

Bibliographical Comments on Part I

We refer to [46] for a comprehensive overview of the history of operads. We only give brief indications on matters related to our approach on algebras over operads.

In most applications, authors deal with algebras and operads within a fixed usual category, like the category of topological spaces, the category of dg-modules, the category of simplicial sets, or the category of simplicial modules. Nevertheless, homology theories, and other usual functors which change the underlying category, are often used in the study of operads.

In the original reference [47], the structure of an operad is defined within the category of topological spaces as a device to model the structure of iterated loop spaces. The definition of operads in the framework of symmetric monoidal categories occurs explicitly in [17]. But, as far as we know, the thesis [54] is the first reference which formalizes the naturality of operad structures with respect to functors of symmetric monoidal categories.

In almost all references, the algebras over an operad are defined within the same category as the operad. But, in problems of stable homotopy, authors consider naturally spectra acted on by operads in simplicial sets (see for instance [21]). However, the general axiomatic background of symmetric monoidal categories over a base, which axiomatizes this relationship, has not been formalized in the operadic literature yet.

The composition product \circ is defined in [32] for Σ_* -objects in sets. The introduction of this operation in the operad literature goes back to [56]. Left modules over operads appear explicitly in [57] in a realization of the cotriple construction of [3, 47] at the operad level. The notion of a left module over operads is studied more thoroughly in [54] and in [14]. The point of view of [54] is to identify algebras over operads with constant left modules. This approach is the usual one of the literature.

The converse point of view adopted in this book, according to which left modules are algebras over operads in a category over the base category, has not been fully exploited yet. However, the equivalence between left modules over operads and algebras in Σ_* -objects is made explicit in [38]. Note further

that the generalization of Morita's theorem to modules over operads in [33] uses such an equivalence implicitly.

In the literature, the first examples of left module structures have appeared as generalized algebras: Lie algebras in Σ_* -objects are introduced in [2] (under the name twisted Lie algebras) to study total Hopf invariants. The study of these generalized Lie algebras structures is carried on in [19, 61], and in [11, 12, 13] and [39] in the language of operads.

Representations and enveloping algebras of algebras over an operads are defined in [18]. The idea of the enveloping operad goes back to [17]. The definition adopted in §4.1 is borrowed from [13]. The notion of a Kähler differential for algebras over an operad is introduced in [26] in a definition of the cohomology of algebras over operads.