A

Problems and Questions

Chapter 3

1. (Section 3.3) The symmetric q-β-chain is defined by the following Figure A.1.

   Its real limit is the η-β-chain (which is symmetric in (α, β)), and its p-adic limit is the symmetric p-β-chain (Sect. 2.1.3). Understand the Martin kernel and the boundary of this symmetric q-β-chain. Note that for this chain $(P^*)\delta_{(0,0)}$ is a probability measure supported at $\{(i,j) \mid N \leq i+j, \max\{i,j\} \leq N\}$, its real limit is supported at $\{(i,j) \mid i+j = N\}$, and its p-adic limit supported at $\{(i,j) \mid \max\{i,j\} = N\}$.

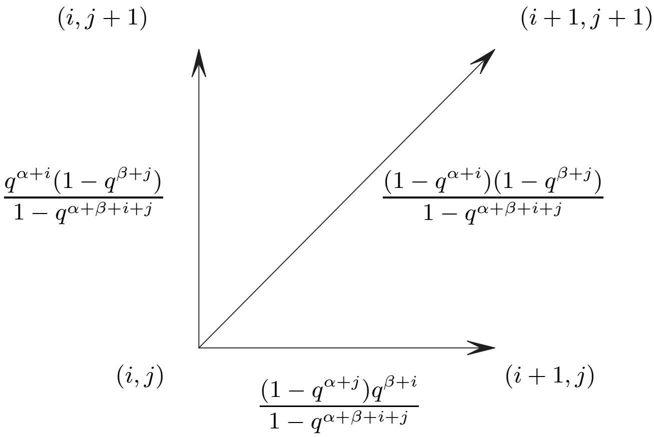


Fig. A.1. The symmetric q-β-chain

Chapter 4

1. (Section 4.5) Find the real γ-chain. Find the η-finite Laguerre basis.

Chapter 6

1. (Section 6.1) Find the q-pure basis. See remark on pp. 112–113 and p. 128 of [Har5].

Chapter 8

1. Determine the idempotent explicitly.

Chapter 9

1. Determine the idempotent explicitly (i.e., find the inverse matrix $(A^*_{\lambda,\alpha})$ of $(A_{\lambda,\alpha})$).

Chapter 10

1. Get the direct proof of Theorem 10.3.1 without going through the Koornwinder polynomials.
2. There is no q-chain and no η-chain which is analogue of the p-adic chain (like we had in the rank 1 case). The problem is that dividing $GL_d/B_m = \mathrm{Grass}(m,d)$ by B_m kills the Schubert cells but $B_{1,\dots,1}\backslash GL_d/B_m$ preserves them. However $B_{1,\dots,1}\backslash GL_d/B_m\big(\mathbb{Z}/p^N\big)$ depends on p (and not just on N). Does there exist some combinatorial quotient $\mathcal{C}_N^{d,m}$ of $B_{1,\dots,1}\backslash GL_d/B_m\big(\mathbb{Z}/p^N\big)$ and Markov chain on $\sqcup_N \mathcal{C}_N^{d,m}$? Namely, begin at the closed point and look in which Schubert cell you fall modulo p^N, $N = 1, 2, \dots$ (eventually end up in big open cell with probability 1).
3. There is no explicit description of the idempotent such that

$$\varphi_\lambda = (D^+)^{\lambda_1-\lambda_2}\rho_{m-1}(D^+)^{\lambda_2-\lambda_3}\rho_{m-2}\cdots\rho_2(D^+)^{\lambda_m}\mathbf{1}$$

as we had for intertwiner $GL_d/B_{1,d-1} \longrightarrow GL_d/B_{1,\dots,1}$ in Chapter 7. We need a factorization of the (radial parts) Laplacian Δ as $\Delta = D^+D$. More generally, let Δ be the Koornwinder 2-nd order differential operator,

$$\Delta = \sum_{1\le j\le m} \phi_j^+(T_j^+ - I) + \phi_j^-(T_j^- - I),$$

where $T_j^{\pm}\varphi(y_1,\dots,y_m) := \varphi(y_1,\dots,q^{\pm 1}y_j,\dots,y_m)$ and

$$\phi_j^+ := \frac{\prod_{0\le i\le 3}(1-a_iy_j)}{(1-y_j^2)(1-qy_j^2)}\prod_{i\neq j}\frac{(1-ty_iy_j)(1-ty_i^{-1}y_j)}{(1-y_iy_j)(1-y_i^{-1}y_j)},$$

$$\phi_j^- := \frac{\prod_{0\le i\le 3}(a_i-y_j)}{(1-y_j^2)(q-y_j^2)}\prod_{i\neq j}\frac{(t-y_iy_j)(t-y_i^{-1}y_j)}{(1-y_iy_j)(1-y_i^{-1}y_j)}$$

with $a_i = q^{\alpha_i}$ $(i = 0,\dots,3)$ and $t = q^{\gamma}$ being parameters. In the case of $m = 1$, Δ is the Askey–Wilson operator which depends on parameters a_i ($i = 0,\dots,3$ but not on $t = q^{\gamma}$) and there is such a factorization $\Delta = D^+D$ (cf. [Har5] pp. 126–127). The Askey–Wilson polynomial φ_λ can be written as $\varphi_\lambda = (D^+)^\lambda \mathbf{1}$ for $\lambda \in \mathbb{N}$.

4. Normalize the q-algebra of this chapter using the non-symmetric q-numbers (as we do in Chapter 11) so as to have a p-adic limit of the algebra.

Chapter 11

1. Is there a more refined p-adic limit, one that will preserve more of the SL_2-structure than the limit of (11.2)?

B

Orthogonal Polynomials

Let p be a prime number. Given a probability measure μ on $\mathbb{Z}_p$, we have its image on $\mathbb{Z}/p^N$ for any N, $\mu_N(a) = \mu(a + p^N\mathbb{Z}_p)$, and the isometric embeddings, and dual orthogonal projection:

$$H_N = \ell^2(\mathbb{Z}/p^N, \mu_N) \underset{\hookrightarrow}{\twoheadleftarrow} L^2(\mathbb{Z}_p, \mu) = H$$

We want the real analogue of this, and the idea is simple: We replace the locally constant function (defined modulo p^N) by the polynomials (of degree $\leq N$). We shall show that the theory of orthogonal polynomials gives the real analogue.

Let μ be a probability measure on $[-1, 1]$, $H = L^2\big([-1, 1], \mu\big)$. Applying the Gram–Schmidt process to the monomials $1, x, x^2, \ldots$, we get a sequence of polynomials which are orthogonal in H:

$$\begin{aligned}
p_0(x) &= 1, \\
p_1(x) &= x - (x, 1) \cdot 1, \\
p_2(x) &= x^2 - \frac{(x^2, p_1)}{(p_1, p_1)} p_1 - \frac{(x^2, 1)}{(1, 1)} \cdot 1, \\
&\vdots \\
p_n(x) &= x^n - \frac{(x^n, p_{n-1})}{(p_{n-1}, p_{n-1})} p_{n-1} - \cdots - \frac{(x^2, 1)}{(1, 1)} \cdot 1.
\end{aligned}$$

Here $(\cdot, \cdot)$ denotes the inner product of H; $(\varphi_1, \varphi_2) := \int_{-1}^{1} \varphi_1(x)\overline{\varphi_2(x)}\mu(dx)$. We here normalize the orthogonal polynomials $p_n(x)$ to have the leading coefficient 1. We have

Theorem B.1.

$$p_n(x) = \det \begin{pmatrix} (1,1) & (1,x) & \cdots & (1,x^n) \\ (x,1) & (x,x) & \cdots & (x,x^n) \\ \vdots & \vdots & & \vdots \\ (x^{n-1},1) & (x^{n-1},x) & \cdots & (x^{n-1},x^n) \\ 1 & x & \cdots & x^n \end{pmatrix} \cdot \frac{1}{G_{n-1}},$$

where $G_{n-1} = \det\big((x^i, x^j)\big)_{0\le i,j\le n-1}$.

Proof. Indeed, the inner product of the determinant with x^j is the same determinant with the bottom row replaced by (x^i, x^j), and for $j < n$ this row already appears in the determinant, whence it vanishes. So the determinant above is a polynomial of degree n which is orthogonal to all polynomials of degree $< n$ and it has the leading coefficient 1 because of the $1/G_{n-1}$ normalization factor. □

Note that if we denote the moments of the measure μ by

$$c_n := (x^n, 1) = \int_{-1}^{1} x^n \mu(dx),$$

we have $(x^i, x^j) = c_{i+j}$.

Theorem B.2. *All the zeros of* $p_n(x)$ *are simple and are contained in* $(-1, 1)$.

Proof. Otherwise $p_n(x)$ changes sign in $(-1, 1)$ only in $m<n$ point $\alpha_1, \ldots, \alpha_m$. Then $\pm p_n(x) \cdot \prod_{j=1}^m (x - \alpha_j) \ge 0$ in $(-1, 1)$, whence $\pm\big(p_n, \prod_{j=1}^m (x-\alpha_j)\big) > 0$. This contradicts the orthogonality of $p_n(x)$ to polynomials of degree $< n$. □

Theorem B.3. *We have the recursion equation*

$$p_{n+1}(x) = (x + b_n)p_n(x) - d_n p_{n-1}(x),$$

where

$$d_n = \frac{h_n}{h_{n-1}}, \qquad h_n = ||p_n||^2 = (p_n, p_n),$$
$$b_n = k_{n+1} - k_n, \qquad p_n(x) \equiv x^n + k_n x^{n-1} \pmod{x^{n-2}}.$$

Proof. The polynomial $p_{n+1} - x \cdot p_n$ is of degree $\le n$ and is orthogonal to polynomials of degree $\le n-2$, hence has the form $b_n \cdot p_n - d_n \cdot p_{n-1}$. We get

$$d_n = \frac{(x\cdot p_n - p_{n+1}, p_{n-1})}{(p_{n-1}, p_{n-1})} = \frac{(p_n, x\cdot p_{n-1})}{(p_{n-1}, p_{n-1})} = \frac{(p_n, p_n + (\text{degree} < n))}{(p_{n-1}, p_{n-1})} = \frac{(p_n, p_n)}{(p_{n-1}, p_{n-1})},$$

and modulo x^{n-1}

$$b_n \cdot x^n \equiv b_n \cdot p_n(x) \equiv p_{n+1}(x) - x \cdot p_n(x)$$
$$\equiv (x^{n+1} + k_{n+1}x^n) - x(x^n + k_n x^{n-1}) \equiv (k_{n+1} - k_n) \cdot x^n.$$

□

Corollary B.4 (Christoffel–Derboux).

$$D_n(x,y) := \sum_{j=0}^{n} \frac{1}{h_j} p_j(x)p_j(y) = \frac{1}{h_n} \frac{p_{n+1}(x)p_n(y) - p_n(x)p_{n+1}(y)}{x-y}, \quad (x \neq y).$$

and for $y \to x$:

$$D_n(x,x) := \sum_{j=0}^{n} \frac{1}{h_j} p_j(x)^2 = \frac{1}{h_n}\big(p'_{n+1}(x)p_n(x) - p'_n(x)p_{n+1}(x)\big).$$

Proof. By induction on n. The induction step follows from the equality

$$\begin{aligned}
&h_n(x-y)\big(D_n(x,y) - D_{n-1}(x,y)\big) = (x-y)p_n(x)p_n(y)\\
&\quad = \Big(p_{n+1}(x) - b_n p_n(x) + \frac{h_n}{h_{n-1}} p_{n-1}(x)\Big)p_n(y)\\
&\qquad - p_n(x)\Big(p_{n+1}(y) - b_n p_n(y) + \frac{h_n}{h_{n-1}} p_{n-1}(y)\Big)\\
&\quad = \big(p_{n+1}(x)p_n(y) - p_n(x)p_{n+1}(y)\big) - \frac{h_n}{h_{n-1}}\big(p_n(x)p_{n-1}(y) - p_{n-1}(x)p_n(y)\big).
\end{aligned}$$

□

Remark B.5. For $\varphi \in H$, its orthogonal projection $D_n\varphi$ to the subspace $H_n := \operatorname{Span}\{1, x, \dots, x^n\}$ of H spanned by polynomials of degree $\leq n$ is given by

$$\begin{aligned}
D_n\varphi(y) :&= \sum_{j=0}^{n} \frac{(\varphi, p_j)}{(p_j, p_j)} p_j(y)\\
&= \int_{-1}^{1} \varphi(x) \sum_{j=0}^{n} \frac{p_j(x)p_j(y)}{h_j} \mu(dx)\\
&= \int_{-1}^{1} \varphi(x) D_n(x,y)\mu(dx) = (\varphi, D_n(\cdot, y)).
\end{aligned}$$

Corollary B.6. *$p_{n+1}(t)$ is the characteristic polynomial of the operator $D_n x$ of multiplication by x restricted to H_n;*

$$p_{n+1}(t) = \det\big(t \cdot I_n - D_n x \,\big|\, H_n\big).$$

Proof. The matrix corresponding to the operator $D_n x$ in the basis $\{p_0, p_1, \dots, p_n\}$ of H_n is by the recursion $x \cdot p_j = p_{j+1} - b_j p_j + d_j p_{j-1}$, given by

$$\begin{pmatrix} -b_0 & d_1 & & & & & \\ 1 & -b_1 & & \vdots & & & \\ 0 & 1 & \ddots & d_j & & \vdots & \\ \vdots & 0 & & -b_j & & 0 & \vdots \\ & \vdots & & 1 & \ddots & d_{n-1} & 0 \\ & & & 0 & & -b_{n-1} & d_n \\ & & & \vdots & & 1 & -b_n \end{pmatrix}.$$

Expanding the determinant of the characteristic polynomial by the last raw, we have

$$\begin{aligned} Q_{n+1}(t) := \det & \begin{pmatrix} t+b_0 & -d_1 & & & & & \\ -1 & t+b_1 & & \vdots & & & \\ 0 & -1 & \ddots & -d_j & & \vdots & \\ \vdots & 0 & & t+b_j & & 0 & \vdots \\ & \vdots & & -1 & \ddots & -d_{n-1} & 0 \\ & & & 0 & & t+b_{n-1} & -d_n \\ & & & \vdots & & -1 & t+b_n \end{pmatrix} \\ &= (t+b_n)Q_n(t) - d_n Q_{n-1}(t). \end{aligned}$$

Hence $Q_{n+1}(t) = p_{n+1}(t)$. □

Given a sequence of points $-1 < \alpha_0 < \alpha_1 < \cdots < \alpha_n < 1$, let

$$L(x) := \prod_{j=0}^{n}(x-\alpha_j) \quad \text{and} \quad L_j(x) := \frac{L(x)}{(x-\alpha_j)L'(\alpha_j)} = \prod_{i\neq j}\frac{(x-\alpha_i)}{(\alpha_j-\alpha_i)},$$

so that $L_j(\alpha_i) = \delta_{ij}$. Given a function $\varphi(x)$ on $[-1,1]$, we approximate it by the polynomial of degree $\le n$,

$$\mathcal{L}_n\varphi(x) = \sum_{j=0}^{n}\varphi(\alpha_j)L_j(x).$$

Note that if $\varphi(x)$ is a polynomial of degree $\le n$, then $\mathcal{L}_n\varphi(x) = \varphi(x)$ and hence

$$\int_{-1}^{1}\varphi(x)\mu(dx) = \int_{-1}^{1}\mathcal{L}_n\varphi(x)\mu(dx) = \sum_{j=0}^{n}\varphi(\alpha_j)\int_{-1}^{1}L_j(x)\mu(dx).$$

We do better if we choose the α_j's to be the zeros of $p_{n+1}(x)$, that is, $L(x) = p_{n+1}(x)$:

Theorem B.7 (Mechanical Quadrature). *For $L(x) = p_{n+1}(x)$, we have*

$$\int_{-1}^{1} \varphi(x)\mu(dx) = \int_{-1}^{1} \mathcal{L}_n\varphi(x)\mu(dx)$$

for all polynomials $\varphi(x)$ of degree $\leq 2n+1$.

Proof. $\varphi(x) - \mathcal{L}_n\varphi(x)$ is a polynomial of degree $\leq 2n+1$ and it vanishes at the α_j's, so

$$\varphi(x) - \mathcal{L}_n\varphi(x) = p_{n+1}(x)f(x)$$

where $f(x)$ is a polynomial of degree $\leq n$. Therefore

$$(\varphi - \mathcal{L}_n\varphi, 1) = (p_{n+1}f, 1) = (p_{n+1}, f) = 0. \qquad \square$$

Thus for $\{\alpha_j^{(n+1)}\}_{j=0,1,\ldots,n}$ the zeros of $p_{n+1}(x)$ and the Christoffel numbers

$$\lambda_j^{(n+1)} := \int_{-1}^{1} \frac{p_{n+1}(x)}{(x-\alpha_j^{(n+1)})p'_{n+1}(x)}\mu(dx) = \int_{-1}^{1} L_j(x)\mu(dx),$$

we have for any polynomial $\varphi(x)$ of degree $\leq 2n+1$

$$\int_{-1}^{1} \varphi(x)\mu(dx) = \int_{-1}^{1} \varphi(x)\mu_n(dx)$$

with the finite probability measure

$$\mu_n := \sum_{j=0}^{n} \lambda_j^{(n+1)} \delta_{\alpha_j^{(n+1)}}.$$

Remark that for $\varphi(x) = L_j(x)^2$ we get

$$\lambda_j^{(n+1)} = \int_{-1}^{1} L_j(x)^2\mu_n(x) = \int_{-1}^{1} L_j(x)^2\mu(dx) > 0,$$

and for $\varphi(x) = 1$,

$$\sum_{j=0}^{n} \lambda_j^{(n+1)} = \int_{-1}^{1} 1\cdot\mu_n(x) = \int_{-1}^{1} 1\cdot\mu(dx) = 1.$$

In particular, for polynomials φ_1, φ_2 of degree $\leq n$, we have

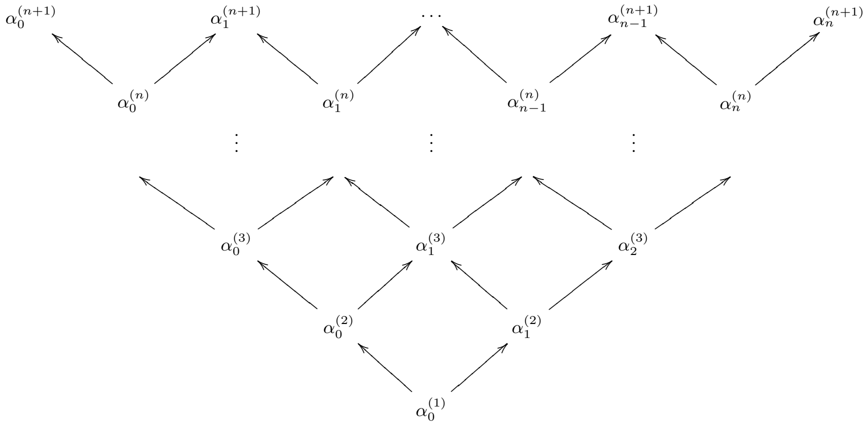
$$\int_{-1}^{1} \varphi_1(x)\overline{\varphi_2(x)}\mu_n(x) = \sum_{j=0}^{n} \lambda_j^{(n+1)}\varphi_1(\alpha_j^{(n+1)})\overline{\varphi_2}(\alpha_j^{(n+1)}) = (\varphi_1, \varphi_2).$$

Thus we can identify H_n isometrically with $\ell_2(\mu_n)$.

Remark also that the zeros $\alpha_j^{(n+1)}$ of $p_{n+1}(x)$ and the zeros $\alpha_j^{(n)}$ of $p_n(x)$, interlace:

$$-1 < \alpha_0^{(n+1)} < \alpha_0^{(n)} < \alpha_1^{(n+1)} < \alpha_1^{(n)} < \cdots < \alpha_{n-1}^{(n)} < \alpha_n^{(n+1)} < 1.$$

Thus we have the picture



This is the picture of our Markov chain.

The *classical* orthogonal polynomials $\{p_n(x)\}$ are characterized (modulo translation and dilation $\{p_n(x)\} \sim \{p_n(ax+b)\}$ changing the interval $[-1,1]$ to an arbitrary interval $[b-a, b+a]$) by any of the following equivalent condition:

1. Hahn: The sequence $\left\{p_n'(x) = \frac{\partial}{\partial x} p_n(x)\right\}_{n \geq 1}$ are again a sequence of orthogonal polynomials (with respect to another measure μ').
2. Bochner: $p_n(x)$ are the eigenfunctions of a second order differential operator

$$\Big(a(x)\frac{\partial^2}{\partial x^2} + b(x)\frac{\partial}{\partial x} + c(x)\Big)p_n(x) = \lambda_n p_n(x).$$

3. Tricomi: $p_n(x)$ can be expressed by a Rodriguez equation

$$p_n(x) = \frac{1}{\gamma_n}\frac{1}{\mu(x)}\frac{\partial^n}{\partial x^n}\mu(x) f(x)^n.$$

These polynomials are either the Jacobi polynomials $p_n^{\alpha,\beta}(x)$, $\mu^{\alpha,\beta}(x) := (1-x)^\alpha (1+x)^\beta$ $(\alpha, \beta > -1)$ or their $\beta \to \infty$ limit, the Laguerre polynomials, or their $\alpha = \beta \to \infty$ limit, the Hermite polynomials. Replacing $\frac{\partial}{\partial x}$ in the above by

$$D_q \varphi(x) = \frac{\varphi(x) - \varphi(qx)}{(1-q)x},$$

we get a similar characterization of the q-classical orthogonal polynomials.

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