

A

Appendix

A.1 Basic facts concerning the Ford domain

In this appendix, we give a proof to some of the basic facts concerning the Ford domain, the proof of which could not be found in the literature. Throughout this appendix, Γ denotes a non-elementary Kleinian group, such that the stabilizer Γ_∞ of ∞ contains parabolic transformations, and H_∞ denotes a fixed horoball centered at ∞ which is precisely (Γ, Γ_∞) -invariant. The following well-known observation plays an important role in this section.

Lemma A.1.1. (1) *The Euclidean radii of the horoballs in $\Gamma H_\infty - \{H_\infty\}$ is bounded from the above.*

(2) *For any compact subset K in \mathbb{H}^3 , only finitely many horoballs in ΓH_∞ intersect K .*

Proof. (1) follows from the fact that all horoballs in $\Gamma H_\infty - \{H_\infty\}$ are disjoint from H_∞ and hence their Euclidean radii are less than the half of the “Euclidean height”, t , of ∂H_∞ , where t is the positive real number such that $\partial H_\infty = \mathbb{C} \times \{t\} \subset \mathbb{H}^3$.

(2) Since K is a compact subset of \mathbb{H}^3 , its Euclidean height is bounded below, i.e., there is a positive constant c such that $K \subset \mathbb{C} \times [c, \infty) \subset \mathbb{H}^3$. On the other hand, by virtue of (1), there is a compact subset, L , of \mathbb{C} which contains the centers of all horoballs in $\Gamma H_\infty - \{H_\infty\}$ intersecting K . (If r is the upper bound obtained in (1), then we may set L to be the closed r -neighborhood of $\text{proj}(K)$ in \mathbb{C} with respect to the Euclidean metric.) Thus if a horoball in $\Gamma H_\infty - \{H_\infty\}$ intersects K , then its Euclidean radius is $\geq c/2$ and its center is contained in the compact set L . Since ΓH_∞ consists of disjoint horoballs, only finitely many of its members satisfy these conditions. Hence we obtain the conclusion.

We rephrase Proposition 1.1.3. part of which is proved in [72, Lemma 5.20] under the additional assumption that ∞ is a bounded parabolic fixed point.

Proposition A.1.2. *The Ford domain $Ph(\Gamma)$ is a “fundamental polyhedron of Γ modulo Γ_∞ ”, in the following sense.*

1. $\mathbb{H}^3 = \cup\{A(Ph(\Gamma)) \mid A \in \Gamma\}$.
2. $\text{int } Ph(\Gamma)$ is precisely (Γ, Γ_∞) -invariant.
3. For any compact set K of \mathbb{H}^3 , only finitely many images $A(Ph(\Gamma))$ ($A \in \Gamma$) can intersect K , namely the set $\{A\Gamma_\infty \in \Gamma/\Gamma_\infty \mid A(Ph(\Gamma)) \cap K \neq \emptyset\}$ is finite.
4. $Ph(\Gamma)$ is a closed convex polyhedron (Definition 3.4.1(2)).

Proof. (1) By applying Lemma A.1.1(1), to a (hyperbolic) closed ball $B(x, r)$ with center $x \in \mathbb{H}^3$ and radius $r > 0$, we see that the minimal distance

$$d(x, \Gamma H_\infty) := \min\{d(x, AH_\infty) \mid A \in \Gamma\}$$

is well-defined, i.e., there is an element $A_x \in \Gamma$ such that $d(x, A_x H_\infty) \leq d(x, AH_\infty)$ for every $A \in \Gamma$. This implies that $A_x^{-1}(x) \in Ph(\Gamma) = \{x \in \mathbb{H}^3 \mid d(x, H_\infty) = d(x, \Gamma H_\infty)\}$. Hence we have $\mathbb{H}^3 = \cup\{A(Ph(\Gamma)) \mid A \in \Gamma\}$.

(2) This is a direct consequence of Lemma 4.1.1(1).

(3) We show that for every $x \in \mathbb{H}^3$ and $\epsilon > 0$, the compact set $B(x, \epsilon)$ satisfies the conclusion. To this end, put $r = d(x, \Gamma H_\infty)$. By Lemma A.1.1(2), $B(x, r + 2\epsilon)$ intersects only finitely many horoballs $A_1(H_\infty), A_2(H_\infty), \dots, A_n(H_\infty)$ in ΓH_∞ . Suppose $A(Ph(\Gamma)) \cap B(x, \epsilon) \neq \emptyset$ for some $A \in \Gamma$. Pick a point y from the intersection. Then

$$\begin{aligned} d(x, A^{-1}(H_\infty)) &\leq \epsilon + d(y, A^{-1}(H_\infty)) = \epsilon + d(y, \Gamma H_\infty) \\ &\leq 2\epsilon + d(x, \Gamma H_\infty) = 2\epsilon + r. \end{aligned}$$

Here the identity in the above follows from the assumption that $y \in A(Ph(\Gamma))$. Hence $B(x, r + 2\epsilon) \cap A(H_\infty) \neq \emptyset$ and therefore $A^{-1}(H_\infty) = A_j(H_\infty)$ for some j . This implies $A^{-1}(Ph(\Gamma)) = A_j(Ph(\Gamma))$. Hence the desired result holds for the compact set $B(x, \epsilon)$.

(4) Let K be a compact subset of \mathbb{C} , and consider the isometric hemispheres $Ih(A)$ ($A \in \Gamma - \Gamma_\infty$) which intersect K . Then as in the proof of Lemma A.1.1(2), we see that their Euclidean radii are bounded below, their centers are contained in a compact subset of \mathbb{C} , and that the radii of the corresponding horoballs $A(H_\infty)$ are bounded below. Hence we see that there are only finitely many such isometric hemispheres. (We can also deduce this by using Lemma 2.5.2(2)). Hence $Ph(\Gamma)$ satisfies the condition for a convex polyhedron in Definition 3.4.1(2).

The following lemma is proved by imitating the argument of [55, Proof of Lemma 2.13] for the Dirichlet domain (cf. [72, Lemma 5.37]).

Lemma A.1.3. *For every point ξ in the Ford polygon $P(\Gamma)$, there is a horoball H_ξ centered at ξ such that $H_\xi \cap H_\infty$ is a singleton and $(\text{int } H_\xi) \cap \Gamma H_\infty = \emptyset$. In particular, any point of $P(\Gamma) \cap \Lambda(\Gamma)$ is not a horospherical limit point of Γ ([55, Definition in p.51]).*

Proof. Let ξ be a point in $P(\Gamma)$. Then the vertical geodesic (ξ, ∞) is contained in $Ph(\Gamma)$. Hence, for each $x \in (\xi, \infty) - H_\infty$, $\text{int } B(x, d(x, H_\infty))$ is disjoint from ΓH_∞ . Thus the open horoball $\cup \{\text{int } B(x, d(x, H_\infty)) \mid x \in (\xi, \infty) - H_\infty\}$, centered at ξ , is disjoint from ΓH_∞ . Moreover its closure, H_ξ , intersect H_∞ precisely at the point $\partial H_\infty \cap (\xi, \infty)$. Thus we obtain the first assertion. To see the second assertion, pick a point x from $\text{int } H_\infty$. Then its orbit Γx is disjoint from H_ξ . Hence ξ is not a horospherical limit point.

Corollary A.1.4. *If Γ is geometrically finite, then any point of $P(\Gamma) \cap \Lambda(\Gamma)$ is the (parabolic) fixed point of a parabolic element of Γ which is not conjugate to an element of Γ_∞ . In particular, if Γ is a quasifuchsian punctured torus group, then $P(\Gamma) \subset \Omega(\Gamma)$.*

Proof. Suppose Γ is geometrically finite and let ξ be a point in $P(\Gamma) \cap \Lambda(\Gamma)$. Then ξ is not a horospherical limit point and hence it is a bounded parabolic fixed point (see [55, Theorem 3.7]). On the other hand, we see that the orbit $\Gamma \infty$ is disjoint from $P(\Gamma)$ as follows. Suppose to the contrary that $A(\infty)$ belongs to $P(\Gamma) = \overline{Ph}(\Gamma) \cap \mathbb{C}$. Then $A(\text{int } H_\infty) \cap \text{int } Ph(\Gamma) \neq \emptyset$ and hence $A \in \Gamma_\infty$ by Proposition A.1.2(2). Thus $\infty = A(\infty) \in P(\Gamma)$, a contradiction. Hence ξ does not belong to the orbit $\Gamma \infty$, and we obtain the first assertion. The second assertion follows from the fact that every parabolic transformation of a quasifuchsian punctured torus group is conjugate to an element of Γ_∞ .

The following lemma is a refinement of Lemma A.1.3.

Lemma A.1.5. *A point $\xi \in \mathbb{C}$ belongs to $P(\Gamma)$, if and only if there is a horoball H_ξ centered at ξ such that $H_\xi \cap \Gamma H_\infty = \emptyset$ and $d(H_\xi, H_\infty) \leq d(H_\xi, A(H_\infty))$ for every $A \in \Gamma$.*

Proof. Suppose that ξ belongs to $P(\Gamma)$. Then by Lemma A.1.3, there is a horoball H_ξ centered at ξ such that $H_\xi \cap H_\infty$ is a singleton and $(\text{int } H_\xi) \cap \Gamma H_\infty = \emptyset$. Reset H_ξ to be a horoball contained in the interior of this horoball. Then we see that this new horoball H_ξ satisfies the desired conditions.

Conversely, suppose that the latter condition is satisfied. We show that $(\xi, \infty) \cap H_\xi \subset Ph(\Gamma)$. To this end, pick a point $x \in (\xi, \infty) \cap H_\xi$ and $A \in \Gamma$. Let γ be the shortest geodesic segment joining x to $A(H_\infty)$. Then

$$\begin{aligned} d(x, A(H_\infty)) &= \text{length}(\gamma) \\ &= \text{length}(\gamma \cap H_\xi) + \text{length}(\gamma \cap (\mathbb{H}^3 - \text{int } H_\xi)) \\ &\geq d(x, \partial H_\xi) + d(H_\xi, A(H_\infty)) \\ &\geq d(x, \partial H_\xi) + d(H_\xi, H_\infty) \\ &= d(x, H_\infty) \end{aligned}$$

(This inequality is easily seen by performing a coordinate change so that H_ξ is centered at ∞ .) This implies $x \in Ph(\Gamma)$. Thus we have $(\xi, \infty) \cap H_\xi \subset Ph(\Gamma)$. Hence $\xi \in P(\Gamma)$.

The above lemma immediately implies the following corollary.

Corollary A.1.6. *For each $A_0 \in \Gamma$, a point $\xi \in \mathbb{C}$ belongs to $A_0(P(\Gamma))$, if and only if there is a horoball H_ξ centered at ξ such that $H_\xi \cap \Gamma H_\infty = \emptyset$ and $d(H_\xi, A_0(H_\infty)) \leq d(H_\xi, A(H_\infty))$ for every $A \in \Gamma$.*

We now prove the following proposition.

Proposition A.1.7. *The intersection, $P(\Gamma) \cap \Omega(\Gamma)$, of the Ford polygon and the domain of discontinuity is a “fundamental polygon of Γ modulo Γ_∞ ”, for the action of Γ on $\Omega(\Gamma)$ in the following sense.*

1. $\Omega(\Gamma) = \cup\{A(P(\Gamma) \cap \Omega(\Gamma)) \mid A \in \Gamma\}$.
2. $\text{int } P(\Gamma) = \text{int}(P(\Gamma) \cap \Omega(\Gamma))$ is precisely (Γ, Γ_∞) -invariant.
3. For any compact set K of $\Omega(\Gamma)$, only finitely many images $A(P(\Gamma))$ ($A \in \Gamma$) can intersect K , namely the set $\{A\Gamma_\infty \in \Gamma/\Gamma_\infty \mid A(P(\Gamma)) \cap K \neq \emptyset\}$ is finite.

Proof. (1) Let ξ be a point in $\Omega(\Gamma)$. Then some neighborhood of ξ in \mathbb{C} is disjoint from the centers of the horoballs in ΓH_∞ , because they are contained in $\Lambda(\Gamma)$. Since the Euclidean radii of the horoballs in $\Gamma H_\infty - \{H_\infty\}$ are bounded above, we can find a closed neighborhood D of ξ in $\overline{\mathbb{H}}^3$ such that $D \cap \Gamma H_\infty = \emptyset$.

Claim A.1.8. Any horoball H_ξ centered at $\xi \in \Omega(\Gamma)$ can intersect only finitely many horoballs in ΓH_∞ .

Proof. Let H_ξ be a horoball centered at ξ . Then the closure of $H_\xi - D$ in \mathbb{H}^3 is compact and a horoball in ΓH_∞ intersects H_ξ if and only if it intersects the relatively compact set $H_\xi - D$. Hence we have the claim by Lemma A.1.1(2).

Now pick a small horoball H_ξ centered at ξ contained in D . Then by applying the above claim to a closed r -neighborhood of H_ξ in \mathbb{H}^3 , which is again a horoball centered at ξ , for sufficiently large r , we see that

$$d(H_\xi, \Gamma H_\infty) := \min\{d(H_\xi, AH_\infty) \mid A \in \Gamma\}$$

is a well-defined positive number, i.e., there is an element $A_\xi \in \Gamma$ such that $d(H_\xi, A_\xi(H_\infty)) \leq d(H_\xi, A(H_\infty))$ for every $A \in \Gamma$. Thus we see $\xi \in A_\xi(P(\Gamma))$ by Corollary A.1.6, and hence $A_\xi^{-1}(\xi) \in P(\Gamma) \cap \Omega(\Gamma)$. So we have $\Omega(\Gamma) = \cup\{A(P(\Gamma) \cap \Omega(\Gamma)) \mid A \in \Gamma\}$.

(2) This is a direct consequence of Lemma 4.1.1(1).

(3) Let K be a compact subset of $\Omega(\Gamma)$. Then by the argument in the proof of (1), there is a compact neighborhood D of K in $\overline{\mathbb{H}}^3$ such that $D \cap \Gamma H_\infty = \emptyset$. Pick a constant $c > 0$ such that each horoball, H_ξ , with Euclidean radius c centered at a point $\xi \in K$ is contained in $\text{int}(D \cap \mathbb{H}^3)$. Throughout the proof we reserve the symbol H_ξ to denote these horoballs, and set $H_K = \cup\{H_\xi \mid \xi \in K\}$. By using the compactness of K , we can

find a constant $r > 0$ such that $d(H_\xi, \Gamma H_\infty)$, which is well-defined by (the proof of) (1), is at most r for every $\xi \in K$. Consider the closed r -neighborhood $B(H_K, r)$ of H_K in \mathbb{H}^3 . Then $B(H_K, r) - D$ is relatively compact in \mathbb{H}^3 . Thus we see, by Lemma A.1.1(2), that $B(H_K, r)$ intersects only finitely many horoballs, $A_1(H_\infty), A_2(H_\infty), \dots, A_n(H_\infty)$, in ΓH_∞ . Now suppose $K \cap A(P(\Gamma)) \neq \emptyset$. Pick a point ξ from the intersection. Then, by Corollary A.1.6, $d(H_\xi, A(H_\infty)) = d(H_\xi, \Gamma H_\infty) \leq r$. Hence $B(H_K, r) \cap A(H_\infty) \neq \emptyset$. Thus $A(H_\infty) = A_j(H_\infty)$ for some j . This implies $A(P(\Gamma)) = A_j(P(\Gamma))$. Hence we obtain the desired result.

Remark A.1.9. (1) Since two “edges” of $\text{fr } P(\Gamma)$ may be tangent, $\text{fr } P(\Gamma)$ is not necessarily a 1-dimensional manifold.

(2) $\text{fr } P(\Gamma) \cap \Omega(\Gamma)$ is locally finite in the sense that any point $x \in \text{fr } P(\Gamma) \cap \Omega(\Gamma)$, there is a neighborhood U of x in $\Omega(\Gamma)$ such that $\text{fr } P(\Gamma) \cap U$ is a finite union of circular arcs. However, $\text{fr } P(\Gamma)$ is not necessarily locally finite around points in $\text{fr } P(\Gamma) \cap \Lambda(\Gamma)$.

At the end of this appendix, we prove the following finiteness property for the Ford domain.

Lemma A.1.10. *For a point p in $\overline{Ph}(\Gamma)$, let $[p]$ be the set of points in $\overline{Ph}(\Gamma)$ which are Γ -equivalent to p , namely*

$$[p] = \Gamma p \cap \overline{Ph}(\Gamma) = \{x \in \overline{Ph}(\Gamma) \mid x = A(p) \text{ for some } A \in \Gamma\}.$$

Then the quotient set $[p]/\Gamma_\infty$ is finite provided that $p \in \mathbb{H}^3 \cup \Omega(\Gamma)$ or p is a bounded parabolic fixed point of Γ . In particular, if Γ is geometrically finite, then $[p]/\Gamma_\infty$ is a finite set for every $p \in \overline{Ph}(\Gamma)$.

Proof. We prove the lemma only for the case when p is a point in \mathbb{C} (and hence in $P(\Gamma)$). (The proof for the case $p \in \mathbb{H}^3$ is parallel to this case and is much simpler.) Let ξ be a point in $P(\Gamma)$, and let H_ξ be a horoball centered at ξ satisfying the condition in Lemma A.1.5. Then $r := d(H_\xi, \Gamma H_\infty)$ is well-defined and is equal to $d(H_\xi, H_\infty)$. By Corollary A.1.6, for each $A \in \Gamma$, we have $A(\xi) \in P(\Gamma)$ if and only if $d(H_\xi, A^{-1}(H_\infty)) = r$. The latter condition holds if and only if $A^{-1}(H_\infty)$ intersects the horoball $B(H_\xi, r)$. Hence we see

$$[p] = \{A(p) \mid B(H_\xi, r) \cap A^{-1}(H_\infty) \neq \emptyset\}.$$

Now suppose that $\xi \in \Omega(\Gamma)$. Then by Claim A.1.8, only finitely many horoballs in ΓH_∞ can intersect $B(H_\xi, r)$. Moreover, for two elements A_1 and A_2 of Γ , $A_1^{-1}(H_\infty) = A_2^{-1}(H_\infty)$ if and only if $A_1^{-1}\Gamma_\infty = A_2^{-1}\Gamma_\infty \in \Gamma/\Gamma_\infty$. Hence the set

$$\{A^{-1}\Gamma_\infty \in \Gamma/\Gamma_\infty \mid B(H_\xi, r) \cap A^{-1}(H_\infty) \neq \emptyset\}$$

is finite. By the observation in the previous paragraph, the correspondence $A^{-1}\Gamma_\infty \mapsto \Gamma_\infty A(\xi)$ determines a surjective map from the above finite set to

the quotient set $[p]/\Gamma_\infty$. (To be precise, the symbol $[p]/\Gamma_\infty$ should be denoted by $\Gamma_\infty \backslash [p]$, because the action of Γ_∞ on $[p]$ is a left action.) Hence $[p]/\Gamma_\infty$ is finite.

Next suppose that ξ is a bounded parabolic fixed point. Then we can easily see that, modulo the action of the parabolic stabilizer Γ_ξ of ξ , only finitely many horoballs in ΓH_∞ can intersect $B(H_\xi, r)$. Thus the set

$$\{\Gamma_p A^{-1} \Gamma_\infty \in \Gamma_p \backslash \Gamma / \Gamma_\infty \mid B(H_\xi, r) \cap A^{-1}(H_\infty) \neq \emptyset\}$$

is finite. Hence we obtain the finiteness of $[p]/\Gamma_\infty$ as in the previous case.

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Notation

- General topology
 - $\text{int } X$: the interior of X
 - $\text{fr } X$: the frontier of X
 - \overline{X} : the closure of X
 - $\pi_1(X)$: the fundamental group of X
- Cellular complex
 - $C^{(k)}$: the k -skeleton of a complex C
 - $\text{lk}(\xi, \mathcal{L})$: the link of ξ in \mathcal{L}
 - $\text{st}_0(\xi, \mathcal{L})$: the subcomplex of \mathcal{L} spanned by ξ and $\text{lk}(\xi, \mathcal{L})$
- Abstract group
 - $[X, Y] := XYX^{-1}Y^{-1}$
 - $X^Y := YXY^{-1}$
 - $\langle\langle X \rangle\rangle$: the normal closure of X
- Numbers
 - \mathbb{N} : the set of natural numbers
 - \mathbb{Z} : the set of integral numbers
 - \mathbb{Q} : the set of rational numbers
 - \mathbb{R} : the set of real numbers
 - \mathbb{C} : the set of complex numbers
 - $\hat{\mathbb{Q}}$: the union $\mathbb{Q} \cup \{\infty\}$
 - $\hat{\mathbb{C}}$: the Riemann sphere $\mathbb{C} \cup \{\infty\}$
 - $[j]$: the integer in $\{0, 1, 2\}$ such that $[j] \equiv j \pmod{3}$
 - ϵ : a sign $-$ or $+$
- Surfaces
 - T : the once-punctured torus
 - S : the four-times punctured sphere
 - \mathcal{O} : the $(2, 2, 2, \infty)$ -orbifold over S^2
 - $\text{Teich}(X)$: the Teichmüller space of a surface X
- Model spaces
 - \mathbb{H}^n : the hyperbolic n -space

- $\overline{\mathbb{H}^n}$: the closure, $\mathbb{H}^n \cup \partial\mathbb{H}^n$, of \mathbb{H}^n
- $\overline{\mathbb{H}^3}$: the closure, $\mathbb{H}^3 \cup \mathbb{C}$, of the upper half space model of \mathbb{H}^3
- $\mathbb{E}^{1,n}$: the $(n+1)$ -dimensional Minkowski space
- $\mathcal{P}(\mathbb{H}^3)$: the space of convex polyhedra in \mathbb{H}^3
- $\mathcal{P}(\overline{\mathbb{H}^3})$: the space of convex polyhedra in $\overline{\mathbb{H}^3}$
- \mathbb{E} : the symbol to mean something which is Euclidean
- $\mathcal{C}(X)$: the closed convex hull of X
- Matrix groups: for $R = \mathbb{Z}, \mathbb{R}$ or \mathbb{C}
 - $SL(2, R)$: the set of all 2×2 -matrices with determinant 1
 - $PSL(2, R) := SL(2, R)/\{\pm 1\}$
- Farey triangulation
 - \mathcal{D} : the Farey triangulation (the modular diagram)
 - $\langle v_0, \dots, v_k \rangle$: the k -simplex in \mathcal{D} spanned by vertices $v_0, \dots, v_k \in \mathcal{D}^{(0)}$
 - $s(X) \in \mathcal{D}^{(0)}$: the slope of X , where X is a generator of $\pi_1(T)$ or an elliptic generator of $\pi_1(\mathcal{O})$
 - \mathcal{T} : the binary tree dual to \mathcal{D}
 - $\overrightarrow{E}(\mathcal{T})$: the set of directed edges of \mathcal{T}
 - \overrightarrow{e} : a directed edge of \mathcal{T}
 - \mathcal{EG} : the set of elliptic generators
 - $\Sigma = (\sigma_1, \dots, \sigma_m)$: a chain of triangles
 - σ^ϵ : the ϵ -terminal triangle of a chain
 - $\Sigma(\nu)$: the chain of triangles determined by a label ν
 - $\sigma^\epsilon(\nu)$: the ϵ -terminal triangle of $\Sigma(\nu)$
- Ideal triangulation of $T \times [-1, 1]$
 - $\text{trg}(\sigma)$: the topological ideal triangulation of T determined by σ
 - $\overline{\text{trg}}(\sigma)$: the topological ideal triangulation of $\mathbb{R}^2 - \mathbb{Z}^2$ determined by σ
 - $\text{Trg}(\nu)$: the layer of topological ideal triangulations of T determined by ν
 - $\overline{\text{Trg}}(\nu)$: the layer of topological ideal triangulations of $\mathbb{R}^2 - \mathbb{Z}^2$ determined by ν
 - $\text{spine}(\sigma)$: the spine of T determined by σ
 - $\text{Spine}(\delta^-, \delta^+)$: the “trace of spines” of T in $T \times [-1, 1]$
- For an element X of $SL(2, \mathbb{C})$ or $PSL(2, \mathbb{C})$
 - $\text{tr } X$: the trace of X
 - $\text{Axis } X$: the axis of $X \subset \mathbb{H}^3$
 - $\overline{\text{Axis}} X$: the closure of $\text{Axis } X$ in $\overline{\mathbb{H}^3}$
 - $\text{Fix } X$: the fixed point of X in \mathbb{C}
 - $I(X)$: the isometric circle of X
 - $E(X)$: the exterior of $I(X)$
 - $Ih(X)$: the isometric hemisphere of X
 - $Eh(X)$: the exterior of $Ih(X)$
 - $\overline{Ih}(X)$: the closure of $Ih(X)$ in $\overline{\mathbb{H}^3}$
 - $\overline{Eh}(X)$: the closure of $Eh(X)$ in $\overline{\mathbb{H}^3}$
 - $\overline{Dh}(X)$: the closure of $Dh(X)$ in $\overline{\mathbb{H}^3}$

- $c(X)$: the center of $I(X)$
- $r(X)$: the radius of $I(X)$
- For a Kleinian group Γ
 - $\text{Stab}_\Gamma(x)$: the stabilizer of x with respect to the action of Γ
 - $\Omega(\Gamma)$: the domain of discontinuity of Γ
 - $\Omega^\epsilon(\Gamma)$: the ϵ -component of $\Omega(\Gamma)$
 - $\Lambda(\Gamma)$: the limit set of Γ
 - $M(\Gamma)$: the hyperbolic manifold (or orbifold) \mathbb{H}^3/Γ
 - $\bar{M}(\Gamma)$: the quotient manifold (or orbifold) $(\mathbb{H}^3 \cup \Omega(\Gamma))/\Gamma$
 - $P(\Gamma)$: the Ford polygon of Γ in $\widehat{\mathbb{C}}$
 - $Ph(\Gamma)$: the Ford domain of Γ in \mathbb{H}^3
 - $\bar{Ph}(\Gamma) := Ph(\Gamma) \cup P(\Gamma)$
 - $\text{Ford}(\Gamma)$: the Ford complex of Γ in $\bar{M}(\Gamma)$
 - $\Delta_{\mathbb{E}}(\Gamma)$: the Euclidean decomposition of $M(\Gamma)$
- Spaces of representations
 - $\text{Hom}_{\text{tp}}(\pi_1(X), PSL(2, \mathbb{C}))$: the space of all type-preserving $PSL(2, \mathbb{C})$ -representations of $\pi_1(X)$ for $X = T, \mathcal{O}$ and S
 - $\mathcal{X} := \text{Hom}_{\text{tp}}(\pi_1(X), PSL(2, \mathbb{C}))/PSL(2, \mathbb{C})$
 - ρ : a type-preserving representation
 - \mathcal{QF} : the space of quasifuchsian representations $\subset \mathcal{X}$
 - $\overline{\mathcal{QF}}$: the closure of \mathcal{QF} in \mathcal{X}
 - Φ : the space of Markoff maps
 - ϕ : a Markoff map
 - Ψ : the space of complex probabilities
 - $\zeta_{\nu, \sigma}^\epsilon$: the polynomial function $\Phi \rightarrow \mathbb{C}$ defined by $\zeta_{\nu, \sigma}^\epsilon(\phi) = \phi(s_0) + \alpha^\epsilon \phi(s_1) + \beta^\epsilon \phi(s_2)$
 - $\Phi_{\nu, \sigma}^\epsilon$: the subvariety $(\zeta_{\nu, \sigma}^\epsilon)^{-1}(0)$ of Φ
 - Φ_ν^ϵ : the “geometric” irreducible component of $\Phi_{\nu, \sigma}^\epsilon$
 - $d_G(\nu)$: the geometric degree of ν
- Side parameter
 - ν : the side parameter
 - $\nu^\epsilon(\rho)$: the ϵ -component of ν
 - ν : used to denote a label $\nu \in \mathcal{D}$
 - $\theta^\epsilon(\rho, \sigma)$: the ϵ -angle invariant of ρ at σ
 - $\theta^\epsilon(\rho, \sigma; s_{[j]})$: the $s_{[j]}$ -component of $\theta^\epsilon(\rho, \sigma)$
 - $\mathcal{J}[\mathcal{QF}] \subset \mathcal{X} \times (\mathbb{H}^2 \times \mathbb{H}^2)$: the space of good labeled representations
 - $\mu_1 : \mathcal{J}[\mathcal{QF}] \rightarrow \mathcal{X}$: the natural projection
 - $\mu_2 : \mathcal{J}[\mathcal{QF}] \rightarrow \mathbb{H}^2 \times \mathbb{H}^2$: the natural projection
 - $\rho = (\rho, \nu)$: a labeled representation
- For a pair (ρ, σ)
 - $I(j) = I(\rho(P_j))$: the isometric circle of $\rho(P_j)$
 - $D(j) = D(\rho(P_j))$: the disk bounded by $I(\rho(P_j))$
 - $E(j) = E(\rho(P_j))$: the exterior of $I(\rho(P_j))$
 - $Th(j) = Th(\rho(P_j))$: the isometric hemisphere of $\rho(P_j)$

- $Dh(j) = Dh(\rho(P_j))$: the half space of \mathbb{H}^3 bounded by $Ih(\rho(P_j))$ whose closure contains $D(\rho(P_j))$
- $Eh(j) = Eh(\rho(P_j))$: the half space $\mathbb{H}^3 - \text{int } Dh(\rho(P_j))$
- $c(j) = c(\rho(P_j))$: the center of $I(\rho(P_j))$
- $\vec{c}(j, j+1) = \vec{c}(\rho; P_j, P_{j+1})$: the vector or oriented line $c(\rho(P_{j+1})) - c(\rho(P_j))$
- $\text{Fix}^\epsilon(j) = \text{Fix}_\sigma^\epsilon(\rho(P_j))$: the fixed point of $\rho(P_j)$ which lies in the ϵ -side of $\mathcal{L}(\rho, \sigma)$
- $\text{Axis}(j) = \text{Axis}(\rho(P_j))$: the axis of $\rho(P_j)$
- $\vec{f}(j)$: the oriented line $\text{Fix}_\sigma^-(\rho(P_j)) \text{Fix}_\sigma^+(\rho(P_j))$.
- $v^\epsilon(j, j+1) = v^\epsilon(\rho; P_j, P_{j+1})$: the point of $I(\rho(P_j)) \cap I(\rho(P_{j+1}))$ which lies in the ϵ -side of $\mathcal{L}(\rho, \sigma)$
- $e^\epsilon(j) = e^\epsilon(\rho, \sigma; P_j)$: the ϵ -ideal edge in $I(\rho(P_j))$ (Notation 4.3.7)
- $\Delta_j^\epsilon = \Delta_j^\epsilon(\rho, \sigma)$: “the j -th triangle” in the ϵ -side of $\mathcal{L}(\rho, \sigma)$ (Definition 4.2.10)
- $\Delta(\rho, \sigma)$: the model triangle for $\Delta_j^\epsilon(\rho, \sigma)$ ’s
- $f_\rho^\epsilon(\xi)$: the ϵ -ideal face determined by ρ and ξ
- $\alpha(\rho, \sigma; s_j)$: the (inner) angle of the triangle at the vertex w_j
- Elliptic generator complex
 - $\mathcal{L}(\rho, \sigma)$: a bi-infinite broken line in \mathbb{C} determined by ρ and σ
 - $\mathcal{L}(\rho, \Sigma)$: the union of bi-infinite broken lines $\mathcal{L}(\rho, \sigma)$ in \mathbb{C} for triangles σ in a chain Σ
 - $\mathcal{L}(\rho)$: the set $\mathcal{L}(\rho, \Sigma(\nu))$ for $\rho = (\rho, \nu)$
 - $\mathcal{L}(\sigma)$: an abstract bi-infinite broken line determined by σ
 - $\mathcal{L}(\Sigma)$: the elliptic generator complex associated with Σ
 - $\mathcal{L}(\nu)$: the elliptic generator complex associated with $\Sigma(\nu)$
 - $\mathcal{L}^*(\nu)$: the augmentation of $\mathcal{L}(\nu)$
 - $\partial_{\text{aug}}^\epsilon \mathcal{L}^*(\nu)$: the “ ϵ -boundary” of $\mathcal{L}^*(\nu)$
- Dual map from \mathcal{L} to ∂Eh
 - $F_\rho : \mathcal{L}(\nu)^{(\leq 2)} \rightarrow \mathcal{P}(\mathbb{H}^3)$: the dual map to \mathbb{H}^3
 - $F_\rho(\xi)$: the image of ξ by F_ρ
 - $\overline{F}_\rho : \mathcal{L}(\nu)^{(\leq 2)} \rightarrow \mathcal{P}(\overline{\mathbb{H}}^3)$: the dual map to $\overline{\mathbb{H}}^3$
 - $\overline{F}_\rho(\xi)$: the image of ξ by \overline{F}_ρ
- Quotient by the action of $\langle K \rangle$
 - $\text{Cusp}(K) := \mathbb{H}^3 / \langle \rho(K) \rangle$
 - $\overline{\text{Cusp}}(K) := \overline{\mathbb{H}}^3 / \langle \rho(K) \rangle$
 - $\partial \text{Cusp}(K) := \mathbb{C} / \langle \rho(K) \rangle$
 - $q_K : \overline{\mathbb{H}}^3 \rightarrow \overline{\text{Cusp}}(K)$: the projection
 - $Eh_K(\rho) := q_K(Eh(\rho)) \subset \text{Cusp}(K)$
 - $E_K(\rho) := q_K(E(\rho)) \subset \partial \text{Cusp}(K)$
 - $F_{K, \rho}(\xi) := q_K(F_\rho(\xi))$

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