A

Summary of Dirichlet Form Theory

Our treatment in this appendix follows that of the standard reference [72] – see also, [106, 3].

A.1 Non-Negative Definite Symmetric Bilinear Forms

Let H be a real Hilbert space with inner product $(\cdot,\cdot)$. We say $\mathcal{E}$ is a *non-negative definite symmetric bilinear form* on H with domain $\mathcal{D}(\mathcal{E})$ if

- $\mathcal{D}(\mathcal{E})$ is a dense linear subspace of H,
- $\mathcal{E} : \mathcal{D}(\mathcal{E}) \times \mathcal{D}(\mathcal{E}) \to \mathbb{R}$,
- $\mathcal{E}(u,v) = \mathcal{E}(v,u)$ for $u,v \in \mathcal{D}(\mathcal{E})$,
- $\mathcal{E}(au+bv,w) = a\mathcal{E}(u,w) + b\mathcal{E}(v,w)$ for $u,v,w \in \mathcal{D}(\mathcal{E})$ and $a,b \in \mathbb{R}$,
- $\mathcal{E}(u,u) \geqslant 0$ for $u \in \mathcal{D}(\mathcal{E})$.

Given a non-negative definite symmetric bilinear form $\mathcal{E}$ on H and $\alpha > 0$, define another non-negative definite symmetric bilinear form $\mathcal{E}_\alpha$ on H with domain $\mathcal{D}(\mathcal{E}_\alpha) := \mathcal{D}(\mathcal{E})$ by

$$\mathcal{E}_\alpha(u,v) := \mathcal{E}(u,v) + \alpha(u,v), \quad u,v \in \mathcal{D}(\mathcal{E}).$$

Note that the space $\mathcal{D}(\mathcal{E})$ is a pre-Hilbert space with inner product $\mathcal{E}_\alpha$, and $\mathcal{E}_\alpha$ and $\mathcal{E}_\beta$ determine equivalent metrics on $\mathcal{D}(\mathcal{E})$ for different $\alpha,\beta > 0$.

If $\mathcal{D}(\mathcal{E})$ is complete with respect to this metric, then $\mathcal{E}$ is said to be *closed*. In this case, $\mathcal{D}(\mathcal{E})$ is then a real Hilbert space with inner product $\mathcal{E}_\alpha$ for each $\alpha > 0$.

A.2 Dirichlet Forms

Now consider a σ-finite measure space $(X,\mathcal{B},m)$ and take H to be the Hilbert space $L^2(X,m)$ with the usual inner product

$$(u, v) := \int_X u(x)v(x)\, m(dx), \quad u, v \in L^2(X, m).$$

Call a non-negative definite symmetric bilinear form $\mathcal{E}$ on $L^2(X, m)$ *Markovian* if for each $\varepsilon > 0$, there exists a real function $\phi_\varepsilon : \mathbb{R} \to \mathbb{R}$, such that

$$\begin{aligned}
&\phi_\varepsilon(t) = t, \quad t \in [0, 1],\\
&-\varepsilon \leqslant \phi_\varepsilon(t) \leqslant 1 + \varepsilon, \quad t \in \mathbb{R},\\
&0 \leqslant \phi_\varepsilon(t) - \phi_\varepsilon(s) \leqslant t - s, \quad -\infty < s < t < \infty,
\end{aligned}$$

and when u belongs to $\mathcal{D}(\mathcal{E})$, $\phi_\varepsilon \circ u$ also belongs to $\mathcal{D}(\mathcal{E})$ with

$$\mathcal{E}(\phi_\varepsilon \circ u, \phi_\varepsilon \circ u) \leqslant \mathcal{E}(u, u).$$

A *Dirichlet form* is a non-negative definite symmetric bilinear form on $L^2(X, m)$ that is Markovian and closed.

A non-negative definite symmetric bilinear form $\mathcal{E}$ on $L^2(X, m)$ is certainly Markovian if whenever u belongs to $\mathcal{D}(\mathcal{E})$, then $v = (0 \vee u) \wedge 1$ also belongs to $\mathcal{D}(\mathcal{E})$ and $\mathcal{E}(v, v) \leqslant \mathcal{E}(u, u)$. In this case say that the *unit contraction* acts on $\mathcal{E}$. It turns out the if the form is closed, then the form is Markovian if and only if the unit contraction acts on it.

Similarly, say that a function v is called a *normal contraction* of a function u if

$$\begin{aligned}
|v(x) - v(y)| &\leqslant |u(x) - u(y)|, \quad x, y \in X,\\
|v(x)| &\leqslant |u(x)|, \quad x \in X,
\end{aligned}$$

and say that $v \in L^2(X, m)$ a normal contraction of $u \in L^2(X, m)$ if some Borel version of v is a normal contraction of some Borel version of u. Say that normal contractions act on $\mathcal{E}$ if whenever v is a normal contraction of $u \in \mathcal{D}(\mathcal{E})$, then $v \in \mathcal{D}(\mathcal{E})$ and $\mathcal{E}(v, v) \leqslant \mathcal{E}(u, u)$. It also turns out that if the form is closed, then the form is Markovian if and only if the unit contraction acts on it.

Example A.1. Let $X \subseteq \mathbb{R}$ be an open subinterval and suppose that m is a Radon measures on X with support all of X. Define a non-negative definite symmetric bilinear form by

$$\mathcal{E}(u, v) := \frac{1}{2} \int_X \frac{du(x)}{dx} \frac{dv(x)}{dx}\, dx$$

on the domain

$$\mathcal{D}(\mathcal{E}) := \{u \in L^2(X, m) : u \text{ is absolutely continuous and } \mathcal{E}(u, u) < \infty\}.$$

We claim that $\mathcal{E}$ is a Dirichlet form on $L^2(X, m)$.

It is easy to check that the unit contraction acts on $\mathcal{E}$. To show the form is closed, take any $\mathcal{E}_1$-Cauchy sequence $\{u_\ell\}$. Then $\{du_\ell/dx\}$ converges to some $f \in L^2(X, dx)$ in $L^2(X, dx)$. Also, $\{u_\ell\}$ converges to some $u \in L^2(X, m)$ in $L^2(X, m)$. From this and the inequality

$$|u(a) - u(b)|^2 \leqslant 2|a - b|\mathcal{E}(u, u), \quad a, b \in X,$$

we conclude that there is a subsequence $\{\ell_k\}$ such that u_{ℓ_k} converges to a continuous function $\tilde{u}$ uniformly on each bounded closed subinterval of X. Obviously $\tilde{u} = u$ m-a.e. For all infinitely differentiable compactly supported functions ϕ on X, an integration by parts shows that

$$\begin{aligned}\int_X f(x)\phi(x)\,dx &= \lim_{\ell_k\to\infty}\int_X \frac{du_{\ell_k}(x)}{dx}\phi(x)\,dx \\ &= -\lim_{\ell_k\to\infty}\int_X u_{\ell_k}(x)\phi'(x)\,dx = -\int_X \tilde{u}(x)\phi'(x)\,dx.\end{aligned}$$

This implies that $\tilde{u}$ is absolutely continuous and $d\tilde{u}/dx = f$. Hence, $\tilde{u} \in \mathcal{D}(\mathcal{E})$ and $\{u_\ell\}$ is $\mathcal{E}_1$-convergent to $\tilde{u}$.

Example A.2. Consider a locally compact metric space (X, ρ) equipped with a Radon measure m. Suppose that we are given a kernel j on $X \times \mathcal{B}(X)$ satisfying the following conditions.

- For any $\varepsilon > 0$, $j(x, X\backslash B_\varepsilon(x))$ is, as a function of $x \in X$, locally integrable with respect to m. Here, as usual, $B_\varepsilon(x)$ is the ball around x of radius ε.
- $\int_X u(x)\,(jv)(x)\,m(dx) = \int_X (ju)(x)\,v(x)\,m(dx)$ for all $u, v \in p\mathcal{B}(X)$.

Then, j determines a symmetric Radon measure J on $X \times X\backslash\varDelta$, where $\varDelta$ is the diagonal, by

$$\int_{X\times X\backslash\varDelta} f(x, y)\,J(dx, dy) := \int_X \left\{\int_X f(x, y)\,j(x, dy)\right\} m(dx).$$

Put

$$\mathcal{E}(u, v) := \int_{X\times X\backslash\varDelta} (u(x) - u(y))(v(x) - v(y))\,J(dx, dy)$$

on the domain

$$\mathcal{D}(\mathcal{E}) := \{u \in L^2(X, m) : \mathcal{E}(u, u) < \infty\}.$$

We claim that $\mathcal{E}$ is a Dirichlet form on $L^2(X, m)$ provided that $\mathcal{D}(\mathcal{E})$ is dense in $L^2(X, m)$.

It is clear that $\mathcal{E}$ is non-negative definite, symmetric, and bilinear. We next show that for a Borel function u that $u = 0$ m-a.e. implies that $\mathcal{E}(u, u) = 0$. Suppose that $u = 0$ m-a.e. Put $\varGamma_{K,\varepsilon} = \{(x, y) \in K \times K : \rho(x, y) > \varepsilon\}$ for $\varepsilon > 0$ and K compact. Then

$$\int_{\Gamma_{K,\varepsilon}} (u(x)-u(y))^2\, J(dx,dy) \leqslant 2\int_{\Gamma_{K,\varepsilon}} (u(x)^2+u(y)^2)\, J(dx,dy)$$
$$= 4\int_{\Gamma_{K,\varepsilon}} u(x)^2 J(dx,dy) \leqslant 4\int_K u(x)^2 j(x, X\backslash B_\varepsilon(x))\, m(dx) = 0.$$

Letting $\varepsilon \downarrow 0$ and $K \uparrow X$ gives $\mathcal{E}(u,u) = 0$.

It is clear that every normal contraction operates on the form and so the form is Markovian. To prove that the form is closed, consider a sequence $\{u_\ell\}$ in $\mathcal{D}(\mathcal{E})$ such that $\lim_{\ell,m\to\infty} \mathcal{E}_1(u_\ell - u_m, u_\ell - u_m) \to 0$. Since $\{u_\ell\}$ converges in $L^2(X,m)$, there is a subsequence $\{\ell_k\}$ and a set $N \in \mathcal{B}(X)$ with $m(N) = 0$ such that $\{u_{\ell_k}(x)\}$ converges on $X\backslash N$. Put $\tilde{u}_{\ell_k}(x) = u_{l_k}(x)$ on $X\backslash N$ and $\tilde{u}_{\ell_k}(x) = 0$ on N. Then $\tilde{u}_{\ell_k}(x)$ has a limit $u(x)$ everywhere and u_ℓ converges to u in $L^2(X,m)$. Moreover,

$$\begin{aligned}
&\mathcal{E}(u-u_m, u-u_m)\\
&\quad = \int_{X\times X\backslash \Delta} \lim_{\ell_k\to\infty} \{(u_{\ell_k}(x) - u_{\ell_k}(y)) - (u_m(x) - u_m(y))\}^2\, J(dx,dy)\\
&\quad \leqslant \liminf_{l_k\to\infty} \mathcal{E}(u_{l_k} - u_m, u_{l_k} - u_m).
\end{aligned}$$

The last term can be made arbitrarily small for sufficiently large m. Thus, u_m is $\mathcal{E}_1-$convergent to $u \in \mathcal{D}(\mathcal{E})$, as required.

A.3 Semigroups and Resolvents

Suppose again that we have a real Hilbert space H with inner product $(\cdot,\cdot)$. Consider a family $\{T_t\}_{t>0}$ of linear operators on H satisfying the following conditions:

- each T_t is a self-adjoint operator with domain H,
- $T_sT_t = T_{s+t}$, $s,t>0$ (that is, $\{T_t\}_{t>0}$ is a *semigroup*),
- $(T_tu, T_tu) \leqslant (u,u)$, $t>0$, $u \in H$ (that is, each T_t is a *contraction*).

We say that $\{T_t\}_{t>0}$ is *strongly continuous* if, in addition,

- $\lim_{t\downarrow 0}(T_tu - u, T_tu - u) = 0$ for all $u \in H$.

A *resolvent* on H is a family $\{G_\alpha\}_{\alpha>0}$ of linear operators on H satisfying the following conditions:

- G_α is a self-adjoint operator with domain H,
- $G_\alpha - G_\beta + (\alpha-\beta)G_\alpha G_\beta = 0$ (the resolvent equation),
- each operator αG_α is a contraction.

The resolvent is said to be *strongly continuous* if, in addition,

- $\lim_{\alpha\to\infty}(\alpha G_\alpha u - u, \alpha G_\alpha u - u) = 0$ for all $u \in H$.

Example A.3. Given a strongly continuous semigroup $\{T_t\}_{t>0}$ on H, the family of operators

$$G_\alpha u := \int_0^\infty e^{-\alpha t} T_t u\, dt$$

is a strongly continuous resolvent on H called the resolvent of the given semigroup. The semigroup may be recovered from the resolvent via the *Yosida approximation*

$$T_t u = \lim_{\beta\to\infty} e^{-t\beta} \sum_{n=0}^{\infty} \frac{(t\beta)^n}{n!} (\beta G_\beta)^n u, \quad u \in H.$$

A.4 Generators

The *generator* A of a strongly continuous semigroup $\{T_t\}_{t>0}$ on H is defined by

$$Au := \lim_{t\downarrow 0} \frac{T_t u - u}{t}$$

on the domain $\mathcal{D}(A)$ consisting of those $u \in H$ such that the limit exists.

Suppose that $\{G_\alpha\}_{\alpha>0}$ is a strongly continuous resolvent on H. Note that if $G_\alpha u = 0$, then, by the resolvent equation, $G_\beta u = 0$ for all $\beta > 0$, and, by strong continuity, $u = \lim_{\beta\to\infty} \beta G_\beta u = 0$. Thus, the operator G_α is invertible and we can set

$$Au := \alpha u - G_\alpha^{-1} u$$

on the domain $\mathcal{D}(A) := G_\alpha(H)$. This operator A is easily seen to be independent of $\alpha > 0$ and is called the *generator* of the resolvent. $\{G_\alpha\}_{\alpha>0}$.

Lemma A.4. *The generator of a strongly continuous semigroup on H coincides with the generator of its resolvent, and the generator is a non-positive definite self-adjoint operator.*

A.5 Spectral Theory

A self-adjoint operator S on H with domain H satisfying $S^2 = S$ is called a *projection.* A family $\{E_\lambda\}_{\lambda\in\mathbb{R}}$ of projection operators on H is called a *spectral family* if

$$\begin{aligned}
&E_\lambda E_\mu = E_\lambda, \quad \lambda \leqslant \mu,\\
&\lim_{\lambda'\downarrow\lambda} E_{\lambda'} u = E_\lambda u, \quad u \in H,\\
&\lim_{\lambda\to-\infty} E_\lambda u = 0, \quad u \in H,\\
&\lim_{\lambda\to\infty} E_\lambda u = u, \quad u \in H.
\end{aligned}$$

Note that $0 \leqslant (E_\lambda u, u) \uparrow (u, u)$ as $\lambda \uparrow \infty$, for $u \in H$, and, by polarization, $\lambda \mapsto (E_\lambda u, v)$ is a function of bounded variation for $u, v \in H$.

Suppose we are given a spectral family $\{E_\lambda\}_{\lambda \in \mathbb{R}}$ on H and a continuous function $\phi(\lambda)$ on $\mathbb{R}$. We can then define a self-adjoint operator A on H, denoted by $\int_{-\infty}^{\infty} \phi(\lambda)\, dE_\lambda$, by requiring that

$$(Au, v) = \int_{-\infty}^{\infty} \phi(\lambda)\, d(E_\lambda u, v), \quad \forall v \in H,$$

where the domain of A is $\mathcal{D}(A) := \{u \in H : \int_{-\infty}^{\infty} \phi(\lambda)\, d(E_\lambda u, u) < \infty\}$.

Conversely, given a self-adjoint operator A on H, there exists a unique spectral family $\{E_\lambda\}_{\lambda \in \mathbb{R}}$ such that $A = \int_{-\infty}^{\infty} \lambda\, dE_\lambda$. This is called the spectral representation of A. If A is non-negative definite, then the corresponding spectral family satisfies $E_\lambda = 0$ for $\lambda < 0$.

Let $-A$ be a non-negative definite self-adjoint operator on H and let $-A = \int_0^{\infty} \lambda\, dE_\lambda$ be its spectral representation. For any non-negative continuous function ϕ on $\mathbb{R}_+$, we define the self-adjoint operator $\phi(-A)$ by $\phi(-A) := \int_0^{\infty} \phi(\lambda)\, dE_\lambda$. Note that $\phi(-A)$ is again non-negative definite.

A.6 Dirichlet Form, Generator, Semigroup, Resolvent Correspondence

Lemma A.5. *Let $-A$ be a non-negative definite self-adjoint operator on H. The family $\{T_t\}_{t>0} := \{\exp(tA)\}_{t>0}$ is a strongly continuous semigroup, and the family $\{G_\alpha\}_{\alpha>0} := \{(\alpha - A)^{-1}\}_{\alpha>0}$ is a strongly continuous resolvent. The generator of $\{T_t\}_{t>0}$ is A and $\{T_t\}_{t>0}$ is the unique strongly continuous semigroup with generator A. A similar statement holds for the resolvent.*

Theorem A.6. *There is a bijective correspondence between the family of closed non-negative definite symmetric bilinear forms $\mathcal{E}$ on H and the family of non-positive definite self-adjoint operators A on H. The correspondence is given by*

$$\mathcal{D}(\mathcal{E}) = \mathcal{D}(\sqrt{-A})$$

and

$$\mathcal{E}(u, v) = (\sqrt{-A}u, \sqrt{-A}v).$$

Consider a σ-finite measure space $(X, \mathcal{B}, m)$. A linear operator S on $L^2(X, m)$ with domain $L^2(X, m)$ is *Markovian* if $0 \leqslant Su \leqslant 1$ m-a.e. whenever $u \in L^2(X, m)$ and $0 \leqslant u \leqslant 1$ m-a.e.

Theorem A.7. *Let $\mathcal{E}$ be a closed non-negative definite symmetric bilinear form on $L^2(X, m)$. Write $\{T_t\}_{t>0}$ and $\{G_\alpha\}_{\alpha>0}$ for the corresponding strongly continuous semigroup and the strongly continuous resolvent on $L^2(X, m)$. The following five conditions are equivalent.*

(a) T_t *is Markovian for each* $t > 0$.
(b) αG_α *is Markovian for each* $\alpha > 0$.
(c) $\mathcal{E}$ *is Markovian.*
(d) The unit contraction operates on $\mathcal{E}$.
(e) Normal contractions operate on $\mathcal{E}$.

A.7 Capacities

Suppose that X is a Lusin space and m is a Radon measure. There is a set function associated with a Dirichlet form $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ on $L^2(X, m)$ called the (1)-capacity and denoted by Cap. If $U \subseteq X$ is open, then

$$\mathrm{Cap}(U) := \inf \{\mathcal{E}_1(f, f) \,:\, f \in \mathcal{D}(\mathcal{E}),\, f(x) \geqslant 1,\, m-\text{a.e. } x \in U\}\,.$$

More generally, if $V \subseteq X$ is an arbitrary subset, then

$$\mathrm{Cap}(V) := \inf \{\mathrm{Cap}(U) :\, V \subseteq U,\, U \text{ is open}\}\,.$$

The set function Cap is a Choquet capacity.

We say that some property holds quasi-everywhere or, equivalently, for quasi-every $x \in X$, if the set $x \in X$ where the property fails to hold has capacity 0. We abbreviate this by saying that the property holds q.e. or for q.e. every $x \in X$.

A.8 Dirichlet Forms and Hunt Processes

A *Hunt process* is a strong Markov process

$$\mathbf{X} = (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \{\mathbb{P}^x\}_{x\in E}, \{X_t\}_{t\geqslant 0})$$

on a Lusin state space E that has right-continuous, left-limited sample paths and is also quasi-left-continuous. Write $\{P_t\}_{t\geqslant 0}$ for the transition semigroup of $\mathbf{X}$. That is, $P_t f(x) = \mathbb{P}^x[f(X_t)]$ for $f \in b\mathcal{B}(E)$. If μ is a Radon measure on $(E, \mathcal{B}(E))$, we say that $\mathbf{X}$ is *μ-symmetric* if $\int_E f(x)\, P_t g(x)\, \mu(dx) = \int_E P_t f(x)\, g(x)\, \mu(dx)$ for all $f, g \in b\mathcal{B}(E)$. Intuitively, if the process $\mathbf{X}$ is started according to the initial "distribution" μ, then reversing the direction of time leaves finite-dimensional distributions unchanged.

Theorem A.8. *Let $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ be a Dirichlet form on $L^2(E, \mu)$, where E is Lusin and μ is Radon. Write $\{T_t\}_{t>0}$ for the associated strongly continuous contraction semigroup of Markovian operators. Suppose that there exists a collection $\mathcal{C} \subseteq L^2(E, \mu)$ and a sequence of compact sets $K_1 \subseteq K_2 \subseteq \ldots$ such that:*

(a) $\mathcal{C}$ *is a countably generated subalgebra of* $\mathcal{D}(\mathcal{E}) \cap bC(E)$,

(b) $\mathcal{C}$ *is* $\mathcal{E}_1$*-dense in* $\mathcal{D}(\mathcal{E})$*,*
(c) $\mathcal{C}$ *separates points of* E *and, for any* $x \in E$*, there is an* $f \in \mathcal{C}$ *such that* $f(x) \neq 0$*,*
(d) $\lim_{n\to\infty} \mathrm{Cap}(E\backslash K_n) = 0$.

Then there is a μ*-symmetric Hunt process* $\mathbf{X}$ *on* E *with transition semigroup* $\{P_t\}_{t\geqslant 0}$ *such that* $P_t f(x) = T_t f(x)$ *for* $f \in b\mathcal{B}(E) \cap L^2(E,\mu)$*.*

Remark A.9. The theory in [72] for symmetric Hunt processes associated with Dirichlet forms is developed under the hypothesis that the state space is locally compact. However, the embedding results outlined in Section 7.3 of [72], shows that the results developed under the hypothesis of local compactness still holds if the state space is Lusin and the hypotheses of Theorem A.8 hold.

Lemma A.10. *Suppose that* $\mathbf{X}$ *is the* μ*-symmetric Hunt process constructed from a Dirichlet form* $(\mathcal{E},\mathcal{D}(\mathcal{E}))$ *satisfying the conditions of Theorem A.8 and* $B \in \mathcal{B}(E)$*. Then* $\mathbb{P}^x\{\exists t > 0 : X_t \in B\} = 0$ *for* μ*-a.e.* $x \in E$ *if and only if* $\mathrm{Cap}(B) = 0$.

B

Some Fractal Notions

This appendix is devoted to recalling briefly some definitions about various ways of assigning sizes and dimensions to metric spaces and then applying this theory to the ultrametric completions of $\mathbb{N}$ obtained in Example 3.41 from the $\mathbb{R}$-tree associated with a non-increasing family of partitions of $\mathbb{N}$.

B.1 Hausdorff and Packing Dimensions

Let $(\mathcal{X}, \rho)$ be a compact metric space. Given a set $A \subseteq \mathcal{X}$ and $\epsilon > 0$, a countable collection of balls $\{B_i\}$ is said to be an *ϵ-covering* of A if $A \subseteq \bigcup_i B_i$ and each ball has diameter at most ϵ. Note that if $\epsilon' < \epsilon''$, then an ϵ'-covering of A is also an ϵ''-covering. For $\alpha > 0$, the *α-dimensional Hausdorff measure* on $\mathcal{X}$ is the Borel measure that assigns mass

$$\mathcal{H}^\alpha(A) := \sup_{\epsilon>0} \inf \left\{ \sum_i \operatorname{diam}(B_i)^\alpha \ : \ \{B_i\} \text{ is an } \epsilon\text{-covering of } A \right\}$$

to a Borel set A. The *Hausdorff dimension* of A is the infimum of those α such that the corresponding α-dimensional Hausdorff measure is zero.

A countable collection of balls $\{B_i\}$ is said to be an *ϵ-packing* of a set $A \subseteq \mathcal{X}$ if the balls are disjoint, the center of each ball belongs to A, each ball has diameter at most ϵ. Note that if $\epsilon' < \epsilon''$, then an ϵ''-packing of A is also an ϵ'-packing. For $\alpha > 0$, the *α-dimensional packing pre-measure* on $\mathcal{X}$ assigns mass

$$P^\alpha(A) := \inf_{\epsilon>0} \sup \left\{ \sum_i \operatorname{diam}(B_i)^\alpha \ : \ \{B_i\} \text{ is an } \epsilon\text{-packing of } A \right\}$$

to a set A. The *α-dimensional packing measure* on $\mathcal{X}$ is the Borel measure that assigns mass

$$\mathcal{P}^{\alpha}(A) := \inf\left\{\sum_i P^{\alpha}(A_i) \,:\, A \subseteq \bigcup_i A_i\right\}$$

to a Borel set A where the infimum is over all countable collections of Borel sets $\{A_i\}$ such that $A \subseteq \bigcup_i A_i$. The *packing dimension* of A is the infimum of those α such that the corresponding α-dimensional packing measure is zero.

Theorem B.1. *The packing dimension of a set is always at least as great as its Hausdorff dimension.*

We refer the reader to [107] for more about and properties of Hausdorff and packing dimension.

B.2 Energy and Capacity

Let $(\mathcal{X}, \rho)$ be a compact metric space. Write $M_1(\mathcal{X})$ for the collection of Borel probability measures on $\mathcal{X}$. A *gauge* is a function $f : [0, \infty[\to [0, \infty]$, such that:

- f is continuous and non-increasing,
- $f(0) = \infty$,
- $f(r) < \infty$ for $r > 0$,
- $\lim_{r\to\infty} f(r) = 0$.

Given $\mu \in M_1(\mathcal{X})$ and a gauge f, the *energy of* μ *in the gauge* f is the quantity

$$\mathcal{E}_f(\mu) := \int \mu(dx) \int \mu(dy)\, f(\rho(x, y)).$$

The *capacity of* $\mathcal{X}$ *in the gauge* f is the quantity

$$\mathrm{Cap}_f(\mathcal{X}) := \left(\inf\{\mathcal{E}_f(\mu) : \mu \in M_1(\mathcal{X})\}\right)^{-1}$$

(note by our assumptions on f that we need only consider diffuse $\mu \in M_1(\mathcal{X})$ in the infimum).

The *capacity dimension* of $\mathcal{X}$ is the supremum of those $\alpha > 0$ such that $\mathcal{X}$ has strictly positive capacity in the gauge $f(x) = x^{-\alpha}$ (where we adopt the convention that the supremum of the empty set is 0).

Theorem B.2. *The Hausdorff and capacity dimensions of a compact metric space always coincide.*

We again refer to [107] for more about capacities and their connection to Hausdorff dimension.

B.3 Application to Trees from Coalescing Partitions

Recall the construction in Example 3.41 of a $\mathbb{R}$-tree and an associated ultrametric completion $(\mathbb{S}, \delta)$ of $\mathbb{N}$ from a coalescing family $\{\Pi(t)\}_{t>0}$ of partitions of $\mathbb{N}$. We will assume that $\Pi(t)$ has finitely many blocks for $t > 0$, so that $(\mathbb{S}, \delta)$ is compact.

Write $N(t)$ for the number of blocks of $\Pi(t)$ and for $k \in \mathbb{N}$ put $\sigma_k := \inf\{t \geqslant 0 : N(t) \leqslant k\}$. The non-increasing function Π is constant on each of the intervals $[\sigma_k, \sigma_{k-1}[$, $k > 1$. Write $1 = I_1(t) < \cdots < I_{N(t)}(t)$ for an ordered listing of the least elements of the various blocks of $\Pi(t)$.

We can associate each partition $\Pi(t)$ with an equivalence relation $\sim_{\Pi(t)}$ on $\mathbb{N}$ by declaring that $i \sim_{\Pi(t)} j$ if i and j are in the same block of $\Pi(t)$.

Given $B \subseteq \mathbb{S}$, write $\mathrm{cl}B$ for the closure of B. Each of the sets

$$\begin{aligned} U_i(t) &= \mathrm{cl}\{j \in \mathbb{N} : j \sim_{\Pi(t)} I_i(t)\} \\ &= \mathrm{cl}\{j \in \mathbb{N} : \delta(j, I_i(t)) \leqslant 2t\} \\ &= \{y \in \mathbb{S} : \delta(y, I_i(t)) \leqslant 2t\} \end{aligned}$$

is a closed ball with diameter at most t (in an ultrametric space, the diameter and radius of a ball are equal). The closed balls of $\mathbb{S}$ are also the open balls and every ball is of the form $U_i(t)$ for some $t > 0$ – see, for example, Proposition 18.4 of [123] – and, in fact, every ball is of the form $U_i(\sigma_k)$ for some $k \in \mathbb{N}$ and $1 \leqslant i \leqslant k$. In particular, the collection of balls is countable. Any ball of diameter at most $2t$ is contained in a unique one of the $U_i(t)$, and any ball of diameter at least $2t$ contains one or more of the $U_i(t)$ – see, for example, Proposition 18.5 of [123].

We need to adapt to our setting the alternative expression for energy obtained by summation–by–parts in Section 2 of [112]. For $t > 0$ write $\mathcal{U}(t)$ for the collection of balls $\{U_1(t), \ldots, U_{N(t)}(t)\}$. Let $\mathcal{U}$ denote the union of these collections over all $t > 0$, so that $\mathcal{U}$ is just the countable collection of all balls of $\mathbb{S}$. Given $U \in \mathcal{U}$ with $U \neq \mathbb{S}$, let $U^{\rightarrow}$ denote the unique element of $\mathcal{U}$ such that there exists no $V \in \mathcal{U}$ with $U \subsetneq V \subsetneq U^{\rightarrow}$. More concretely, such a ball U is in $\mathcal{U}(\sigma_k)$ but not in $\mathcal{U}(\sigma_{k-1})$ for some unique $k > 1$, and $U^{\rightarrow}$ is the unique element of $\mathcal{U}(\sigma_{k-1})$ such that $U \subset U^{\rightarrow}$. Define $\mathbb{S}^{\rightarrow} := \dagger$, where $\dagger$ is an adjoined symbol. Put $\mathrm{diam}(\dagger) = \infty$.

Given a gauge f, write φ_f for the diffuse measure on $[0, \infty[$ such that $\varphi_f([r, \infty[) = \varphi_f(]r, \infty[) = f(r)$, $r \geqslant 0$. For a diffuse probability measure $\mu \in M_1(\mathbb{S})$ we have, with the convention $f(\infty) = 0$,

$$
\begin{aligned}
\mathcal{E}_f(\mu) &= \int \mu(dx) \int \mu(dy)\, f(\delta(x,y)) \\
&= \int \mu(dx) \int \mu(dy) \sum_{U \in \mathcal{U},\, \{x,y\} \subseteq U} f(\operatorname{diam}(U)) - f(\operatorname{diam}(U^{\rightarrow})) \\
&= \sum_{U \in \mathcal{U}} (f(\operatorname{diam}(U)) - f(\operatorname{diam}(U^{\rightarrow}))) \\
&\quad \times \int \mu(dx) \int \mu(dy)\, \mathbf{1}\{\{x,y\} \subseteq U\} \qquad \text{(B.1)} \\
&= \sum_{U \in \mathcal{U}} (f(\operatorname{diam}(U)) - f(\operatorname{diam}(U^{\rightarrow})))\, \mu(U)^2 \\
&= \sum_{U \in \mathcal{U}} \int_{[0,\infty[} \varphi_f(dt) \mathbf{1}\{U \in \mathcal{U}(t)\} \mu(U)^2 \\
&= \int_{[0,\infty[} \varphi_f(dt) \sum_{U \in \mathcal{U}(t)} \mu(U)^2.
\end{aligned}
$$

Proposition B.3. *Suppose for all $t > 0$ that the asymptotic block frequencies*

$$F_i(t) := \lim_{n \to \infty} n^{-1} \left|\{0 \leqslant j \leqslant n-1 : j \sim_{\Pi(t)} I_i(t)\}\right|, \; 1 \leqslant i \leqslant N(t),$$

exist and

$$F_1(t) + \cdots + F_{N(t)}(t) = 1.$$

Suppose also that for some $\alpha > 0$ that

$$0 < \liminf_{t \downarrow 0} t^{\alpha} N(t) \leqslant \limsup_{t \downarrow 0} t^{\alpha} N(t) < \infty$$

and

$$0 < \liminf_{t \downarrow 0} t^{-\alpha} \sum_{i=1}^{N(t)} F_i(t)^2 \leqslant \limsup_{t \downarrow 0} t^{-\alpha} \sum_{i=1}^{N(t)} F_i(t)^2 < \infty.$$

Then the Hausdorff and packing dimensions of $\mathbb{S}$ are both α and there are constants $0 < c' \leqslant c'' < \infty$ such that for any gauge f

$$c' \left(\int_0^1 f(t) t^{\alpha-1}\, dt \right)^{-1} \leqslant \operatorname{Cap}_f(\mathbb{S}) \leqslant c'' \left(\int_0^1 f(t) t^{\alpha-1}\, dt \right)^{-1}.$$

Proof. In order to establish that both the Hausdorff and packing dimensions of $\mathbb{S}$ are at most α it suffices to consider the packing dimension, because packing dimension always dominates Hausdorff dimension.

By definition of packing dimension, in order to establish that the packing dimension is at most α it suffices to show for each $\eta > \alpha$ that there is a constant $c < \infty$ such that for any packing $B_1, B_2, \ldots$ of $\mathbb{S}$ with balls of diameter at most 1, we have $\sum_k \operatorname{diam}(B_k)^{\eta} \leqslant c$. If $2 \cdot 2^{-p} \leqslant \operatorname{diam}(B_k) < 2 \cdot 2^{-(p-1)}$ for some $p \in \{0, 1, 2, \ldots\}$, then B_k contains one or more of the balls $U_i(2^{-p})$. Thus

$$|\{k \in \mathbb{N} : 2 \cdot 2^{-p} \leqslant \operatorname{diam}(B_k) < 2 \cdot 2^{-(p-1)}\}| \leqslant N(2^{-p})$$

and

$$\sum_k \operatorname{diam}(B_k)^\eta \leqslant \sum_{p=0}^{\infty} N(2^{-p}) 2^{-(p-1)\eta} < \infty,$$

as required.

If we establish the claim regarding capacities, then this will establish that the capacity dimension of $\mathbb{S}$ is α. This then gives the required lower bound on the packing and Hausdorff dimensions because the Hausdorff measure equals the capacity dimension and the packing dimension dominates Hausdorff dimension.

In order to establish the claimed lower bound on $\operatorname{Cap}_f(\mathbb{S})$ it appears, a priori, that for each gauge f we might need to find a probability measure μ depending on f such that $(\mathcal{E}_f(\mu))^{-1}$ is at least the left–hand side of the inequality. It turns out, however, that we can find a measure that works simultaneously for all gauges f. We construct this measure as follows.

Let $\mathcal{A}$ denote the algebra of subsets of $\mathbb{S}$ generated by the collection of balls $\mathcal{U}$. Thus, $\mathcal{A}$ is just the countable collection of finite unions of balls. The σ–algebra generated by $\mathcal{A}$ is the Borel σ–algebra of $\mathbb{S}$. The sets in $\mathcal{A}$ are compact, and, moreover, for all $k \in \mathbb{N}$ and indices $1 \leqslant i \leqslant k$ if $U_i(\sigma_k) = U_{i_1}(\sigma_{k+1}) \cup U_{i_2}(\sigma_{k+1}) \cup \cdots \cup U_{i_m}(\sigma_{k+1})$ (that is, if $\{I_{i_1}(\sigma_{k+1}), I_{i_2}(\sigma_{k+1}), \ldots, I_{i_m}(\sigma_{k+1})\} = \{I_\ell(\sigma_{k+1}) : I_\ell(\sigma_{k+1}) \sim_{\Pi(\sigma_k)} I_i(\sigma_k)\}$), then $F_i(\sigma_k) = F_{i_1}(\sigma_{k+1}) + F_{i_2}(\sigma_{k+1}) + \cdots + F_{i_m}(\sigma_{k+1})$. It is, therefore, possible to define a finitely additive set function ν on $\mathcal{A}$ such that

$$\nu(U_i(t)) = F_i(t), \; t > 0, \; 1 \leqslant i \leqslant N(t), \tag{B.2}$$

and

$$\nu(\mathbb{S}) = 1. \tag{B.3}$$

Furthermore, if $A_1 \supseteq A_2 \supseteq \ldots$ is a decreasing sequence of sets in the algebra $\mathcal{A}$ such that $\bigcap_n A_n = \varnothing$, then, by compactness, $A_n = \varnothing$ for all n sufficiently large and it is certainly the case that $\lim_{n\to\infty} \nu(A_n) = 0$. A standard extension theorem – see, for example, Theorems 3.1.1 and 3.1.4 of [48] – gives that the set function ν extends to a probability measure (also denoted by ν) on the Borel σ–algebra of $\mathbb{S}$.

From (B.1) we see that for some constant $0 < c' < \infty$ (not depending on f) we have

$$\begin{aligned}
\mathrm{Cap}_f(\mathbb{S}) \geqslant (\mathcal{E}_f(\nu))^{-1} &= \left(\int \varphi_f(dt) \sum_{U \in \mathcal{U}(t)} \nu(U)^2 \right)^{-1} \\
&= \left(\int \varphi_f(dt) \sum_{i=1}^{N(t)} F_i(t)^2 \right)^{-1} \\
&\geqslant c' \left(\int \varphi_f(dt)(t \wedge 1)^{\alpha} \right)^{-1} = c' \left(\int_0^1 f(t) t^{\alpha-1}\, dt \right).
\end{aligned}$$

Turning to the upper bound on $\mathrm{Cap}_f(\mathbb{S})$, note from the Cauchy-Schwarz inequality that for any $\mu \in M_1(\mathbb{S})$

$$1 = \left(\sum_{U \in \mathcal{U}(t)} \mu(U) \right)^2 \leqslant N(t) \sum_{U \in \mathcal{U}(t)} \mu(U)^2,$$

and so, by (B.1),

$$\begin{aligned}
\mathrm{Cap}_f(\mathbb{S}) &\leqslant \left(\int \varphi_f(dt) N(t)^{-1} \right)^{-1} \\
&\leqslant c'' \left(\int \varphi_f(dt)(t \wedge 1)^{\alpha} \right)^{-1} = c'' \left(\int_0^1 f(t) t^{\alpha-1}\, dt \right),
\end{aligned}$$

for some constant $0 < c'' < \infty$. □

Remark B.4. By the Cauchy-Schwarz inequality,

$$1 = \left(\sum_{i=1}^{N(t)} F_i(t) \right)^2 \leqslant \left(\sum_{i=1}^{N(t)} F_i(t)^2 \right) N(t).$$

Thus,

$$\limsup_{t \downarrow 0} t^{\alpha} N(t) < \infty \Longrightarrow \liminf_{t \downarrow 0} t^{-\alpha} \sum_{i=1}^{N(t)} F_i(t)^2 > 0$$

and

$$\limsup_{t \downarrow 0} t^{-\alpha} \sum_{i=1}^{N(t)} F_i(t)^2 < \infty \Longrightarrow \liminf_{t \downarrow 0} t^{\alpha} N(t) > 0.$$

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