Bibliographical Comments on Part III

First applications of model categories in algebra occur in Quillen's original monograph [50] for certain categories of simplicial algebras, including monoids, commutative algebras, ... (see also the article [52]).

The (semi-)model category of algebras over an operad is studied in several references in various contexts: the case of algebras in differential graded modules is addressed in [17, 26]; the case of algebras in simplicial sets and simplicial modules in [54]; the case of algebras in spectra in [21, 23, 36]. The articles [4] and [58] handle the definition of the (semi-)model structure in a general axiomatic setting. In the reference [58], the axioms of (semi-)model categories are proved by a technical study of pushouts in categories of algebras over operads. In [4], it is observed that verifications of [58] can be simplified for certain operads, including cofibrant operads, if the underlying category has a monoidal fibrant replacement functor and comes equipped with a good interval object (see *loc. cit.* for precise requirements).

The paper [22] includes another general construction of model structures for algebras and left modules over non-symmetric operads.

The definition of the (semi-)model category of operads occur in [26] for operads in dg-modules, and in [4, 58] in a more general setting. Note however that the homotopy theory of dg-operads was studied before by methods of differential graded algebra. In particular, the paper [45] gives a generalization to operads of the minimal models of rational homotopy. The Koszul duality of [18] has an interpretation in terms of these minimal models.

The notion of a semi-model category arises from [29] and gives the basis of the theory of [58]. Note however that our axioms differ slightly from these original references. The article [41] uses the semi-model structures of [29, 58], but most authors still prefer to deal with full model structures.

The definition of the homology with trivial coefficients occurs in [17, 18] for algebras over operads in dg-modules. The (co)homology with non-trivial coefficients is defined in [1]. In all these references, the authors deal with algebras in non-negatively graded dg-modules. The generalization of the (co)homology to algebras in unbounded dg-modules occurs in [26]. Universal coefficients

spectral sequences are defined in [1] (for algebras in non-negatively graded dg-modules) by using the cotriple construction.

The work [20, 21, 62, 63] tackles applications of the cohomology of algebras over operads to realization problems in stable homotopy. In this setting, it is more appropriate to deal with simplicial objects and simplicial algebras rather than differential graded algebras, but this does not change much the definition of the (co)homology of algebras over operads.

The definition of the cotriple construction goes back to [3] and [47]. The idea to represent the cotriple construction of functors at the operad level occurs in [56, 57] and in [54]. The two-sided version of the cotriple construction is also studied in [14] in the differential graded context and is related to other complexes of the theory of operads, notably the Koszul complex of [18]. The recent preprint [24] focuses on applications of the operadic cotriple construction in the context of spectra.