}$ is a twistor-spinor on $(\overline{M}^n, \overline{g})$. In dimension $n \geq 3$ since it has constant norm it is the sum of two real Killing spinors (Proposition A.2.1.3.e)); furthermore

$$\begin{aligned}
\overline{D}\,\overline{\phi} &\overset{(1.18)}{=} |\psi|^2(\overline{D\phi} - \frac{n-1}{2}\overline{\frac{\mathrm{grad}(|\psi|^2)}{|\psi|^2} \cdot \phi}) \\
&= |\psi|^2(-\overline{\frac{\mathrm{grad}(|\psi|)}{|\psi|^2} \cdot \psi} + \frac{1}{|\psi|}\overline{D\psi} - \frac{n-1}{2}\overline{\frac{\mathrm{grad}(|\psi|^2)}{|\psi|^2} \cdot \phi}) \\
&= |\psi|\overline{D\psi} - \frac{n}{2}\overline{\mathrm{grad}(|\psi|^2) \cdot \phi},
\end{aligned}$$

so that, for any $p \in Z_\psi$, $|\overline{D}\,\overline{\phi}|(x) \underset{x \to p}{\longrightarrow} 0$ and by (A.10) for $\overline{g}$ and $\overline{\phi}$ one obtains $S_{\overline{g}}(x) \underset{x \to p}{\longrightarrow} 0$ (both w.r.t. the topology given by g on M). Applying again Proposition A.2.1.3.e), since $S_{\overline{g}}$ is constant it must vanish identically, hence $\overline{\phi}$ is parallel on $(\overline{M}^n, \overline{g})$ as soon as $Z_\psi \neq \varnothing$. $\square$

Note that the equivalent statement in dimension $n=2$ does not hold because of Proposition A.2.3.

A.3 Classification results for manifolds with twistor-spinors

Corollary A.2.4 induces a dichotomy in the classification of $n(\geq 3)$-dimensional Riemannian spin manifolds M carrying a non-zero twistor-spinor ψ: either $Z_\psi = \varnothing$ and then up to conformal change of metric M belongs to the class of manifolds admitting Killing spinors (which is studied in greater detail in Section A.4), or $Z_\psi \neq \varnothing$. In the latter case and for closed M, using the solution to the Yamabe problem about the existence of a constant scalar curvature metric in a conformal class A. Lichnerowicz showed:

Theorem A.3.1 (A. Lichnerowicz [178]) *Let (M^n, g), $n \geq 2$, be a closed Riemannian spin manifold carrying a non-trivial twistor-spinor ψ with non-empty zero-set Z_ψ. Then $|Z_\psi| = 1$ and (M^n, g) is conformally equivalent to $(\mathbb{S}^n, \mathrm{can})$.*

A relatively simple proof of Theorem A.3.1 can be found in [172].

For general M W. Kühnel and H.-B. Rademacher proved that the Ricci-flat metric $\frac{1}{|\psi|^4}g$ on $M^n \setminus Z_\psi$ is either flat or locally irreducible, more precisely:

Theorem A.3.2 (W. Kühnel and H.-B. Rademacher [171]) *Let (M^n, g) be a simply-connected Riemannian spin manifold carrying a non-trivial twistor-spinor ψ with non-empty zero-set Z_ψ and assume that the metric is not conformally flat. Then the following holds:*

1. *Every non-zero twistor-spinor on (M^n, g) vanishes exactly at Z_ψ.*
2. *For $N := \dim(\mathrm{Ker}(P))$ and the reduced holonomy group $\overline{\mathrm{Hol}} := \mathrm{Hol}(\overline{M}^n, \overline{g})$ of the Ricci-flat metric $\overline{g} := \frac{1}{|\psi|^4}g$ on $\overline{M}^n := M^n \setminus Z_\psi$ one has one of the following:*

   a) $n = 2m \geq 4$, $\overline{\mathrm{Hol}} = \mathrm{SU}_m$ *and* $N = 2$.
   b) $n = 4m \geq 8$, $\overline{\mathrm{Hol}} = \mathrm{Sp}_m$ *and* $N = m + 1$.
   c) $n = 7$, $\overline{\mathrm{Hol}} = \mathrm{G}_2$ *and* $N = 1$.
   d) $n = 8$, $\overline{\mathrm{Hol}} = \mathrm{Spin}_7$ *and* $N = 1$.

Theorem A.3.2, a proof of which can be found in the beautiful paper [172], actually requires Mc.K. Wang's classification of manifolds with non-zero parallel spinors, see Theorem A.4.2 below. Besides, we mention that up to now no example with reduced holonomy of type b), c) or d) has been described (an example with $\overline{\mathrm{Hol}} = \mathrm{SU}_m$ is constructed in [170]).

A.4 Classification results for manifolds with Killing spinors

We now come to the geometric properties specifically implied by the existence of a non-zero Killing spinor (Definition A.1.1.*ii*)).

Proposition A.4.1 *Let (M^n, g) be an $n(\geq 2)$-dimensional Riemannian spin manifold admitting a non-zero α-Killing spinor ψ for some $\alpha \in \mathbb{C}$.*

1. *The zero-set of ψ is empty. If furthermore α is real then $|\psi|$ is constant on M.*
2. *The space of α-Killing spinors on (M^n, g) is at most $2^{[\frac{n}{2}]}$-dimensional.*
3. *The manifold (M^n, g) is Einstein with scalar curvature $S = 4n(n-1)\alpha^2$. In particular α must be real or purely imaginary.*

Proof: By definition ψ is an α-Killing spinor if and only if it is a parallel section of ΣM w.r.t. the covariant derivative $X \mapsto \nabla_X - \alpha X\cdot$; moreover that covariant derivative is metric as soon as α is real. This shows 1. and 2.
Assuming $n \geq 3$ it follows from (A.4) that, for every $X \in TM$,

$$\mathrm{Ric}(X) \cdot \psi = (2(n-2)\alpha^2 + \frac{S}{2(n-1)})X \cdot \psi$$

(remember that ψ is a twistor-spinor satisfying $D\psi = -n\alpha\psi$). Since ψ has no zero we obtain that (M^n, g) is Einstein with scalar curvature $S = 4n(n-1)\alpha^2$. In dimension $n = 2$ the equation (A.7) for ψ is of the form

$$R^{\nabla}_{X,Y}\psi = \alpha^2(X \cdot Y - Y \cdot X) \cdot \psi,$$

hence comparing with (1.8) we obtain $S = 8\alpha^2$, which concludes the proof of 3. $\square$

In particular Myers' theorem implies that a complete Riemannian spin manifold without boundary and carrying a non-zero real Killing spinor must be compact.

Complete simply-connected Riemannian spin manifolds (M^n, g) carrying a non-zero space of α-Killing spinors have been classified by Mc.K. Wang [232] for $\alpha = 0$, C. Bär [37] for $\alpha \in \mathbb{R}^*$ and H. Baum [50] for $\alpha \in i\mathbb{R}^*$ respectively. In the first case, a parallel spinor must be a fixed point of the action of the lift of the reduced holonomy group to Spin_n. Excluding the trivial example $(M^n, g) = (\mathbb{R}^n, \mathrm{can})$, which has a maximal number of linearly independent parallel spinors, as well as local products (products of manifolds with parallel spinors carry themselves parallel spinors), the classification can be deduced from Berger-Simons' list of Riemannian holonomy groups.

Theorem A.4.2 (McK. Wang[232]) *Let (M^n, g) be an $(n \geq 2)$-dimensional simply-connected complete irreducible Riemannian spin manifold without boundary. Let N denote the dimension of* $\mathrm{Ker}(\nabla)$. *Then the manifold (M^n, g) carries a non-zero parallel spinor if and only if its reduced holonomy group* $\mathrm{Hol} := \mathrm{Hol}(M, g)$ *belongs to the following list:*

a) $\mathrm{Hol} = \mathrm{SU}_m$, *$n = 2m \geq 4$, and in that case $N = 2$.*
b) $\mathrm{Hol} = \mathrm{Sp}_m$, *$n = 4m \geq 8$, and in that case $N = m + 1$.*
c) $\mathrm{Hol} = \mathrm{G}_2$, *$n = 7$, and in that case $N = 1$.*
d) $\mathrm{Hol} = \mathrm{Spin}_7$, *$n = 8$, and in that case $N = 1$.*

There also exists a classification in the non-flat non-simply-connected case in terms of lifts the holonomy group to the spin group, see [203] where the proof of Theorem A.4.2 can also be found.

The classification when $\alpha \in \mathbb{R}^*$ relies on Mc.K. Wang's one using the following clever remark due to C. Bär and based on a geometric construction by S. Gallot (see reference in [37]): a spinor field is a $\frac{1}{2}$-Killing spinor on the manifold (M^n, g) if and only if the induced spinor field on its Riemannian cone $(M \times \mathbb{R}_+^*, t^2 g \oplus dt^2)$ is parallel. Hence C. Bär proved:

Theorem A.4.3 (C. Bär [37]) *Let (M^n, g) be an $(n \geq 2)$-dimensional simply-connected closed Riemannian spin manifold. Let p (resp. q) denote the dimension of the space of $\frac{1}{2}$- (resp. $-\frac{1}{2}$-) Killing spinors on (M^n, g). Then the manifold (M^n, g) carries up to scaling a non-zero $\pm\frac{1}{2}$-Killing*

spinor if and only if it is either the round sphere $(\mathbb{S}^n, \mathrm{can})$ *(in which case* $(p,q) = (2^{[\frac{n}{2}]}, 2^{[\frac{n}{2}]})$*) or one of the following:*

a) $(4m+1)$*-dimensional Einstein-Sasaki,* $m \geq 1$*, and in that case* $(p,q) = (1,1)$*.*
b) $(4m+3)$*-dimensional Einstein-Sasaki but not* 3*-Sasaki,* $m \geq 2$*, and in that case* $(p,q) = (0,2)$*.*
c) $(4m+3)$*-dimensional* 3*-Sasaki,* $m \geq 2$*, and in that case* $(p,q) = (0, m+2)$*.*
d) 6*-dimensional nearly Kähler non-Kähler, and in that case* $(p,q) = (1,1)$*.*
e) 7*-dimensional with a nice* 3*-form* ϕ *satisfying* $\nabla\phi = *\phi$ *but not Sasaki, and in that case* $(p,q) = (0,1)$*.*
f) 7*-dimensional Sasaki but not* 3*-Sasaki, and in that case* $(p,q) = (0,2)$*.*
g) 7*-dimensional* 3*-Sasaki, and in that case* $(p,q) = (0,3)$*.*

For the definitions of 3-Sasaki structures and nice forms as well as the proof of Theorem A.4.3 we refer to [37]. Parts of this classification had already been obtained in [89, 129, 90, 91, 92, 117, 87]. As an interesting fact, two higher eigenvalues of $(n = 4m+3)$-dimensional 3-Sasaki manifolds can be explicitly computed in terms of the scalar curvature: A. Moroianu showed [199] that on such manifolds both $-\sqrt{\frac{nS}{4(n-1)}} - 1$ and $\sqrt{\frac{nS}{4(n-1)}} + 2$ are Dirac eigenvalues with multiplicities at least $3m$ and m respectively. The proof relies on a clever combination of the Killing vector fields provided by the 3-Sasaki structure and the Killing spinors.

In the last case ($\alpha \in i\mathbb{R}^*$) the classification turns out to rely on totally different arguments. Studying in detail the level sets of the length function of an imaginary Killing spinor H. Baum proved the following theorem, which relies on Theorem A.4.2 but where the assumption $\pi_1(M) = 1$ turns out not to be necessary.

Theorem A.4.4 (H. Baum [50]) *Let* (M^n, g) *be an* $(n \geq 2)$*-dimensional connected complete Riemannian spin manifold without boundary. Then* (M^n, g) *admits a non-trivial* α*-Killing spinor with* $\alpha \in i\mathbb{R}^*$ *if and only if it is isometric to a warped product of the form*

$$(N \times \mathbb{R}, e^{4i\alpha t} h \oplus dt^2),$$

where (N^{n-1}, h) *is a complete connected Riemannian spin manifold carrying a non-zero parallel spinor.*

Of course the n-dimensional real hyperbolic space can be obtained as a warped product of this form (take $(N,h) = (\mathbb{R}^{n-1}, \mathrm{can})$); in the disk model, this corresponds to the foliation by horospheres tangential to a fixed point on the boundary at infinity.

It was noticed by O. Hijazi, S. Montiel and A. Roldán [140] that the geometric part of Theorem A.4.4 - i.e., that (M^n, g) must be a pseudo-hyperbolic space - follows from a classical argument by Yoshihiro Tashiro (see reference

in [140]), namely from the existence of a smooth non-zero real-valued function f on M such that

$$\mathrm{Hess}(f) - fg = 0.$$

Here, up to rescaling g, the function $f := |\psi|^2$, where ψ is a non-zero α-Killing spinor on (M^n, g), satisfies that equation (use (A.9) when $n \geq 3$). Nevertheless this argument does not describe the correspondence between spinor fields on M and those on the warped product, see [50] for a rigorous treatment of that point.

Theorem A.4.4 generalizes to the situation where the constant α is replaced by a smooth imaginary-valued function, in which case a similar statement on the structure of the underlying manifold holds.

Theorem A.4.5 (H.-B. Rademacher [209]) *Let (M^n, g) be an $n(\geq 2)$-dimensional connected complete Riemannian spin manifold without boundary. For a given non-zero $\alpha \in C^\infty(M, i\mathbb{R})$ assume the existence of a non-zero section ψ of ΣM satisfying*

$$\nabla_X \psi = \alpha X \cdot \psi$$

for all $X \in TM$. Then (M^n, g) is isometric either to the real hyperbolic space of constant sectional curvature $4\alpha^2$ (in particular α must be constant) or to a warped product of the form $(N \times \mathbb{R}, e^{4i\int_0^t \alpha(s)ds} h \oplus dt^2)$, where (N^{n-1}, h) is a complete connected Riemannian spin manifold admitting a non-zero parallel spinor and $\alpha \in C^\infty(\mathbb{R}, i\mathbb{R})$.

Conversely, for any given $\alpha \in C^\infty(\mathbb{R}, i\mathbb{R})$ and (N^{n-1}, h) as above, the warped product $(N \times \mathbb{R}, e^{4i\int_0^t \alpha(s)ds} h \oplus dt^2)$ admits a non-zero section ψ of ΣM satisfying $\nabla_X \psi = \alpha X \cdot \psi$ for all $X \in TM$, where α is extended by a constant onto the N-factor. Interestingly enough, there exist compact quotients admitting such spinors for some necessarily non-constant α's, see [209, Thm. 1] and references therein. The proof of Theorem A.4.5 relies on the classification of complete Riemannian manifolds carrying a non-isometric conformal closed Killing field, see [209] for details.

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