

---

## Appendix

### A Some Consequences of Egorov's Theorem

Let us first recall the statement of Egorov's theorem :

**Theorem A.1** *Let  $(X, \mu)$  be some measure space with finite positive measure. Consider a sequence  $(g_n)$  such that  $g_n \rightarrow g$  almost everywhere. Then, for each  $\varepsilon > 0$ , there exists a measurable  $E \subset X$  such that*

$$\mu(X \setminus E) < \varepsilon \quad \text{and } g_n \rightarrow g \text{ uniformly on } E.$$

#### A.1 The Product Limit Theorem

We now give a corollary of Egorov's theorem, established by DiPerna and Lions [44], which is used repeatedly for the study of the Boltzmann equation and its hydrodynamic limits.

**Proposition A.2** *Let  $(X, \mu)$  be some measure space with finite positive measure. Consider two sequences of real-valued measurable functions defined on  $X$ , denoted  $(f_n)$  and  $(g_n)$ . If  $(g_n)$  is bounded in  $L^\infty(X)$  such that  $g_n \rightarrow g$  almost everywhere, and  $f_n \rightharpoonup f$  weakly in  $L^1(X)$  then  $f_n g_n \rightharpoonup fg$  weakly in  $L^1(X)$ .*

*Proof.* Without loss of generality, we can assume that  $g = 0$ .

Let  $\delta$  be any fixed nonnegative constant. The sequence  $(f_n)$ , being relatively weakly compact in  $L^1(X)$ , is equiintegrable. Thus, by picking  $\alpha > 0$  sufficiently small, one has for every measurable set  $A$  such that  $\mu(A) < \alpha$ ,

$$\int_A |f_n - f|(x) d\mu(x) < \delta \text{ uniformly in } n. \quad (\text{A.1})$$

Fix such a constant  $\alpha$ . By Egorov's theorem, as  $(g_n)$  is bounded in  $L^\infty(X)$ , and  $g_n \rightarrow 0$  almost everywhere, there exists a measurable set  $A$  such that  $\mu(A) < \alpha$  and

$$g_n \rightarrow 0 \text{ as } n \rightarrow \infty \text{ uniformly on } X \setminus A. \quad (\text{A.2})$$

Fix such a set  $A$ . Then,

$$\int |(f_n - f)g_n| d\mu = \int_A |(f_n - f)g_n| d\mu + \int_{X \setminus A} |(f_n - f)g_n| d\mu$$

By (A.1) the first term in the right-hand side satisfies

$$\int_A |(f_n - f)g_n| d\mu \leq \|g\|_{L^\infty(X)} \int_A |f_n - f| d\mu \leq \|g\|_{L^\infty(X)} \delta,$$

whereas by (A.2) the second term converges to 0 as  $n \rightarrow \infty$  :

$$\int_{X \setminus A} |(f_n - f)g_n| d\mu \leq \left( \sup_n \|f_n - f\|_{L^1(X)} \right) \|g_n\|_{L^\infty(X \setminus A)} \rightarrow 0.$$

Finally

$$\lim_{n \rightarrow \infty} \int |(f_n - f)g_n| d\mu \leq \|g\|_{L^\infty(X)} \delta,$$

but  $\delta$  was arbitrary, whence Proposition A.2 holds.  $\square$

## A.2 An Asymptotic Result of Variables Separating

For the study of boundary conditions, we also need the following variant of the Product Limit theorem, which has been proved in [82] :

**Proposition A.3** *Let  $(X, \mu_X)$  and  $(Y, \mu_Y)$  be two measure spaces of finite measures. Consider a family of nonnegative functions  $(\chi_\varepsilon)$  uniformly bounded in  $L^\infty(X \times Y)$  converging almost everywhere to 1 on  $X \times Y$ , and a family  $(\rho_\varepsilon)$  of nonnegative functions of  $L^1(X)$  such that*

$$(\chi_\varepsilon \rho_\varepsilon) \text{ is relatively weakly compact in } L^1(X \times Y)$$

*Then any limit point  $\rho$  of  $(\chi_\varepsilon \rho_\varepsilon)$  belongs to  $L^1(X)$ , namely does not depend on  $y \in Y$ .*

Before giving the proof, let us notice that if  $(\rho_\varepsilon)$  were supposed to be relatively weakly compact in  $L^1(X \times Y)$  then the conclusion of the lemma would be straightforward.

*Proof.* Consider a subsequence of  $(\chi_\varepsilon \rho_\varepsilon)$  (still denoted  $(\chi_\varepsilon \rho_\varepsilon)$ ) converging to  $\rho$  as  $\varepsilon \rightarrow 0$ . Let  $\delta$  be any fixed nonnegative constant. As  $\chi_\varepsilon$  converges almost everywhere to 1, there exist  $A \subset X \times Y$  with  $|X \times Y \setminus A| \leq \delta$ , and  $\varepsilon_0 > 0$  such that

$$\forall \varepsilon \leq \varepsilon_0, \quad \chi_\varepsilon|_A \geq \frac{1}{2}$$

By the Product Limit theorem, as  $\left(\frac{\mathbf{1}_A}{\chi_\varepsilon}\right)_{\varepsilon \leq \varepsilon_0}$  is bounded in  $L^\infty(X \times Y)$  and converges a.e. to  $\mathbf{1}_A$ ,

$$\rho_\varepsilon \mathbf{1}_A = \rho_\varepsilon \chi_\varepsilon \frac{\mathbf{1}_A}{\chi_\varepsilon} \rightharpoonup \rho \mathbf{1}_A \text{ weakly in } L^1(X \times Y).$$

Define

$$A_X = \left\{ x \in X / \int \mathbf{1}_A(x, y) d\mu_Y(y) \geq \frac{\mu_Y(Y)}{2} \right\}.$$

From Bienaymé Tchebichev's inequality

$$\begin{aligned} \mathbf{1}_{A_X}(x) \rho_\varepsilon(x) &\leq \mathbf{1}_{A_X}(x) \rho_\varepsilon(x) \frac{2 \int \mathbf{1}_A(x, y) d\mu_Y(y)}{\mu_Y(Y)} \\ &= \frac{2 \mathbf{1}_{A_X}}{\mu_Y(Y)} \int \rho_\varepsilon(x) \mathbf{1}_A(x, y) d\mu_Y(y) \end{aligned} \quad (\text{A.3})$$

we deduce that  $(\mathbf{1}_{A_X} \rho_\varepsilon)$  is weakly compact in  $L^1(X)$ . Then, up to extraction of a subsequence,

$$\mathbf{1}_{A_X} \rho_\varepsilon \rightharpoonup \tilde{\rho} \text{ weakly in } L^1(X)$$

By the Product Limit theorem, as  $(\chi_\varepsilon)$  is bounded in  $L^\infty(X \times Y)$  and converges a.e. to 1,

$$\mathbf{1}_{A_X} \rho_\varepsilon \chi_\varepsilon \rightharpoonup \tilde{\rho} \text{ weakly in } L^1(X \times Y)$$

from which we deduce that

$$\rho \mathbf{1}_{A_X} = \tilde{\rho} \in L^1(X) \quad (\text{A.4})$$

On the other hand, if  $x \notin A_X$ ,

$$\int \mathbf{1}_{X \times Y \setminus A}(x, y) d\mu_Y(y) = \int (1 - \mathbf{1}_A)(x, y) d\mu_Y(y) \geq \frac{\mu_Y(Y)}{2}$$

Then,

$$\begin{aligned} \int \mathbf{1}_{X \setminus A_X}(x) d\mu_X(x) &\leq \int \mathbf{1}_{X \setminus A_X}(x) \frac{2 \int \mathbf{1}_{X \times Y \setminus A}(x, y) d\mu_Y(y)}{\mu_Y(Y)} d\mu_X(x) \\ &\leq \frac{2}{\mu_Y(Y)} \delta \end{aligned} \quad (\text{A.5})$$

As there exists  $A_X$  satisfying (A.4) and (A.5) for all  $\delta > 0$ ,  $\rho$  depends only on the variable  $x$ , and thus  $\rho \in L^1(X)$ .  $\square$

## B Classical Trace Results on the Solutions of Transport Equations

In order to deal with kinetic equations involving reflection conditions at the boundary, we need the following fundamental result due to Cessenat [32] following Bardos [3] and Ukai [104], which allows to define the trace of any weak solution to the free-transport equation in a very general setting.

### B.1 Definition of the Trace

**Proposition B.1** *For any smooth subset  $\Omega$  of  $\mathbf{R}^3$ , denote by  $W^p(\Omega)$  the functional space*

$$\{f \in L^p(\mathbf{R} \times \Omega \times \mathbf{R}^3) / (\text{St}\partial_t + v \cdot \nabla_x)f \in L^p(\mathbf{R} \times \Omega \times \mathbf{R}^3)\}.$$

*Then the trace operator*

$$\gamma : f \in W^p(\Omega) \mapsto f|_{\partial\Omega} \in L^p(\mathbf{R} \times \partial\Omega \times \mathbf{R}^3, dtd\sigma_x |v \cdot n(x)|^2 (1 + |v|)^{-1} dv)$$

*is continuous.*

*Proof.* Without loss of generality, we can restrict our attention to nonnegative functions.

By Green's formula, we have for any bounded function  $\varphi \in C^1(\bar{\Omega} \times \mathbf{R}^3)$ ,

$$\begin{aligned} & p \iiint \varphi(x, v) f^{p-1} (\text{St}\partial_t + v \cdot \nabla_x) f(t, x, v) dtdxdv \\ & + \iiint (v \cdot \nabla_x) \varphi(x, v) f^p(t, x, v) dtdxdv \\ & = \iiint \varphi(x, v) f^p(t, x, v) (v \cdot n(x)) dtd\sigma_x dv \end{aligned} \quad (\text{B.6})$$

As  $\Omega$  is assumed to be smooth, there exists some vector field  $n \in W^{1,\infty}(\bar{\Omega})$  which coincides with the outward unit normal at the boundary. Thus, choosing

$$\varphi(x, v) = \frac{(v \cdot n(x))}{(1 + |v|^2)^{1/2}}$$

we get

$$\begin{aligned} & \|f|_{\partial\Omega}\|_{L^p(dtd\sigma_x |v \cdot n(x)|^2 (1 + |v|)^{-1} dv)} \\ & \leq C \left( \|f\|_{L^p(dtdxdv)} + \|(\text{St} + v \cdot \nabla_x)f\|_{L^p(dtdxdv)} \right), \end{aligned} \quad (\text{B.7})$$

which concludes the proof.  $\square$

### B.2 Free Transport with Reflection at the Boundary

In order to extend regularity and dispersion results to free transport with reflection at the boundary, we have then to establish a priori estimates on the incoming flux (which is defined in terms of the outgoing flux).

**Proposition B.2** *Let  $\Omega$  be any smooth subset of  $\mathbf{R}^3$ , and  $f \in L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)$  be a solution to the free-transport equation*

$$\text{St}\partial_t f + v \cdot \nabla_x f = S$$

*supplemented with Maxwell's boundary condition*

$$f|_{\Sigma_-} = (1 - \alpha)Lf|_{\Sigma_+} + \alpha Kf|_{\Sigma_+} \quad \text{on } \Sigma_-$$

where the outgoing/incoming sets  $\Sigma_+$  and  $\Sigma_-$  at the boundary  $\partial\Omega$  are defined by

$$\Sigma_{\pm} = \{(x, v) \in \partial\Omega \times \mathbf{R}^3, \quad \pm n(x) \cdot v > 0\},$$

the local reflection operator  $L$  is given by

$$Lf(x, v) = f(x, v - 2(v \cdot n(x))n(x)),$$

and the diffuse reflection operator  $K$  is given by

$$Kf(x, v) = M_w(v) \int_{v' \cdot n(x) > 0} f(x, v') (v' \cdot n(x)) dv'$$

for some normalized Maxwellian distribution  $M_w$  characterizing the state of the wall.

Then, there exists some nonnegative constant  $C$  (depending on the Lipschitz norm of  $n$ ) such that

$$\iiint f|_{\Sigma} |v \cdot n(x)| dv dx dt \leq \frac{C}{\alpha} (\|f\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)} + \|S\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)}).$$

*Proof.* The smoothness assumption made on the boundary implies the existence of a vector field  $n$  which belongs to  $W^{1,\infty}(\bar{\Omega})$  and coincides with the outward unit normal vector at the boundary. Therefore, multiplying the transport equation by  $(n(x) \cdot v)/(1 + |v|)$  and integrating with respect to all variables, we get

$$\iiint \frac{v \cdot n(x)}{1 + |v|} (v \cdot \nabla_x) f dv dx dt = \iiint \frac{v \cdot n(x)}{1 + |v|} S dv dx dt$$

Then, using Green's formula, we get

$$\iiint f|_{\Sigma} \frac{(v \cdot n(x))^2}{1 + |v|} dv d\sigma_x dt \leq C(\|f\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)} + \|S\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)}).$$

In particular

$$\alpha \iiint_{v \cdot n < 0} Kf|_{\Sigma_+} \frac{(v \cdot n(x))^2}{1 + |v|} dv d\sigma_x dt \leq C(\|f\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)} + \|S\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)}).$$

By definition of  $K$ , we have the spreading condition

$$\int_{v \cdot n(x) > 0} f|_{\Sigma_+} (v \cdot n(x)) dv \leq \kappa_0 \int_{v \cdot n(x) < 0} Kf|_{\Sigma_+} \frac{(v \cdot n(x))^2}{1 + |v|} dv.$$

We then deduce that

$$\iiint_{v \cdot n(x) > 0} f|_{\Sigma_+} (v \cdot n(x)) dv d\sigma_x dt \leq \frac{C\kappa_0}{\alpha} (\|f\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)} + \|S\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)}).$$

On the other hand, the normalization condition on  $M_w$  implies

$$-\int_{v \cdot n(x) < 0} f|_{\Sigma}(v \cdot n(x)) dv = \int_{v \cdot n(x) > 0} f|_{\Sigma}(v \cdot n(x)) dv.$$

We therefore have

$$\iiint f|_{\Sigma} |v \cdot n(x)| dv d\sigma_x dt \leq \frac{2C\kappa_0}{\alpha} (\|f\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)} + \|S\|_{L^1(\mathbf{R} \times \Omega \times \mathbf{R}^3)}).$$

which is the expected inequality.  $\square$

**Remark B.3** *Note that, in the case of a purely specular reflection, we do not obtain such a bound on the trace. Nevertheless, because the specular reflection is completely transparent in the weak formulation of the transport equation (by obvious symmetry properties), the study is actually much easier (very similar to the case when there is no boundary). We refer to the work [63] by Hamdache for a careful treatment of that case.*

## C Some Consequences of Chacon's Biting Lemma

Let us first recall the statement of Chacon's Biting Lemma [21]:

**Theorem C.1** *Let  $(X, \mu)$  be some measure space with finite positive measure. Consider a sequence  $(g_n)$  bounded in  $L^1(X)$ . Then, there exists a subsequence  $(g'_n)$  of  $(g_n)$  and some function  $g \in L^1(X)$  such that  $g'_n \rightarrow g$  in the sense of Chacon, meaning that for each  $\varepsilon > 0$ , there exists a measurable  $E \subset X$  such that  $\mu(X \setminus E) \leq \varepsilon$  and*

$$g'_n \rightarrow g \text{ in } L^1(E).$$

### C.1 From Renormalized Convergence to Chacon's Convergence

We now give an extension of Chacon's Biting Lemma to the space of measurable and almost everywhere finite functions, established by Mischler [85] to study the traces of kinetic equations in the framework of renormalized solutions.

This requires the following definition of *renormalized convergence* (see [85] and the references therein for basic results concerning renormalized convergence) :

**Definition C.2** *A sequence  $(g_n)$  of measurable and almost everywhere finite functions is said to converge in renormalized sense to some measurable and almost everywhere finite function  $g$ , if for any increasing sequence  $\Gamma_M \in C \cap L^\infty(\mathbf{R}^+)$  converging simply to  $\text{Id}|_{\mathbf{R}^+}$  as  $M \rightarrow \infty$ , and any subsequence  $(g'_n)$  of  $(g_n)$ , there exists a sequence  $\gamma_M$  and a subsequence  $(g''_n)$  of  $(g'_n)$  such that*

$$\Gamma_M(g''_n) \rightharpoonup \gamma_M \text{ weakly-}^* \text{ in } L^\infty(X) \text{ and } \gamma_M \rightarrow g \text{ a.e. in } X.$$

We have then the following relation between renormalized convergence and convergence in the sense of Chacon (see [85]) :

**Proposition C.3** *Let  $(X, \mu)$  be some measure space with finite positive measure. Consider a sequence  $(g_n)$  of measurable and almost everywhere finite functions, such that*

$$g_n \rightarrow g \text{ in renormalized sense,}$$

*where  $g$  is some measurable and almost everywhere finite function. Then,*

$$\lim_{M \rightarrow +\infty} \sup_n \mu(\{g_n \geq M\}) = 0,$$

*and there exists a subsequence  $(g'_n)$  of  $(g_n)$  such that  $g'_n \rightarrow g$  in the sense of Chacon.*

*Proof.* Without loss of generality, we can restrict our attention to nonnegative functions.

- We first prove the  $L^0$  bound, arguing by contradiction. Since  $g$  is a measurable and almost everywhere finite function, for any arbitrary  $\varepsilon > 0$ , there exists  $E \subset X$  such that  $\mu(X \setminus E) < \varepsilon$  and  $g \in L^1(E)$ .

If there is no  $m$  such that

$$\sup_n \mu(\{g_n \geq m\}) < \varepsilon$$

there exists an increasing sequence  $(n_m)$  such that

$$\forall m \in \mathbf{N}, \quad \mu(\{g_{n_m} \geq m\}) \geq \varepsilon.$$

Therefore, for any  $l \in \mathbf{N}$  and any  $m \geq l$ ,

$$\int_E \Gamma_l(g_{n_m}) d\mu \geq l\varepsilon,$$

where  $\Gamma_l$  is some smooth version of the truncation  $x \mapsto \min(x, l)$ . Passing to the limit  $m \rightarrow \infty$  leads to

$$\int_E g d\mu \geq \int_E \gamma_l d\mu \geq \varepsilon l,$$

by definition of the renormalized convergence. Thus

$$\int_E g d\mu \geq \lim_{l \rightarrow \infty} \varepsilon l,$$

which gives the expected contradiction.

- We have then to prove the convergence in the sense of Chacon. Given  $\varepsilon > 0$ , one can choose  $E \subset X$  such that  $\mu(X \setminus E) < \varepsilon$  and  $g \in L^1(E)$ .

We construct a first subsequence  $(n_l)$  such that

$$\int_E \Gamma_l(g_{n_l}) d\mu \leq \int_E g d\mu + \frac{1}{l}.$$

Then, for any  $m \leq l$ ,  $\Gamma_m(g_{n_l}) \leq \Gamma_l(g_{n_l})$  so that

$$g = \limsup_{m \rightarrow \infty} \gamma_m \leq \limsup_{m \rightarrow \infty} \liminf_{l \rightarrow \infty} \Gamma_l(g_{n_l}),$$

where the  $\liminf$  is taken in the sense of Chacon. By Fatou's lemma and the definition of the renormalized convergence, we also have

$$\forall A \subset X, \quad \int_A \limsup_{l \rightarrow \infty} \Gamma_l(g_{n_l}) d\mu \leq \limsup_{l \rightarrow \infty} \int_A \Gamma_l(g_{n_l}) d\mu \leq \int_A g d\mu.$$

Combining all these results show that

$$\Gamma_l(g_{n_l}) \rightarrow g \text{ in the sense of Chacon on } E.$$

In other words, there exists  $E'$  such that  $\mu(E \setminus E') < \varepsilon$  and

$$\Gamma_l(g_{n_l}) \rightarrow g \text{ weakly in } L^1(E').$$

Furthermore, since  $(g_n)$  is bounded in  $L^0$ , one can choose a second subsequence (still denoted  $(g_{n_l})$ ) such that the sets  $Z_L$  defined by

$$Z_L = \{\exists l \geq L / g_{n_l} \neq \Gamma_l(g_{n_l})\}$$

satisfy

$$\mu(Z_L) \leq \sum_{l \geq L} \mu(\{g_{n_l} > l\}) \rightarrow 0 \text{ as } L \rightarrow \infty.$$

Finally, choosing  $L$  large enough such that  $\mu(Z_L) < \varepsilon$ , and setting  $E'' = E' \setminus Z_L$ , we obtain

$$\mu(X \setminus E'') < 3\varepsilon \text{ and } g_{n_l} \rightarrow g \text{ weakly in } L^1(E'').$$

We conclude thanks to a diagonal process.  $\square$

## C.2 A Result of Partial Equiintegrability

In order to characterize the limiting incoming flux in Maxwell's boundary condition, we also need the following variant of the previous result, also established in [85].

**Proposition C.4** *Let  $(X, \mu_X)$  be some measure space with finite positive measure, and  $(\mu_Y(x))_{x \in X}$  a family of probability measures on the space  $Y$ , and denote  $\langle g \rangle_Y = \int g d\mu_Y$ .*



Let  $h \in C(\mathbf{R}^+, \mathbf{R}^+)$  be some convex function of class  $C^2(\mathbf{R}_*^+)$  with superlinear growth at infinity, and such that  $H(s, t) \equiv (h(t) - h(s))(t - s)$  is convex.

Consider a sequence  $(g_n)$  of measurable nonnegative and almost everywhere finite functions on  $X \times Y$ , such that

$$\int \left( \langle h(g_n) \rangle_Y - h(\langle g_n \rangle_Y) \right) d\mu_X \leq C, \quad (C.8)$$

$\langle g_n \rangle_Y \rightarrow \bar{g}$  in renormalized sense on  $X$

Then there exists  $g \in L^1(X \times Y, d\mu_X(x)d\mu_Y(y))$  and a subsequence  $(g'_n)$  of  $(g_n)$  such that, for every  $\varepsilon > 0$ , one can find some  $E \subset X$  with

$$\mu_X(X \setminus E) < \varepsilon \text{ and } g'_n \rightharpoonup g \text{ weakly in } L^1(E \times Y).$$

In particular,  $\bar{g}(x) = \langle g \rangle_Y$  almost everywhere.

Furthermore

$$\int \left( \langle h(g) \rangle_Y - h(\langle g \rangle_Y) \right) d\mu_X \leq C.$$

*Proof.* The point to be understood here is how the (convex) functional which generalizes the Darrozes-Guiraud information allows to gain some equiintegrability with respect to  $y$ , and thus to establish the convergence of some integral quantities.

• By Proposition C.3, we deduce from the renormalized convergence in (C.8) that, up to extraction of a subsequence,

$$\langle g_n \rangle_Y \rightarrow \bar{g} \text{ in the sense of Chacon.}$$

In particular, for any  $\varepsilon > 0$ , there exists  $A \subset X$  such that

$$\mu_X(X \setminus A) < \varepsilon \quad \text{and} \quad \langle g_n \rangle_Y \rightarrow \bar{g} \text{ weakly in } L^1(A).$$

Thanks to Dunford-Pettis' lemma, there is therefore a (nonnegative increasing) convex function  $\Phi$  with superlinear growth at infinity such that  $\Phi(0) = 0$ ,  $\Phi'(0) > 0$  and

$$\int_A \Phi(\langle g_n \rangle_Y) d\mu_X \leq C_1.$$

We are then able to build some (nonnegative increasing) convex function  $\Psi$  with superlinear growth at infinity such that  $\Psi(0) = 0$ ,  $\Psi'(0) > 0$  and

$$\begin{aligned} \Psi &\leq \Phi, \\ h - \Psi &\text{ is convex.} \end{aligned}$$

Jensen's inequality, written for the function  $h - \Psi$ , combined with the uniform bound in (C.8), gives therefore

$$\int \left( \langle \Psi(g_n) \rangle_Y - \Psi(\langle g_n \rangle_Y) \right) d\mu_X \leq C,$$

and thus

$$\iint_{A \times Y} \Psi(g_n) d\mu_Y d\mu_X \leq C + C_1.$$

By Dunford-Pettis' lemma, we obtain that  $(g_n)$  belongs to a weak compact subset of  $L^1(A \times Y)$ .

We conclude, by a diagonal process, that there is a function  $g$  in  $L^1(X \times Y)$  and a subsequence of  $(g_n)$  which converges to  $g$  in the sense stated in Proposition C.4. In particular, for any  $\varepsilon > 0$ , there exists  $A \subset X$  such that

$$\mu_X(X \setminus A) < \varepsilon \quad \text{and} \quad \langle g_n \rangle_Y \rightarrow \langle g \rangle_Y \text{ in } L^1(A).$$

Identifying the limit leads to  $\bar{g} = \langle g \rangle_Y$  for almost every  $x \in X$ .

- It remains then to take limits in the uniform bound in (C.8).

We start by proving that, for all  $x \in X$ ,

$$E : g \in L^1(Y) \mapsto E(g) = \left\langle h(g) \right\rangle_Y - h\left(\langle g(y) \rangle_Y\right)$$

is a convex functional. We proceed indeed by approximation replacing  $h$  by  $h_\varepsilon : z \mapsto h(z + \varepsilon) - h(\varepsilon)$ . As  $h_\varepsilon \in C^2(\mathbf{R}^+)$ ,

$$DE_\varepsilon(g_1) \cdot g_2 = \left\langle h'_\varepsilon(g_1) \cdot g_2 \right\rangle_Y - h'_\varepsilon\left(\langle g_1 \rangle_Y\right) \langle g_2 \rangle_Y.$$

Therefore, by Jensen's inequality, we have for  $g_1, g_2 \in L^\infty(Y)$

$$\begin{aligned} & \left( DE_\varepsilon(g_1) - DE_\varepsilon(g_2) \right) \cdot (g_1 - g_2) \\ &= \left\langle H(g_1, g_2) \right\rangle_Y - H\left(\langle g_1 \rangle_Y, \langle g_2 \rangle_Y\right) \geq 0, \end{aligned}$$

so that  $DE_\varepsilon$  is monotone and  $E_\varepsilon$  is convex on  $L^\infty(Y)$ . Passing to the limit  $\varepsilon \rightarrow 0$ , we then obtain that  $E$  is convex on  $L^\infty(Y)$  :

$$\forall t \in [0, 1], \quad E(tg_1 + (1-t)g_2) \leq tE(g_1) + (1-t)E(g_2).$$

Now let  $g_1, g_2 \in L^1(Y)$ . If  $h(g_1)$  or  $h(g_2)$  does not belong to  $L^1(Y)$  the convex inequality obviously holds. In the other case, we choose two sequences  $(g_{1\varepsilon})$  and  $(g_{2\varepsilon})$  of  $L^\infty(Y)$  such that  $g_{1\varepsilon} \nearrow g_1$  and  $g_{2\varepsilon} \nearrow g_2$  almost everywhere, and passing to the limit  $\varepsilon \rightarrow 0$  in the convex inequality written for  $g_{1\varepsilon}$  and  $g_{2\varepsilon}$ , we get by Lebesgue's theorem and Fatou's lemma that

$$\forall t \in [0, 1], \quad E(tg_1 + (1-t)g_2) \leq tE(g_1) + (1-t)E(g_2).$$

Now, if  $g_1, g_2 \in L^1(X \times Y)$ , then  $g_1(x, \cdot), g_2(x, \cdot) \in L^1(Y)$  for almost all  $x \in X$  and, integrating the previous convex inequality, we obtain that the functional

$$g \in L^1(X \times Y) \mapsto \int E(g) d\mu_X$$

is convex. Furthermore, by Fatou's Lemma, this functional is lower semi-continuous.

From the convergence stated in Proposition C.4 and established previously, we then deduce that

$$\int \left( \langle h(g) \rangle_Y - h(\langle g \rangle_Y) \right) d\mu_X \leq C,$$

which concludes the proof.  $\square$

---

## References

1. V.I. Agoshkov: *Spaces of functions with differential difference characteristics and smoothness of the solutions of the transport equations*, Dokl. Akad. Nauk SSSR **276** (1984), 1289–1293.
2. J.-P. Aubin: *Un théorème de compacité.*, C. R. Acad. Sci. Paris **256** (1963), 5042–5044.
3. C. Bardos : *Problèmes aux limites pour les équations aux dérivées partielles du premier ordre à coefficients réels; théorèmes d'approximation; application à l'équation de transport*, Ann. Scient. Ec. Norm. Sup. **3** (1970), 185–233.
4. C. Bardos, F. Golse, C.D. Levermore: *Fluid Dynamic Limits of the Boltzmann Equation I*, J. Stat. Phys. **63** (1991), 323–344.
5. C. Bardos, F. Golse, C.D. Levermore: *Fluid Dynamic Limits of Kinetic Equations II: Convergence Proofs for the Boltzmann Equation*, Comm. Pure & Appl. Math **46** (1993), 667–753.
6. C. Bardos, F. Golse, C.D. Levermore. *The acoustic limit for the Boltzmann equation*. Arch. Ration. Mech. Anal. **153** (2000), 177–204.
7. C. Bardos, S. Ukai: *The classical incompressible Navier-Stokes limit of the Boltzmann equation*, Math. Models and Methods in the Appl. Sci. **1** (1991), 235–257.
8. C. Bardos, O. Pironneau. *A formalism for the differentiation of conservation laws*, C. R. Math. Acad. Sci. Paris **335** (2002), 839–845.
9. C. Bardos, E. Titi: *Euler Equations of Incompressible Ideal Fluids*, Preprint (2008).
10. J.T. Beale, T. Kato, A.J. Majda: *Remarks on the breakdown of smooth solutions for the 3D Euler equations*, Commun. Math. Phys. **1994** (1984), 61–66.
11. A. Bobylev. *The theory of the nonlinear, spatially uniform Boltzmann equation for Maxwellian molecules*, Sov. Sci. Rev. C. Math. Phys. **7** (1998), 111–233.
12. A. Bobylev, C. Cercignani. *On the rate of entropy production for the Boltzmann equation*, J. Statis. Phys **94** (1999), 603–618.
13. A. Bobylev, D. Levermore., in preparation.
14. L. Boltzmann. *Über die Prinzipien der Mechanik: Zwei Akademische Antrittsreden*. Leipzig : S. Hirzel, 1903.
15. F. Bouchut, L. Desvillettes. *A proof of the smoothing properties of the positive part of Boltzmann's kernel*, Rev. Mat. Iberoamericana **14** (1998), 47–61.

16. F. Bouchut, F. Golse, M. Pulvirenti: "Kinetic Equations and Asymptotic Theory", L. Desvillettes & B. Perthame ed., Editions scientifiques et médicales Elsevier, Paris, 2000.
17. Y. Brenier. *Convergence of the Vlasov-Poisson system to the incompressible Euler equations*, Comm. Partial Differential Equations **25** (2000), 737–754.
18. A. Bressan, R. Colombo. *The semigroup generated by  $2 \times 2$  conservation laws*, Arch. Rational Mech. Anal. **133** (1995), 1–75.
19. A. Bressan, S. Bianchini. *Vanishing viscosity solutions of nonlinear hyperbolic systems*, Ann. of Math. **161** (2005), 223–342.
20. H. Brézis, J.P. Bourguignon. *Remarks on the Euler equation*. J. Functional Analysis **15** (1974), 341–363.
21. J. Brooks, R. Chacon. *Continuity and compactness of measures*. Adv. in Math. **37** (1980), 16–26.
22. L. Caffarelli, R. Kohn, L. Nirenberg. *Partial regularity of suitable weak solutions of the Navier-Stokes equations*, Comm. Pure Appl. Math. **35** (1982), 771–831.
23. R.E. Caflisch: *The Boltzmann equation with a soft potential. I. Linear, spatially-homogeneous*, Commun. Math. Phys. **74** (1980), 71–95.
24. R.E. Caflisch: *The fluid dynamic limit of the nonlinear Boltzmann equation*, Comm. on Pure and Appl. Math. **33** (1980), 651–666.
25. M. Cannone, Y. Meyer, F. Planchon. *Solutions auto-similaires des équations de Navier-Stokes*, Séminaire sur les Equations aux Dérivées Partielles **8**, Ecole Polytech., Palaiseau, 1994.
26. F. Castella, B. Perthame. *Estimations de Strichartz pour les équations de transport cinétique*, C. R. Acad. Sci. Paris Sr. I Math. **322** (1996), 535–540.
27. J.-Y. Chemin. *Fluides parfaits incompressibles [Incompressible perfect fluids]*, Astérisque **230**, 1995.
28. C. Cercignani: "The Boltzmann Equation and Its Applications" Springer-Verlag, New-York NY, 1988.
29. C. Cercignani. *Global weak solutions of the Boltzmann equation*, J. Stat. Phys. **118** (2005), 333–342.
30. C. Cercignani, R. Illner. *Global weak solutions to the Boltzmann equation in a slab with diffusive boundary conditions*, Arch. Rational Mech. Anal. **134** (1996), 1–16.
31. C. Cercignani, R. Illner, M. Pulvirenti, *The Mathematical Theory of Dilute Gases*, Springer Verlag, New York NY, 1994.
32. M. Cessenat. *Théorèmes de traces pour des espaces de fonctions de la neutronique*, C. R. Acad. Sci. Paris **300** (1985), 89–92.
33. S. Chapman, T.G. Cowling: "The mathematical theory of non-uniform gases: An account of the kinetic theory of viscosity, thermal conduction, and diffusion in gases". Cambridge University Press, New York, 1960.
34. P. Constantin, C. Foias: "Navier-Stokes equations." Chicago Lectures in Mathematics. University of Chicago Press, Chicago, 1988.
35. R. Courant, K.O. Friedrichs: "Supersonic flow and shock waves." Springer-Verlag, New York-Heidelberg, 1976.
36. C.M. Dafermos: "Hyperbolic conservation laws in continuum physics." Grundlehren der Mathematischen Wissenschaften **325**, Springer-Verlag, Berlin, 2000.
37. Darrozès, J.S.; Guiraud, J.P. *Généralisation formelle du théorème H en présence de parois*, C.R. Acad. Sci. Paris **262** (1966), 368–371.

38. Dautray, R.; Lions, J.L.. “Analyse mathématique et calcul numérique pour les sciences et les techniques” **9**, Masson, Paris, 1988.
39. C. De Lellis: *Notes on hyperbolic systems of conservation laws and transport equations*, Preprint (2006).
40. C. De Lellis, L. Székelyhidi: *On admissibility criteria for weak solutions of the Euler equations*, Preprint (2008).
41. J.-M. Delort. *Existence de nappes de tourbillon en dimension deux [Existence of vortex sheets in dimension two]*, J. Amer. Math. Soc. **4** (1991), 553–586.
42. A. DeMasi, R. Esposito, J. Lebowitz, *Incompressible Navier-Stokes and Euler Limits of the Boltzmann Equation*, Comm Pure Appl. Math. **42** (1990), 1189–1214.
43. L. Desvillettes, F. Golse, *A remark concerning the Chapman-Enskog asymptotics*, in “Advances in kinetic theory and computing”, B. Perthame ed., 191–203, Ser. Adv. Math. Appl. Sci., 22, World Sci. Publishing, River Edge, NJ, 1994.
44. R.J. DiPerna, P.-L. Lions: *On the Cauchy problem for the Boltzmann equation: global existence and weak stability results*, Ann. of Math. **130** (1990), 321–366.
45. R. Ellis & M. Pinsky. *The first and second fluid approximations to the linearized Boltzmann equation.*, J. Math. Pures Appl. **54** (1975), 125–156.
46. R. Esposito, R. Marra, H.T. Yau. *Navier-Stokes equations for stochastic particle systems on the lattice*. Comm. Math. Phys. **182** (1996), 395–456.
47. J. Francheteau & G. Métivier: *Existence de chocs faibles pour des systmes quasi-linéaires hyperboliques multidimensionnels [Existence of weak shocks for multidimensional hyperbolic quasilinear systems]*, Astérisque **268** (2000).
48. H. Fujita, T. Kato. *On the Navier-Stokes initial value problem. I*. Arch. Rational Mech. Anal. **16** (1964), 269–315.
49. T. Gallay, C.E. Wayne, *Invariant manifolds and the long-time asymptotics of the Navier-Stokes and vorticity equations on  $\mathbf{R}^2$* , Arch. Rat. Mech. Anal. **163** (2002), 209–258.
50. J. Glimm: *Solutions in the large for nonlinear hyperbolic systems of equations*, Comm. on Pure and Appl. Math. **18** (1965), 697–715.
51. F. Golse, D. Levermore. *The Stokes-Fourier and Acoustic Limits for the Boltzmann Equation*, Comm. on Pure and Appl. Math. **55** (2002), 336–393.
52. F. Golse, D. Levermore, L. Saint-Raymond. *La méthode de l’entropie relative pour les limites hydrodynamiques de modèles cinétiques Séminaire Equations aux dérivées partielles* (Polytechnique) (1999-2000).
53. F. Golse, P.-L. Lions, B. Perthame, R. Sentis: *Regularity of the moments of the solution of a transport equation*, J. Funct. Anal. **76** (1988), 110–125.
54. F. Golse, L. Saint-Raymond. *The Navier-Stokes limit of the Boltzmann equation for bounded collision kernels*, Invent. Math. **155** (2004), no. 1, 81–161.
55. F. Golse, L. Saint-Raymond. *The Navier-Stokes limit of the Boltzmann equation for hard potentials*, to appear in J. Math. Pures Appl. (2008).
56. F. Golse, L. Saint-Raymond: *Velocity averaging in  $L^1$  for the transport equation*, C. R. Acad. Sci. **334** (2002), 557–562.
57. F. Golse, L. Saint-Raymond. *A remark about the asymptotic theory of the Boltzmann equation*, preprint.
58. F. Golse, L. Saint-Raymond. *Hydrodynamic limits for the Boltzmann equation*, Riv. Mat. Univ. Parma **4** (2005), 1–144.
59. H. Grad. *Asymptotic theory of the Boltzmann equation II Rarefied Gas Dynamics* (Proc. of the 3rd Intern. Sympos. Palais de l’UNESCO, Paris, 1962) Vol. I, 26–59.

60. E. Grenier. *Quelques limites singulières oscillantes*, Séminaire sur les Equations aux Dérivées Partielles **21**, Ecole Polytech., Palaiseau, 1995.
61. E. Grenier. *On the nonlinear instability of Euler and Prandtl equations*, Comm. Pure Appl. Math. **53** (2000), 1067–1091.
62. Y. Guo. *The Boltzmann equation in the whole space*, Indiana Univ. Math. J. **53** (2004), 1081–1094.
63. K. Hamdache: *Problèmes aux limites pour l'équation de Boltzmann: existence globale de solutions [Boundary value problems for the Boltzmann equation: global existence of solutions]*, Comm. Partial Differential Equations **13** (1988), 813–845.
64. H. Hertz. *Die Principien der Mechanik*. Leipzig, 1894.
65. D. Hilbert. *Begründung der kinetischen Gastheorie* Math. Ann. **72** (1912), 562–577.
66. M. Lachowicz, *On the initial layer and the existence theorem for the nonlinear Boltzmann equation*. Math. Methods Appl. Sci. **9** (1987), 342–366.
67. P.D. Lax: *Hyperbolic systems of conservation laws. II*, Comm. Pure Appl. Math. **10** (1957), 537–566.
68. C. Landim, H.T. Yau. *Fluctuation-dissipation equation of asymmetric simple exclusion processes*. Probab. Theory Related Fields **108** (1997), 321–356.
69. O.E. Lanford, *Time evolution of large classical systems* Lect. Notes in Physics **38**, J. Moser ed., 1–111, Springer Verlag (1975).
70. J. Leray. *Etude de diverses équations intégrales non linéaires et quelques problèmes que pose l'hydrodynamique*, J. Math. Pures Appl. **9** (1933), 1–82.
71. L. Lichtenstein: *Über einige Existenz Problem der hydrodynamik homogener unzusammendrückbarer, reibunglosser Flüssigkeiten und die Helmholtzschen Wirbelsätze*, Mat. Zeit. Phys. **23** (1925), 89154; **26** (1927), 193323; **32** (1930), 608.
72. P.-L. Lions, *Compactness in Boltzmann's equation via Fourier integral operators and applications* J. Math. Kyoto Univ. **34** (1994), 391–427, 429–461, 539–584.
73. P.-L. Lions, *Conditions at infinity for Boltzmann's equation*, Comm. Partial Differential Equations **19** (1994), 335–367.
74. P.-L. Lions: “Mathematical Topics in Fluid Mechanics, Vol. 1: Incompressible Models”, The Clarendon Press, Oxford University Press, New York, 1996.
75. P.-L. Lions, N. Masmoudi: *From Boltzmann Equation to the Navier-Stokes and Euler Equations I*, Archive Rat. Mech. & Anal. **158** (2001), 173–193.
76. P.-L. Lions, N. Masmoudi: *From Boltzmann Equation to the Navier-Stokes and Euler Equations II*, Archive Rat. Mech. & Anal. **158** (2001), 195–211.
77. P.-L. Lions, N. Masmoudi: *Une approche locale de la limite incompressible*, C. R. Acad. Sci. Paris Sr. I Math. **329** (1999), 387–392.
78. T. P. Liu. *Solutions in the large for the equations of nonisentropic gas dynamics*, Indiana Univ. Math. J. **26** (1977), 147–177.
79. T.-P. Liu, S.-H. Yu: *Boltzmann equation: micro-macro decompositions and positivity of shock profiles*, Comm. Math. Phys. **246** (2004), 133–179.
80. T.-P. Liu, T. Yang, S.-H. Yu: *Energy method for Boltzmann equation*, Phys. D **188** (2004), 178–192.
81. E. Mach. *Die Mechanik in ihrer Entwicklung*. Leipzig, zweite Auflage, 1889.
82. N. Masmoudi, L. Saint-Raymond. *From the Boltzmann equation to the Stokes-Fourier system in a bounded domain*, Comm. Pure Appl. Math., **56** (2003), 1263–1293.
83. G. Métivier, K. Zumbrun. *Existence of semilinear relaxation shocks*. Preprint 2008.

84. S. Mischler. *On the initial boundary value problem for the Vlasov-Poisson-Boltzmann system*, Comm. Math. Phys. **210** (2000), 447–466.
85. S. Mischler. *Kinetic equations with Maxwell boundary condition*, Preprint (2002).
86. C. B. Morrey. *On the derivation of the equations of hydrodynamics from Statistical Mechanics*, Commun. Pure Appl. Math., **8** (1955), 279–290.
87. C. Mouhot. *Rate of convergence to equilibrium for the spatially homogeneous Boltzmann equation with hard potentials*, Comm. Math. Phys. **261** (2006), 629–672.
88. T. Nishida. *Fluid dynamical limit of the nonlinear Boltzmann equation to the level of the compressible Euler equation*. Comm. Math. Phys. **61** (1978), 119–148.
89. S. Olla, S. Varadhan, H. Yau, *Hydrodynamical limit for a Hamiltonian system with weak noise*, Commun. Math. Phys. **155** (1993), 523–560.
90. B. Perthame: *Introduction to the collision models in Boltzmann’s theory*, in “Modeling of Collisions”, P.-A. Raviart ed., Masson, Paris, 1997.
91. J. Quastel and H.-T. Yau, *Lattice gases, large deviations, and the incompressible Navier-Stokes equations*, Ann. of Math. **148** (1998), 51–108.
92. L. Saint-Raymond: *From the BGK model to the Navier-Stokes equations*, Ann. Sci. Ecole Norm. Sup. (4) **36** (2003), 271–317.
93. L. Saint-Raymond: *Du modèle BGK de l’équation de Boltzmann aux équations d’Euler des fluides incompressibles*, Bull. Sci. Math. **126** (2002), 493–506.
94. L. Saint-Raymond: *Convergence of solutions to the Boltzmann equation in the incompressible Euler limit*, Arch. Ration. Mech. Anal. **166** (2003), 47–80.
95. L. Saint-Raymond: *Hydrodynamic limits: some improvements of the relative entropy method*, Ann. Inst. H. Poincaré, to appear, 2008. doi:10.1016/j.anihpc.2008.01.001.
96. S. Schochet. *Fast singular limits of hyperbolic PDEs*, J. Differential Equations **114** (1994), 476–512.
97. D. Serre: “Systèmes de lois de conservation. I. Hyperbolicité, entropies, ondes de choc. Fondations.” Diderot Editeur, Paris, 1996.
98. D. Serre: “Systèmes de lois de conservation. II. Structures géométriques, oscillation et problèmes mixtes. Fondations.” Diderot Editeur, Paris, 1996.
99. J. Serrin: *On the interior regularity of weak solutions of the Navier-Stokes equations*. Arch. Rational Mech. Anal. **9** (1962), 187–195.
100. T. Sideris: *Formation of Singularities in 3D Compressible Fluids*, Commun. Math. Phys. **101** (1985), 475–485.
101. J. Smoller: “Shock waves and reaction-diffusion equations.” Grundlehren der Mathematischen Wissenschaften, Springer-Verlag, New York-Berlin, 1983.
102. R. Temam: “Navier-Stokes equations. Theory and numerical analysis.” Studies in Mathematics and its Applications, North-Holland Publishing Co., Amsterdam-New York-Oxford, 1977.
103. S. Ukai. *On the existence of global solutions of a mixed problem for the nonlinear Boltzmann equation*, Proc. of the Japan Acad. **50** (1974), 179–184.
104. S. Ukai: *Eigenvalues of the neutron transport operator for a homogeneous finite moderator*, J. Math. Anal. Appl. **18** (1967), 297–314.
105. S. Ukai, K. Asano: *The Euler limit and initial layer of the nonlinear Boltzmann equation*, Hokkaido Math. J. **12** (1983), 311–332.
106. C. Villani: “A review of mathematical topics in collisional kinetic theory.” Handbook of mathematical fluid dynamics, North-Holland, Amsterdam, 2002.
107. P. Volkmann. *Einführung in das Studium der theoretischen Physik*. Leipzig, 1900.



- 108. H. T. Yau. *Relative entropy and hydrodynamics of Ginzburg-Landau models*, Lett. Math. Phys. **22** (1991), 63–80.
- 109. H. T. Yau. *Scaling limit of particle systems, incompressible Navier-Stokes equation and Boltzmann equation*. Proceedings of the International Congress of Mathematicians, Doc. Math. **3** (1998), 193–202.
- 110. V. Yudovitch. *Non stationary flows of an ideal incompressible fluid*, Zh. Vych. Math. **3** (1963), 1032–1066.

---

# Index

- a priori estimates, 47
- accommodation coefficient, 29
- acoustic waves, 27, 104
- adiabaticity, 27
- asymptotic expansion
  - Chapman-Enskog's expansion, 26
  - Hilbert's expansion, 26
- asymptotic expansions
  - Chapman-Enskog's expansion, 8
  - Hilbert's expansion, 8
- Bienaymé-Tchebichev inequality, 73
- Boltzmann's H theorem, 21
- Boltzmann-Grad limit, 15
- boundary condition
  - Dirichlet boundary condition, 29, 81
  - Maxwellian reflection, 19
  - Navier boundary condition, 29, 81
  - Robin boundary condition, 29
  - specular reflection, 18
- bulk velocity, 24
- Carleman's parametrization, 59
- Chacon's Biting Lemma, 41, 174
- chaos assumption, 16
- coercivity estimate, 61
- collision invariants, 20, 60
- collisional cross-section, 17
- compensated compactness, 148
- compressibility, 27
- conservation defects, 93
- conservation laws, 20, 80
- constraints
  - Boussinesq constraint, 90, 131
  - incompressibility constraint, 90, 131
- continuity of the collision operator, 64, 141
- convexity, 38
- Darrozès-Guiraud information, 35, 53
- dispersion properties, 70
- dissipative solution, 125
- Dunford-Pettis criterion, 51, 72
- Egorov's Theorem, 169
- elastic collisions, 15
- entropic convergence, 88
- entropic solution, 165
- entropy dissipation, 21, 36, 54
- entropy-entropy dissipation inequality, 56
- equiintegrability with respect to  $v$ , 72
- ergodicity, 4
- filtering method, 146
- free transport
  - averaging lemma, 67
  - dispersion, 70
  - mixing property, 70, 78
- friction, 81
- gain term, 17, 40
- Grad's cut-off assumption, 18
- Grad's moment method, 9
- grazing collisions, 18

- harmonic analysis, 149
- Hilbert's decomposition, 58
- infinitesimal Maxwellian, 87, 131
- kinetic models
  - collisional models, 14
  - free transport, 14
  - mean-field models, 14
- Knudsen number, 23
- law of large numbers, 2
- linearized collision operator, 31, 58
- loss term, 17, 39
- Mach number, 24
- mean free path, 23
- mean free time, 23
- modulated entropy, 126
- moment method, 89
- potential for interaction, 166
- potential for interaction, 45
- Product Limit theorem, 37, 169
- Rankine-Hugoniot curves, 166
- relative entropy, 36, 48
- relative entropy method, 6, 86, 131
- relaxation layer, 27
- renormalized
  - convergence, 175
  - fluctuation, 52
  - solution, 34
  - trace variations, 53
- scaling invariance, 80
- singularity, 164
- spectral gap, 61
- speed of sound, 22
- state relation, 25
- stochastic lattice models, 7
- strong-weak stability, 83, 123
- Strouhal number, 23
- system of conservation laws, 163
  - hyperbolic, 164
  - symmetrizable, 164
- thermal speed, 22
- velocity averaging, 37, 67
- viscosity, 27
- weak compactness method, 86
- weak solution, 44
- well-prepared initial data, 128
- Young's inequality, 50

# Lecture Notes in Mathematics

For information about earlier volumes  
please contact your bookseller or Springer  
LNM Online archive: [springerlink.com](http://springerlink.com)

- Vol. 1788: A. Vasil'ev, Moduli of Families of Curves for Conformal and Quasiconformal Mappings (2002)
- Vol. 1789: Y. Sommerhäuser, Yetter-Drinfel'd Hopf algebras over groups of prime order (2002)
- Vol. 1790: X. Zhan, Matrix Inequalities (2002)
- Vol. 1791: M. Knebusch, D. Zhang, Manis Valuations and Prüfer Extensions I: A new Chapter in Commutative Algebra (2002)
- Vol. 1792: D. D. Ang, R. Gorenflo, V. K. Le, D. D. Trong, Moment Theory and Some Inverse Problems in Potential Theory and Heat Conduction (2002)
- Vol. 1793: J. Cortés Monforte, Geometric, Control and Numerical Aspects of Nonholonomic Systems (2002)
- Vol. 1794: N. Pytheas Fogg, Substitution in Dynamics, Arithmetics and Combinatorics. Editors: V. Berthé, S. Ferenczi, C. Mauduit, A. Siegel (2002)
- Vol. 1795: H. Li, Filtered-Graded Transfer in Using Non-commutative Gröbner Bases (2002)
- Vol. 1796: J.M. Melenk, hp-Finite Element Methods for Singular Perturbations (2002)
- Vol. 1797: B. Schmidt, Characters and Cyclotomic Fields in Finite Geometry (2002)
- Vol. 1798: W.M. Oliva, Geometric Mechanics (2002)
- Vol. 1799: H. Pajot, Analytic Capacity, Rectifiability, Menger Curvature and the Cauchy Integral (2002)
- Vol. 1800: O. Gabber, L. Ramero, Almost Ring Theory (2003)
- Vol. 1801: J. Azéma, M. Émery, M. Ledoux, M. Yor (Eds.), Séminaire de Probabilités XXXVI (2003)
- Vol. 1802: V. Capasso, E. Merzbach, B. G. Ivanoff, M. Dozzi, R. Dalang, T. Mountford, Topics in Spatial Stochastic Processes. Martina Franca, Italy 2001. Editor: E. Merzbach (2003)
- Vol. 1803: G. Dolzmann, Variational Methods for Crystalline Microstructure – Analysis and Computation (2003)
- Vol. 1804: I. Cherednik, Ya. Markov, R. Howe, G. Lusztig, Iwahori-Hecke Algebras and their Representation Theory. Martina Franca, Italy 1999. Editors: V. Baldoni, D. Barbasch (2003)
- Vol. 1805: F. Cao, Geometric Curve Evolution and Image Processing (2003)
- Vol. 1806: H. Broer, I. Hoveijn, G. Lunther, G. Vegter, Bifurcations in Hamiltonian Systems. Computing Singularities by Gröbner Bases (2003)
- Vol. 1807: V. D. Milman, G. Schechtman (Eds.), Geometric Aspects of Functional Analysis. Israel Seminar 2000-2002 (2003)
- Vol. 1808: W. Schindler, Measures with Symmetry Properties (2003)
- Vol. 1809: O. Steinbach, Stability Estimates for Hybrid Coupled Domain Decomposition Methods (2003)
- Vol. 1810: J. Wengenroth, Derived Functors in Functional Analysis (2003)
- Vol. 1811: J. Stevens, Deformations of Singularities (2003)
- Vol. 1812: L. Ambrosio, K. Deckelnick, G. Dziuk, M. Mimura, V. A. Solonnikov, H. M. Sonner, Mathematical Aspects of Evolving Interfaces. Madeira, Funchal, Portugal 2000. Editors: P. Colli, J. F. Rodrigues (2003)
- Vol. 1813: L. Ambrosio, L. A. Caffarelli, Y. Brenier, G. Buttazzo, C. Villani, Optimal Transportation and its Applications. Martina Franca, Italy 2001. Editors: L. A. Caffarelli, S. Salsa (2003)
- Vol. 1814: P. Bank, F. Baudoin, H. Föllmer, L.C.G. Rogers, M. Sonner, N. Touzi, Paris-Princeton Lectures on Mathematical Finance 2002 (2003)
- Vol. 1815: A. M. Vershik (Ed.), Asymptotic Combinatorics with Applications to Mathematical Physics. St. Petersburg, Russia 2001 (2003)
- Vol. 1816: S. Albeverio, W. Schachermayer, M. Tala-grand, Lectures on Probability Theory and Statistics. Ecole d'Été de Probabilités de Saint-Flour XXX-2000. Editor: P. Bernard (2003)
- Vol. 1817: E. Koelink, W. Van Assche (Eds.), Orthogonal Polynomials and Special Functions. Leuven 2002 (2003)
- Vol. 1818: M. Bildhauer, Convex Variational Problems with Linear, nearly Linear and/or Anisotropic Growth Conditions (2003)
- Vol. 1819: D. Masser, Yu. V. Nesterenko, H. P. Schlickeweï, W. M. Schmidt, M. Waldschmidt, Diophantine Approximation. Cetraro, Italy 2000. Editors: F. Amoroso, U. Zannier (2003)
- Vol. 1820: F. Hiai, H. Kosaki, Means of Hilbert Space Operators (2003)
- Vol. 1821: S. Teufel, Adiabatic Perturbation Theory in Quantum Dynamics (2003)
- Vol. 1822: S.-N. Chow, R. Conti, R. Johnson, J. Mallet-Paret, R. Nussbaum, Dynamical Systems. Cetraro, Italy 2000. Editors: J. W. Macki, P. Zecca (2003)
- Vol. 1823: A. M. Anile, W. Allegretto, C. Ringhofer, Mathematical Problems in Semiconductor Physics. Cetraro, Italy 1998. Editor: A. M. Anile (2003)
- Vol. 1824: J. A. Navarro González, J. B. Sancho de Salas,  $\mathcal{C}^\infty$  – Differentiable Spaces (2003)
- Vol. 1825: J. H. Bramble, A. Cohen, W. Dahmen, Multiscale Problems and Methods in Numerical Simulations, Martina Franca, Italy 2001. Editor: C. Canuto (2003)
- Vol. 1826: K. Dohmen, Improved Bonferroni Inequalities via Abstract Tubes. Inequalities and Identities of Inclusion-Exclusion Type. VIII, 113 p, 2003.
- Vol. 1827: K. M. Pilgrim, Combinations of Complex Dynamical Systems. IX, 118 p, 2003.
- Vol. 1828: D. J. Green, Gröbner Bases and the Computation of Group Cohomology. XII, 138 p, 2003.
- Vol. 1829: E. Altman, B. Gaujal, A. Hordijk, Discrete-Event Control of Stochastic Networks: Multimodularity and Regularity. XIV, 313 p, 2003.

- Vol. 1830: M. I. Gil', Operator Functions and Localization of Spectra. XIV, 256 p, 2003.
- Vol. 1831: A. Connes, J. Cuntz, E. Guentner, N. Higson, J. E. Kaminker, Noncommutative Geometry, Martina Franca, Italy 2002. Editors: S. Doplicher, L. Longo (2004)
- Vol. 1832: J. Azéma, M. Émery, M. Ledoux, M. Yor (Eds.), Séminaire de Probabilités XXXVII (2003)
- Vol. 1833: D.-Q. Jiang, M. Qian, M.-P. Qian, Mathematical Theory of Nonequilibrium Steady States. On the Frontier of Probability and Dynamical Systems. IX, 280 p, 2004.
- Vol. 1834: Yo. Yomdin, G. Comte, Tame Geometry with Application in Smooth Analysis. VIII, 186 p, 2004.
- Vol. 1835: O.T. Izhboldin, B. Kahn, N.A. Karpenko, A. Vishik, Geometric Methods in the Algebraic Theory of Quadratic Forms. Summer School, Lens, 2000. Editor: J.-P. Tignol (2004)
- Vol. 1836: C. Năstăsescu, F. Van Oystaeyen, Methods of Graded Rings. XIII, 304 p, 2004.
- Vol. 1837: S. Tavaré, O. Zeitouni, Lectures on Probability Theory and Statistics. Ecole d'Été de Probabilités de Saint-Flour XXXI-2001. Editor: J. Picard (2004)
- Vol. 1838: A.J. Ganesh, N.W. O'Connell, D.J. Wischik, Big Queues. XII, 254 p, 2004.
- Vol. 1839: R. Gohm, Noncommutative Stationary Processes. VIII, 170 p, 2004.
- Vol. 1840: B. Tsirelson, W. Werner, Lectures on Probability Theory and Statistics. Ecole d'Été de Probabilités de Saint-Flour XXXII-2002. Editor: J. Picard (2004)
- Vol. 1841: W. Reichel, Uniqueness Theorems for Variational Problems by the Method of Transformation Groups (2004)
- Vol. 1842: T. Johnsen, A. L. Knutsen,  $K_3$  Projective Models in Scrolls (2004)
- Vol. 1843: B. Jefferies, Spectral Properties of Noncommuting Operators (2004)
- Vol. 1844: K.F. Siburg, The Principle of Least Action in Geometry and Dynamics (2004)
- Vol. 1845: Min Ho Lee, Mixed Automorphic Forms, Torus Bundles, and Jacobi Forms (2004)
- Vol. 1846: H. Ammari, H. Kang, Reconstruction of Small Inhomogeneities from Boundary Measurements (2004)
- Vol. 1847: T.R. Bielecki, T. Björk, M. Jeanblanc, M. Rutkowski, J.A. Scheinkman, W. Xiong, Paris-Princeton Lectures on Mathematical Finance 2003 (2004)
- Vol. 1848: M. Abate, J. E. Fornæss, X. Huang, J. P. Rosay, A. Tumanov, Real Methods in Complex and CR Geometry, Martina Franca, Italy 2002. Editors: D. Zaitsev, G. Zampieri (2004)
- Vol. 1849: Martin L. Brown, Heegner Modules and Elliptic Curves (2004)
- Vol. 1850: V.D. Milman, G. Schechtman (Eds.), Geometric Aspects of Functional Analysis. Israel Seminar 2002-2003 (2004)
- Vol. 1851: O. Catoni, Statistical Learning Theory and Stochastic Optimization (2004)
- Vol. 1852: A.S. Kechris, B.D. Miller, Topics in Orbit Equivalence (2004)
- Vol. 1853: Ch. Favre, M. Jonsson, The Valuative Tree (2004)
- Vol. 1854: O. Saeki, Topology of Singular Fibers of Differential Maps (2004)
- Vol. 1855: G. Da Prato, P.C. Kunstmann, I. Lasiecka, A. Lunardi, R. Schnaubelt, L. Weis, Functional Analytic Methods for Evolution Equations. Editors: M. Iannelli, R. Nagel, S. Piazzera (2004)
- Vol. 1856: K. Back, T.R. Bielecki, C. Hipp, S. Peng, W. Schachermayer, Stochastic Methods in Finance, Bressanone/Brixen, Italy, 2003. Editors: M. Frittelli, W. Runggaldier (2004)
- Vol. 1857: M. Émery, M. Ledoux, M. Yor (Eds.), Séminaire de Probabilités XXXVIII (2005)
- Vol. 1858: A.S. Cherny, H.-J. Engelbert, Singular Stochastic Differential Equations (2005)
- Vol. 1859: E. Letellier, Fourier Transforms of Invariant Functions on Finite Reductive Lie Algebras (2005)
- Vol. 1860: A. Borisyuk, G.B. Ermentrout, A. Friedman, D. Terman, Tutorials in Mathematical Biosciences I. Mathematical Neurosciences (2005)
- Vol. 1861: G. Benettin, J. Henrard, S. Kuksin, Hamiltonian Dynamics – Theory and Applications, Cetraro, Italy, 1999. Editor: A. Giorgilli (2005)
- Vol. 1862: B. Helffer, F. Nier, Hypocoelliptic Estimates and Spectral Theory for Fokker-Planck Operators and Witten Laplacians (2005)
- Vol. 1863: H. Führ, Abstract Harmonic Analysis of Continuous Wavelet Transforms (2005)
- Vol. 1864: K. Efsthathiou, Metamorphoses of Hamiltonian Systems with Symmetries (2005)
- Vol. 1865: D. Applebaum, B.V. R. Bhat, J. Kustermans, J. M. Lindsay, Quantum Independent Increment Processes I. From Classical Probability to Quantum Stochastic Calculus. Editors: M. Schürmann, U. Franz (2005)
- Vol. 1866: O.E. Barndorff-Nielsen, U. Franz, R. Gohm, B. Kümmerer, S. Thorbjørnsen, Quantum Independent Increment Processes II. Structure of Quantum Lévy Processes, Classical Probability, and Physics. Editors: M. Schürmann, U. Franz, (2005)
- Vol. 1867: J. Sneyd (Ed.), Tutorials in Mathematical Biosciences II. Mathematical Modeling of Calcium Dynamics and Signal Transduction. (2005)
- Vol. 1868: J. Jorgenson, S. Lang,  $\text{Pos}_n(\mathbb{R})$  and Eisenstein Series. (2005)
- Vol. 1869: A. Dembo, T. Funaki, Lectures on Probability Theory and Statistics. Ecole d'Été de Probabilités de Saint-Flour XXXIII-2003. Editor: J. Picard (2005)
- Vol. 1870: V.I. Gurariy, W. Lusky, Geometry of Müntz Spaces and Related Questions. (2005)
- Vol. 1871: P. Constantin, G. Gallavotti, A.V. Kazhikhov, Y. Meyer, S. Ukai, Mathematical Foundation of Turbulent Viscous Flows, Martina Franca, Italy, 2003. Editors: M. Cannone, T. Miyakawa (2006)
- Vol. 1872: A. Friedman (Ed.), Tutorials in Mathematical Biosciences III. Cell Cycle, Proliferation, and Cancer (2006)
- Vol. 1873: R. Mansuy, M. Yor, Random Times and Enlargements of Filtrations in a Brownian Setting (2006)
- Vol. 1874: M. Yor, M. Émery (Eds.), In Memoriam Paul-André Meyer - Séminaire de Probabilités XXXIX (2006)
- Vol. 1875: J. Pitman, Combinatorial Stochastic Processes. Ecole d'Été de Probabilités de Saint-Flour XXXII-2002. Editor: J. Picard (2006)
- Vol. 1876: H. Herrlich, Axiom of Choice (2006)
- Vol. 1877: J. Steuding, Value Distributions of  $L$ -Functions (2007)
- Vol. 1878: R. Cerf, The Wulff Crystal in Ising and Percolation Models, Ecole d'Été de Probabilités de Saint-Flour XXXIV-2004. Editor: Jean Picard (2006)
- Vol. 1879: G. Slade, The Lace Expansion and its Applications, Ecole d'Été de Probabilités de Saint-Flour XXXIV-2004. Editor: Jean Picard (2006)
- Vol. 1880: S. Attal, A. Joye, C.-A. Pillet, Open Quantum Systems I, The Hamiltonian Approach (2006)

- Vol. 1881: S. Attal, A. Joye, C.-A. Pillet, *Open Quantum Systems II, The Markovian Approach* (2006)
- Vol. 1882: S. Attal, A. Joye, C.-A. Pillet, *Open Quantum Systems III, Recent Developments* (2006)
- Vol. 1883: W. Van Assche, F. Marcellàn (Eds.), *Orthogonal Polynomials and Special Functions, Computation and Application* (2006)
- Vol. 1884: N. Hayashi, E.I. Kaikina, P.I. Naumkin, I.A. Shishmarev, *Asymptotics for Dissipative Nonlinear Equations* (2006)
- Vol. 1885: A. Telcs, *The Art of Random Walks* (2006)
- Vol. 1886: S. Takamura, *Splitting Deformations of Degenerations of Complex Curves* (2006)
- Vol. 1887: K. Habermann, L. Habermann, *Introduction to Symplectic Dirac Operators* (2006)
- Vol. 1888: J. van der Hoeven, *Transseries and Real Differential Algebra* (2006)
- Vol. 1889: G. Osipenko, *Dynamical Systems, Graphs, and Algorithms* (2006)
- Vol. 1890: M. Bunge, J. Funk, *Singular Coverings of Toposes* (2006)
- Vol. 1891: J.B. Friedlander, D.R. Heath-Brown, H. Iwaniec, J. Kaczorowski, *Analytic Number Theory*, Cetraro, Italy, 2002. Editors: A. Perelli, C. Viola (2006)
- Vol. 1892: A. Baddeley, I. Bárány, R. Schneider, W. Weil, *Stochastic Geometry*, Martina Franca, Italy, 2004. Editor: W. Weil (2007)
- Vol. 1893: H. Hanßmann, *Local and Semi-Local Bifurcations in Hamiltonian Dynamical Systems, Results and Examples* (2007)
- Vol. 1894: C.W. Groetsch, *Stable Approximate Evaluation of Unbounded Operators* (2007)
- Vol. 1895: L. Molnár, *Selected Preserver Problems on Algebraic Structures of Linear Operators and on Function Spaces* (2007)
- Vol. 1896: P. Massart, *Concentration Inequalities and Model Selection*, Ecole d'Été de Probabilités de Saint-Flour XXXIII-2003. Editor: J. Picard (2007)
- Vol. 1897: R. Doney, *Fluctuation Theory for Lévy Processes*, Ecole d'Été de Probabilités de Saint-Flour XXXV-2005. Editor: J. Picard (2007)
- Vol. 1898: H.R. Beyer, *Beyond Partial Differential Equations, On linear and Quasi-Linear Abstract Hyperbolic Evolution Equations* (2007)
- Vol. 1899: *Séminaire de Probabilités XL*. Editors: C. Donati-Martin, M. Émery, A. Rouault, C. Stricker (2007)
- Vol. 1900: E. Bolthausen, A. Bovier (Eds.), *Spin Glasses* (2007)
- Vol. 1901: O. Wittenberg, *Intersections de deux quadriques et pincesaux de courbes de genre 1, Intersections of Two Quadrics and Pencils of Curves of Genus 1* (2007)
- Vol. 1902: A. Isaev, *Lectures on the Automorphism Groups of Kobayashi-Hyperbolic Manifolds* (2007)
- Vol. 1903: G. Kresin, V. Maz'ya, *Sharp Real-Part Theorems* (2007)
- Vol. 1904: P. Giesl, *Construction of Global Lyapunov Functions Using Radial Basis Functions* (2007)
- Vol. 1905: C. Prévôt, M. Röckner, *A Concise Course on Stochastic Partial Differential Equations* (2007)
- Vol. 1906: T. Schuster, *The Method of Approximate Inverse: Theory and Applications* (2007)
- Vol. 1907: M. Rasmussen, *Attractivity and Bifurcation for Nonautonomous Dynamical Systems* (2007)
- Vol. 1908: T.J. Lyons, M. Caruana, T. Lévy, *Differential Equations Driven by Rough Paths*, Ecole d'Été de Probabilités de Saint-Flour XXXIV-2004 (2007)
- Vol. 1909: H. Akiyoshi, M. Sakuma, M. Wada, Y. Yamashita, *Punctured Torus Groups and 2-Bridge Knot Groups (I)* (2007)
- Vol. 1910: V.D. Milman, G. Schechtman (Eds.), *Geometric Aspects of Functional Analysis*, Israel Seminar 2004-2005 (2007)
- Vol. 1911: A. Bressan, D. Serre, M. Williams, K. Zumbrun, *Hyperbolic Systems of Balance Laws*, Cetraro, Italy 2003. Editor: P. Marcati (2007)
- Vol. 1912: V. Berinde, *Iterative Approximation of Fixed Points* (2007)
- Vol. 1913: J.E. Marsden, G. Misiolek, J.-P. Ortega, M. Perlmutter, T.S. Ratiu, *Hamiltonian Reduction by Stages* (2007)
- Vol. 1914: G. Kutyniok, *Affine Density in Wavelet Analysis* (2007)
- Vol. 1915: T. Bıyıköğlu, J. Leydold, P.F. Stadler, *Laplacian Eigenvectors of Graphs, Perron-Frobenius and Faber-Krahn Type Theorems* (2007)
- Vol. 1916: C. Villani, F. Rezakhanlou, *Entropy Methods for the Boltzmann Equation*. Editors: F. Golse, S. Olla (2008)
- Vol. 1917: I. Veselić, *Existence and Regularity Properties of the Integrated Density of States of Random Schrödinger* (2008)
- Vol. 1918: B. Roberts, R. Schmidt, *Local Newforms for  $\mathrm{GSp}(4)$*  (2007)
- Vol. 1919: R.A. Carmona, I. Ekeland, A. Kohatsu-Higa, J.-M. Lasry, P.-L. Lions, H. Pham, E. Taflin, *Paris-Princeton Lectures on Mathematical Finance 2004*. Editors: R.A. Carmona, E. Çinlar, I. Ekeland, E. Jouini, J.A. Scheinkman, N. Touzi (2007)
- Vol. 1920: S.N. Evans, *Probability and Real Trees*. Ecole d'Été de Probabilités de Saint-Flour XXXV-2005 (2008)
- Vol. 1921: J.P. Tian, *Evolution Algebras and their Applications* (2008)
- Vol. 1922: A. Friedman (Ed.), *Tutorials in Mathematical BioSciences IV. Evolution and Ecology* (2008)
- Vol. 1923: J.P.N. Bishwal, *Parameter Estimation in Stochastic Differential Equations* (2008)
- Vol. 1924: M. Wilson, *Littlewood-Paley Theory and Exponential-Square Integrability* (2008)
- Vol. 1925: M. du Sautoy, L. Woodward, *Zeta Functions of Groups and Rings* (2008)
- Vol. 1926: L. Barreira, V. Claudia, *Stability of Nonautonomous Differential Equations* (2008)
- Vol. 1927: L. Ambrosio, L. Caffarelli, M.G. Crandall, L.C. Evans, N. Fusco, *Calculus of Variations and Non-Linear Partial Differential Equations*. Cetraro, Italy 2005. Editors: B. Dacorogna, P. Marcellini (2008)
- Vol. 1928: J. Jonsson, *Simplicial Complexes of Graphs* (2008)
- Vol. 1929: Y. Mishura, *Stochastic Calculus for Fractional Brownian Motion and Related Processes* (2008)
- Vol. 1930: J.M. Urbano, *The Method of Intrinsic Scaling. A Systematic Approach to Regularity for Degenerate and Singular PDEs* (2008)
- Vol. 1931: M. Cowling, E. Frenkel, M. Kashiwara, A. Valette, D.A. Vogan, Jr., N.R. Wallach, *Representation Theory and Complex Analysis*. Venice, Italy 2004. Editors: E.C. Tarabusi, A. D'Agnolo, M. Picardello (2008)
- Vol. 1932: A.A. Agrachev, A.S. Morse, E.D. Sontag, H.J. Sussmann, V.I. Utkin, *Nonlinear and Optimal*

Control Theory. Cetraro, Italy 2004. Editors: P. Nistri, G. Stefani (2008)

Vol. 1933: M. Petkovic, Point Estimation of Root Finding Methods (2008)

Vol. 1934: C. Donati-Martin, M. Émery, A. Rouault, C. Stricker (Eds.), Séminaire de Probabilités XLI (2008)

Vol. 1935: A. Unterberger, Alternative Pseudodifferential Analysis (2008)

Vol. 1936: P. Magal, S. Ruan (Eds.), Structured Population Models in Biology and Epidemiology (2008)

Vol. 1937: G. Capriz, P. Giovine, P.M. Mariano (Eds.), Mathematical Models of Granular Matter (2008)

Vol. 1938: D. Auroux, F. Catanese, M. Manetti, P. Seidel, B. Siebert, I. Smith, G. Tian, Symplectic 4-Manifolds and Algebraic Surfaces. Cetraro, Italy 2003. Editors: F. Catanese, G. Tian (2008)

Vol. 1939: D. Boffi, F. Brezzi, L. Demkowicz, R.G. Durán, R.S. Falk, M. Fortin, Mixed Finite Elements, Compatibility Conditions, and Applications. Cetraro, Italy 2006. Editors: D. Boffi, L. Gastaldi (2008)

Vol. 1940: J. Banasiak, V. Capasso, M.A.J. Chaplain, M. Lachowicz, J. Mięksis, Multiscale Problems in the Life Sciences. From Microscopic to Macroscopic. Będlewo, Poland 2006. Editors: V. Capasso, M. Lachowicz (2008)

Vol. 1941: S.M.J. Haran, Arithmetical Investigations. Representation Theory, Orthogonal Polynomials, and Quantum Interpolations (2008)

Vol. 1942: S. Alberverio, F. Flandoli, Y.G. Sinai, SPDE in Hydrodynamic. Recent Progress and Prospects. Cetraro, Italy 2005. Editors: G. Da Prato, M. Röckner (2008)

Vol. 1943: L.L. Bonilla (Ed.), Inverse Problems and Imaging. Martina Franca, Italy 2002 (2008)

Vol. 1944: A. Di Bartolo, G. Falcone, P. Plaumann, K. Strambach, Algebraic Groups and Lie Groups with Few Factors (2008)

Vol. 1945: F. Brauer, P. van den Driessche, J. Wu (Eds.), Mathematical Epidemiology (2008)

Vol. 1946: G. Allaire, A. Arnold, P. Degond, T.Y. Hou, Quantum Transport. Modelling, Analysis and Asymptotics. Cetraro, Italy 2006. Editors: N.B. Abdallah, G. Frosali (2008)

Vol. 1947: D. Abramovich, M. Mariño, M. Thaddeus, R. Vakil, Enumerative Invariants in Algebraic Geometry and String Theory. Cetraro, Italy 2005. Editors: K. Behrend, M. Manetti (2008)

Vol. 1948: F. Cao, J.-L. Lisani, J.-M. Morel, P. Musé, F. Sur, A Theory of Shape Identification (2008)

Vol. 1949: H.G. Feichtinger, B. Helffer, M.P. Lamoureux, N. Lerner, J. Toft, Pseudo-Differential Operators, Quantization and Signals. Cetraro, Italy 2006. Editors: L. Rodino, M.W. Wong (2008)

Vol. 1950: M. Bramson, Stability of Queueing Networks, Ecole d'Été de Probabilités de Saint-Flour XXXVI-2006 (2008)

Vol. 1951: A. Moltó, J. Orihuela, S. Troyanski, M. Valdivia, A Non Linear Transfer Technique for Renorming (2009)

Vol. 1952: R. Mikhailov, I.B.S. Passi, Lower Central and Dimension Series of Groups (2009)

Vol. 1953: K. Arwini, C.T.J. Dodson, Information Geometry (2008)

Vol. 1954: P. Biane, L. Bouten, F. Cipriani, N. Konno, N. Privault, Q. Xu, Quantum Potential Theory. Editors: U. Franz, M. Schuermann (2008)

Vol. 1955: M. Bernot, V. Caselles, J.-M. Morel, Optimal Transportation Networks (2008)

Vol. 1956: C.H. Chu, Matrix Convolution Operators on Groups (2008)

Vol. 1957: A. Guionnet, On Random Matrices: Macroscopic Asymptotics, Ecole d'Été de Probabilités de Saint-Flour XXXVI-2006 (2009)

Vol. 1958: M.C. Olsson, Compactifying Moduli Spaces for Abelian Varieties (2008)

Vol. 1959: Y. Nakkajima, A. Shiho, Weight Filtrations on Log Crystalline Cohomologies of Families of Open Smooth Varieties (2008)

Vol. 1960: J. Lipman, M. Hashimoto, Foundations of Grothendieck Duality for Diagrams of Schemes (2009)

Vol. 1961: G. Buttazzo, A. Pratelli, S. Solimini, E. Stepanov, Optimal Urban Networks via Mass Transportation (2009)

Vol. 1962: R. Dalang, D. Khoshnevisan, C. Mueller, D. Nualart, Y. Xiao, A Minicourse on Stochastic Partial Differential Equations (2009)

Vol. 1963: W. Siebert, Local Lyapunov Exponents (2009)

Vol. 1964: W. Roth, Operator-valued Measures and Integrals for Cone-valued Functions and Integrals for Cone-valued Functions (2009)

Vol. 1965: C. Chidume, Geometric Properties of Banach Spaces and Nonlinear Iterations (2009)

Vol. 1966: D. Deng, Y. Han, Harmonic Analysis on Spaces of Homogeneous Type (2009)

Vol. 1967: B. Fresse, Modules over Operads and Functors (2009)

Vol. 1968: R. Weissauer, Endoscopy for GSP(4) and the Cohomology of Siegel Modular Threefolds (2009)

Vol. 1969: B. Roynette, M. Yor, Penalising Brownian Paths (2009)

Vol. 1970: R. Kotecký, Methods of Contemporary Mathematical Statistical Physics (2009)

Vol. 1971: L. Saint-Raymond, Hydrodynamic Limits of the Boltzmann Equation (2009)

Vol. 1972: T. Mochizuki, Donaldson Type Invariants for Algebraic Surfaces (2009)

## Recent Reprints and New Editions

Vol. 1702: J. Ma, J. Yong, Forward-Backward Stochastic Differential Equations and their Applications. 1999 – Corr. 3rd printing (2007)

Vol. 830: J.A. Green, Polynomial Representations of  $GL_n$ , with an Appendix on Schensted Correspondence and Littelmann Paths by K. Erdmann, J.A. Green and M. Schoker 1980 – 2nd corr. and augmented edition (2007)

Vol. 1693: S. Simons, From Hahn-Banach to Monotonicity (Minimax and Monotonicity 1998) – 2nd exp. edition (2008)

Vol. 470: R.E. Bowen, Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms. With a preface by D. Ruelle. Edited by J.-R. Chazottes. 1975 – 2nd rev. edition (2008)

Vol. 523: S.A. Alberverio, R.J. Høegh-Krohn, S. Mazur, Mathematical Theory of Feynman Path Integral. 1976 – 2nd corr. and enlarged edition (2008)

Vol. 1764: A. Cannas da Silva, Lectures on Symplectic Geometry 2001 – Corr. 2nd printing (2008)

Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

**Editorial Policy** (for the publication of monographs)

1. Lecture Notes aim to report new developments in all areas of mathematics and their applications - quickly, informally and at a high level. Mathematical texts analysing new developments in modelling and numerical simulation are welcome.

Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They may be based on specialised lecture courses. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this “lecture notes” character. For similar reasons it is unusual for doctoral theses to be accepted for the Lecture Notes series, though habilitation theses may be appropriate.

2. Manuscripts should be submitted either online at [www.editorialmanager.com/lnm](http://www.editorialmanager.com/lnm) to Springer’s mathematics editorial in Heidelberg, or to one of the series editors. In general, manuscripts will be sent out to 2 external referees for evaluation. If a decision cannot yet be reached on the basis of the first 2 reports, further referees may be contacted: The author will be informed of this. A final decision to publish can be made only on the basis of the complete manuscript, however a refereeing process leading to a preliminary decision can be based on a pre-final or incomplete manuscript. The strict minimum amount of material that will be considered should include a detailed outline describing the planned contents of each chapter, a bibliography and several sample chapters.

Authors should be aware that incomplete or insufficiently close to final manuscripts almost always result in longer refereeing times and nevertheless unclear referees’ recommendations, making further refereeing of a final draft necessary.

Authors should also be aware that parallel submission of their manuscript to another publisher while under consideration for LNM will in general lead to immediate rejection.

3. Manuscripts should in general be submitted in English. Final manuscripts should contain at least 100 pages of mathematical text and should always include
  - a table of contents;
  - an informative introduction, with adequate motivation and perhaps some historical remarks: it should be accessible to a reader not intimately familiar with the topic treated;
  - a subject index: as a rule this is genuinely helpful for the reader.

For evaluation purposes, manuscripts may be submitted in print or electronic form (print form is still preferred by most referees), in the latter case preferably as pdf- or zipped ps-files. Lecture Notes volumes are, as a rule, printed digitally from the authors’ files. To ensure best results, authors are asked to use the LaTeX2e style files available from Springer’s web-server at:

<ftp://ftp.springer.de/pub/tex/latex/svmonot1/> (for monographs) and  
<ftp://ftp.springer.de/pub/tex/latex/svmultt1/> (for summer schools/tutorials).



Additional technical instructions, if necessary, are available on request from:  
lnm@springer.com.

4. Careful preparation of the manuscripts will help keep production time short besides ensuring satisfactory appearance of the finished book in print and online. After acceptance of the manuscript authors will be asked to prepare the final LaTeX source files and also the corresponding dvi-, pdf- or zipped ps-file. The LaTeX source files are essential for producing the full-text online version of the book (see <http://www.springerlink.com/openurl.asp?genre=journal&issn=0075-8434> for the existing online volumes of LNM).

The actual production of a Lecture Notes volume takes approximately 12 weeks.

5. Authors receive a total of 50 free copies of their volume, but no royalties. They are entitled to a discount of 33.3% on the price of Springer books purchased for their personal use, if ordering directly from Springer.
6. Commitment to publish is made by letter of intent rather than by signing a formal contract. Springer-Verlag secures the copyright for each volume. Authors are free to reuse material contained in their LNM volumes in later publications: a brief written (or e-mail) request for formal permission is sufficient.

#### **Addresses:**

Professor J.-M. Morel, CMLA,  
École Normale Supérieure de Cachan,  
61 Avenue du Président Wilson, 94235 Cachan Cedex, France  
E-mail: Jean-Michel.Morel@cmla.ens-cachan.fr

Professor F. Takens, Mathematisch Instituut,  
Rijksuniversiteit Groningen, Postbus 800,  
9700 AV Groningen, The Netherlands  
E-mail: F.Takens@rug.nl

Professor B. Teissier, Institut Mathématique de Jussieu,  
UMR 7586 du CNRS, Équipe “Géométrie et Dynamique”,  
175 rue du Chevaleret,  
75013 Paris, France  
E-mail: teissier@math.jussieu.fr

*For the “Mathematical Biosciences Subseries” of LNM:*

Professor P.K. Maini, Center for Mathematical Biology,  
Mathematical Institute, 24-29 St Giles,  
Oxford OX1 3LP, UK  
E-mail: maini@maths.ox.ac.uk

Springer, Mathematics Editorial, Tiergartenstr. 17,  
69121 Heidelberg, Germany,  
Tel.: +49 (6221) 487-259  
Fax: +49 (6221) 4876-8259  
E-mail: lnm@springer.com