Appendix A
Relative SNCD

Let $f\colon X \longrightarrow S$ be a smooth morphism of schemes and let D be a relative SNCD on X/S. Set $U := X \setminus D$ and let $j\colon U \overset{\subset}{\longrightarrow} X$ be the natural open immersion. Set $N(D) := \mathcal{O}_X \cap j_*(\mathcal{O}_U^*)$ as in §2.1. In this appendix we prove basic properties of D, $M(D)$ and $N(D)$:

(A) We determine the local behavior of the decomposition of D by smooth components ((A.0.1)).

(B) We refine decompositions of D by smooth components as possible ((A.0.3), (A.0.7)).

(C) We prove the equality $N(D) = M(D)$ if S is reduced at any point of $f(D)$ ((A.0.8) (1)).

(D) We show the inequality $N(D) \neq M(D)$ if S is not reduced at $f(x)$ for a point $x \in X$ ((A.0.8) (2)). In fact, we prove that the stalk $(N(D)/\mathcal{O}_X^*)_x$ is not finitely generated ((A.0.9)).

For a morphism $Y \longrightarrow T$ of schemes, recall $\mathrm{Div}(Y/T)_{\geq 0}$ in §2.1. If T is the spectrum of a commutative ring A with unit element, we often denote $\mathrm{Div}(Y/T)_{\geq 0}$ simply by $\mathrm{Div}(Y/A)_{\geq 0}$ in this section. For a non-zero divisor section $y \in \Gamma(Y, \mathcal{O}_Y)$ such that $\underline{\mathrm{Spec}}_Y(\mathcal{O}_Y/y\mathcal{O}_Y)$ is flat over T, we denote by $\mathrm{div}(y)$ the effective Cartier divisor defined by the ideal sheaf $y\mathcal{O}_Y$ of $\mathcal{O}_Y$.

Let us take a diagram (2.1.7.2) (with S_0 replaced by S) and a point z in $V \subseteq X$. Then, we may assume that $z \in \bigcap_{i=1}^{s}\{y_i = 0\}$ by shrinking V and replacing s if necessary. Henceforth we always assume that $z \in \bigcap_{i=1}^{s}\{y_i = 0\}$ when we take a diagram like (2.1.7.2) and a point z in V.

In (2.1.7), there is no relation a priori between a decomposition of D by smooth components and the diagram (2.1.7.2). Though the uniqueness of the decomposition does not necessarily hold, the local decomposition is determined by the diagram (2.1.7.2):

Proposition A.0.1. *Let $f\colon (X, D) \longrightarrow S$ be as in the beginning of this section. Let $\Delta := \{D_\lambda\}_{\lambda \in \Lambda}$ be a decomposition of D by smooth components. Let z be a point of D and assume that we are given a cartesian diagram (2.1.7.2) (with S_0 replaced by S, such that $z \in \bigcap_{i=1}^{s}\{y_i = 0\}$). Then, by*

shrinking V, for any $1 \le i \le s$, there exists a unique element $\lambda_i \in \Lambda$ satisfying $D_{\lambda_i}|_V = \mathrm{div}(y_i)$ in $\mathrm{Div}(V/S)_{\ge 0}$.

Proof. Set $B := \mathcal{O}_{X,z}$ and $X_z := \mathrm{Spec}(B)$. Let $D_1, \dots, D_m$ be the elements in Δ which contain z and let d_i $(1 \le i \le m)$ be elements of B such that $D_i \cap X_z = \mathrm{div}(d_i)$. Then we have the following equality

$$\sum_{i=1}^{m} \mathrm{div}(d_i) = \sum_{i=1}^{s} \mathrm{div}(y_i) \tag{A.0.1.1}$$

in $\mathrm{Div}(X_z/S)_{\ge 0}$ by the diagram (2.1.7.2). We have to prove that $s = m$ and that $y_i B = d_i B$ $(1 \le i \le s)$ up to some renumbering of the indexes.

Case I: First consider the case $S = \mathrm{Spec}(\kappa)$, where κ is a field.

In this case, B is a regular local ring. By Auslander-Buchsbaum's theorem [63, §19, Theorem 48], B is a UFD. Since $\mathcal{O}_{D_i,z} = B/d_iB$ is also a regular local ring, the ideal d_iB of B is prime. Since $y_i \notin B^*$ $(1 \le i \le s)$ (because $z \in \bigcap_{i=1}^{s}\{y_i = 0\}$), the ideal y_iB of B is also prime. Since B is a UFD, the equality (A.0.1.1) implies the equalities $s = m$ and $d_iB = y_iB$ $(1 \le i \le s)$ up to some renumbering of the indexes. This completes the proof in the Case I.

Case II: Next consider the case $S = \mathrm{Spec}(A)$, where A is an integral domain.

Set $K := \mathrm{Frac}(A)$ and let η be the generic point of S. For a scheme T over S, set $T_\eta := T \times_S \eta$. Set $B_K := B \otimes_A K$ and for a point w in $X_{z,\eta} := \mathrm{Spec}(B_K)$, denote by $(B_K)_w$ the localization of B_K at the prime ideal corresponding to w. Then $\mathrm{Spec}((B_K)_w)$ is nothing but the localization of X_η at w, where we identify w with its image in X_η.

Fix an index i $(1 \le i \le m)$ for the moment. Since $D_i \cap X_z$ is flat over S, there exists a point $w \in D_i \cap X_z$ such that w is sent to η by the structural morphism $X_z \longrightarrow S$ (Indeed, we may assume that S is local; in this case the morphism $D_i \cap X_z \longrightarrow S$ is faithfully flat.). Then $w \in D_{i,\eta}$. By the result in the Case I, there exists an index j $(1 \le j \le s)$ such that

$$d_i(B_K)_w = y_j(B_K)_w. \tag{A.0.1.2}$$

Denote by φ the composite morphism $B \longrightarrow B_K \longrightarrow (B_K)_w$ and by $\overline{\varphi}$ the induced morphism $B/y_jB \longrightarrow (B_K)_w/y_j(B_K)_w$ by φ. Then φ is flat. Consider the following commutative diagram

(A.0.1.3)

$$\begin{tikzcd}
0 \arrow[r] & B \arrow[r, "y_j \times"] \arrow[d, "\varphi"'] & B \arrow[r] \arrow[d, "\varphi"'] & B/y_jB \arrow[r] \arrow[d, "\overline{\varphi}"'] & 0 \\
0 \arrow[r] & (B_K)_w \arrow[r, "y_j \times"] & (B_K)_w \arrow[r] & (B_K)_w/y_j(B_K)_w \arrow[r] & 0.
\end{tikzcd}$$

Because the upper horizontal sequence in (A.0.1.3) is exact and because φ is flat, the lower horizontal sequence is also exact. The morphisms $B \longrightarrow B_K$ and $B/y_jB \longrightarrow B_K/y_jB_K$ are injective. Because B_K and B_K/y_jB_K are regular, they are integral domains. Hence the morphisms $B_K \longrightarrow (B_K)_w$ and $B_K/y_jB \longrightarrow (B_K)_w/y_j(B_K)_w$ are also injective. Thus the vertical morphisms in the diagram (A.0.1.3) are injective. By an easy diagram-chasing, we have an equality $y_jB = \varphi^{-1}(y_j(B_K)_w)$. Similarly we have an equality $d_iB = \varphi^{-1}(d_i(B_K)_w)$. Hence $d_iB = y_jB$ by (A.0.1.2).

For each i $(1 \leq i \leq m)$, we may take $j(i)$ with $1 \leq j(i) \leq m$ such that $d_iB = y_{j(i)}B$. Then we obtain the following equality

$$\sum_{j=1}^{s} \operatorname{div}(y_j) = \sum_{i=1}^{m} \operatorname{div}(y_{j(i)}) \quad \text{in} \quad \operatorname{Div}(X_z/S)_{\geq 0}.$$

Set $Y := \bigcap_{i=1}^{m} \operatorname{div}(y_{j(i)}) \subset X_z$. Since z is contained in Y and since Y is flat over S, there exists a point $w \in Y$ such that w is sent to η by the structural morphism $X_z \longrightarrow S$. Hence $w \in Y_\eta$. If we define $(B_K)_w$ as before, we have the equality $\sum_{j=1}^{s} \operatorname{div}(y_j) = \sum_{i=1}^{m} \operatorname{div}(y_{j(i)})$ in $\operatorname{Div}(\operatorname{Spec}((B_K)_w)/K)_{\geq 0}$. Since $(B_K)_w$ is a UFD and since $y_j \notin (B_K)_w^*$ $(1 \leq j \leq m)$, we obtain $s = m$ and $\operatorname{div}(d_i) = \operatorname{div}(y_i)$ up to some renumbering of the indexes.

Case III: Next consider the case where S is a noetherian scheme.

Let t be the image of z by f. Then we may assume that S is the spectrum of a noetherian local ring A with closed point t. Let $\mathfrak{m}$ be the maximal ideal of A. Let $\mathfrak{p}$ be a prime ideal of A. By the result in the Case II, we have $s = m$ and $d_i(B/\mathfrak{p}B) = y_i(B/\mathfrak{p}B)$ $(1 \leq i \leq s)$ up to some renumbering of the indexes. Note that the correspondence $d_i \leftrightarrow y_i$ is independent of the choice of $\mathfrak{p}$. Indeed, $d_i(B/\mathfrak{p}B) = y_i(B/\mathfrak{p}B)$ implies $d_i(B/\mathfrak{m}B) = y_i(B/\mathfrak{m}B)$, and hence y_i is uniquely determined by the result in the Case I.

Now let $\psi\colon B \longrightarrow B/\mathfrak{p}B$ be the projection and by $\overline{\psi}$ the induced morphism $B/y_iB \longrightarrow (B/\mathfrak{p}B)/y_i(B/\mathfrak{p}B)$ by ψ. Consider the following diagram

(A.0.1.4)

$$\begin{array}{ccccccccc}
0 & \longrightarrow & B & \xrightarrow{y_i \times} & B & \longrightarrow & B/y_iB & \longrightarrow & 0 \\
 & & \downarrow{\scriptstyle \psi} & & \downarrow{\scriptstyle \psi} & & \downarrow{\scriptstyle \overline{\psi}} & & \\
0 & \longrightarrow & B/\mathfrak{p}B & \xrightarrow{y_i \times} & B/\mathfrak{p}B & \longrightarrow & (B/\mathfrak{p}B)/y_i(B/\mathfrak{p}B) & \longrightarrow & 0.
\end{array}$$

Since y_i is a non-zero divisor in B and in $B/\mathfrak{p}B$, the horizontal sequences are exact. Because $\operatorname{Ker}(\overline{\psi}) = (\mathfrak{p}B + y_iB)/y_iB$, an easy diagram-chasing gives an equality $\psi^{-1}(y_i(B/\mathfrak{p}B)) = y_iB + \mathfrak{p}B$. Similarly we have an equality $\psi^{-1}(d_i(B/\mathfrak{p}B)) = d_iB + \mathfrak{p}B$, and hence we have an inclusion $d_iB \subset y_iB + \mathfrak{p}B$, that is,

(A.0.1.5) $$d_i(B/y_iB) \subset \mathfrak{p}(B/y_iB)$$

for any $1 \le i \le s$ and for any prime ideal $\mathfrak{p}$ of A. Let $\mathfrak{n}$ be the nilpotent radical of A and let $\mathfrak{n} = \bigcap_{k=1}^{q} \mathfrak{p}_k$ ($q \in \mathbb{Z}_{>0}$) be the primary decomposition of $\mathfrak{n}$, where $\mathfrak{p}_k$ is a prime ideal of A. Then, by (A.0.1.5) for $\mathfrak{p} = \mathfrak{p}_k$ ($1 \le k \le q$), we obtain $d_i(B/y_iB) \subset \bigcap_{k=1}^{q} \mathfrak{p}_k(B/y_iB) = \mathfrak{n}(B/y_iB)$, where the last equality follows from the flatness of B/y_iB over A. Hence we obtain $d_iB \subset y_iB + \mathfrak{n}B$ for $1 \le i \le s$.

Now we prove the inclusion $d_iB \subset y_iB$ ($1 \le i \le s$). To prove this, it suffices to prove the inclusion $d_iB \subset y_iB + \mathfrak{n}^eB$ ($1 \le i \le s$) for any positive integer e since $\mathfrak{n}^e = 0$ for some e. We prove this inclusion by induction on e. We have already proved the inclusion for the case $e = 1$. Assume that we have the inclusion $d_iB \subset y_iB + \mathfrak{n}^eB$ ($1 \le i \le s$) for a positive integer e. Then there exists an element u_i (resp. ϵ_i) of B (resp. $\mathfrak{n}^eB$) such that $d_i = u_iy_i + \epsilon_i$. By the diagram (2.1.7.2), we have

$$(\prod_{i=1}^{s} y_i)B = (\prod_{i=1}^{s} d_i)B = (\prod_{i=1}^{s}(u_iy_i + \epsilon_i))B. \tag{A.0.1.6}$$

By reducing the equality (A.0.1.6) modulo $\mathfrak{n}$, we obtain

$$(\prod_{i=1}^{s} y_i)(B/\mathfrak{n}B) = (\prod_{i=1}^{s} u_i)(\prod_{i=1}^{s} y_i)(B/\mathfrak{n}B).$$

Since $\prod_{i=1}^{s} y_i$ is a non-zero divisor in $B/\mathfrak{n}B$, each u_i is invertible in $B/\mathfrak{n}B$, and hence so is in B. Replacing d_i by $u_i^{-1}d_i$, we may assume that $u_i = 1$ for any i. Fix an index i_0 ($1 \le i_0 \le s$) and consider the equation (A.0.1.6) (with $u_i = 1$) modulo $y_{i_0}B + \mathfrak{n}^{e+1}B$. Then we see that $\epsilon_{i_0}\prod_{i\ne i_0} y_i \in y_{i_0}B + \mathfrak{n}^{e+1}B$. Because the following diagram

$$\begin{tikzcd} B/\mathfrak{n}^{e+1} \arrow[r, "(\prod_{i\ne i_0} y_i)\times"] \arrow[d, "y_{i_0}\times"'] & B/\mathfrak{n}^{e+1} \arrow[d, "y_{i_0}\times"] \\ B/\mathfrak{n}^{e+1} \arrow[r, "(\prod_{i\ne i_0} y_i)\times"] & B/\mathfrak{n}^{e+1} \end{tikzcd} \tag{A.0.1.7}$$

is cartesian (because $B/\mathfrak{n}^{e+1}B$ is flat over $(A/\mathfrak{n}^{e+1})[y_1, \ldots, y_d]$), $\epsilon_{i_0} \in y_{i_0}B + \mathfrak{n}^{e+1}B$. Hence, for any i ($1 \le i \le s$), $\epsilon_i \in y_iB + \mathfrak{n}^{e+1}B$ and consequently $d_i \in y_iB + \mathfrak{n}^{e+1}B$. Therefore we obtain the inclusion $d_iB \subset y_iB + \mathfrak{n}^{e+1}B$, in fact, the inclusion $d_iB \subset y_iB$. By the last inclusion and the equality $(\prod_{i=1}^{s} d_i)B = (\prod_{i=1}^{s} y_i)B$, we can easily deduce the equality $d_iB = y_iB$. We complete the proof in the Case III.

Case IV: Finally we prove the proposition in the general case. By shrinking V, the structural morphism $V \longrightarrow S$ fits into the following cartesian diagram

$$
\begin{array}{ccc}
V & \longrightarrow & \underline{V} \\
\downarrow & & \downarrow \\
S & \longrightarrow & \underline{S}
\end{array}
$$

satisfying the following conditions:

(1) $\underline{S}$ is a noetherian scheme and $\underline{V} \longrightarrow \underline{S}$ is smooth.

(2) There exists an effective Cartier divisor $\underline{D}_i = \operatorname{div}(\underline{d}_i)$ $(1 \leq i \leq m)$ on $\underline{V}$ which is smooth over $\underline{S}$ satisfying $\underline{D}_i \times_{\underline{S}} S = D_i|_V$. Set $\underline{D} := \sum_{i=1}^{m} \underline{D}_i$.

(3) There exists a cartesian diagram

$$
\begin{array}{ccc}
\underline{D} & \xrightarrow{\subset} & \underline{V} \\
\downarrow & & \downarrow g \\
\underline{\operatorname{Spec}}_{\underline{S}}(\mathcal{O}_{\underline{S}}[y_1, \ldots, y_d]/(y_1 \cdots y_s)) & \longrightarrow & \underline{\operatorname{Spec}}_{\underline{S}}(\mathcal{O}_{\underline{S}}[y_1, \ldots, y_d]),
\end{array}
$$

over $\underline{S}$ which is compatible with the diagram (2.1.7.2), where g is an etale morphism. Let $\underline{z}$ be the image of z in $\underline{V}$. Then, by the result in the Case III to $(\underline{V}, \underline{D})$, $s = m$ and $\operatorname{div}(\underline{d}_i) = \operatorname{div}(y_i)$ $(1 \leq i \leq s)$ in $\operatorname{Div}(\operatorname{Spec}(\mathcal{O}_{\underline{V},\underline{z}})/\underline{S})$ up to some renumbering of the indexes. Then $\operatorname{div}(d_i) = \operatorname{div}(y_i)$ $(1 \leq i \leq s)$ in $\operatorname{Div}(X_z/S)$. This equality completes the proof of the proposition. $\square$

To study the global behavior of decompositions of a relative SNCD by smooth components, we introduce the notion of the *refinement* of a decomposition by smooth components.

Definition A.0.2. (1) Let $f\colon (X, D) \longrightarrow S$ be as in the beginning of this section and let $\Delta := \{D_\lambda\}_{\lambda \in \Lambda}$ and $\Delta' := \{D'_\sigma\}_{\sigma \in \Sigma}$ be decompositions of D by smooth components. Then we say that Δ' is a *refinement* of Δ if, for any $\lambda \in \Lambda$, there exists a subset Σ_λ of Σ with $\Sigma = \coprod_{\lambda \in \Lambda} \Sigma_\lambda$ such that $D_\lambda = \sum_{\sigma \in \Sigma_\lambda} D'_\sigma$.

(2) We say that a closed and open set of a topological space is *clopen*.

If X is locally noetherian as a topological space, we can prove the existence of the finest (hence canonical) decomposition of a relative SNCD by smooth components:

Proposition A.0.3. *Let $f\colon (X, D) \longrightarrow S$ be as in the beginning of this section. If X is locally noetherian as a topological space, then there exists a unique decomposition of D by smooth components which is a refinement of any decomposition of D by smooth components.*

Proof. Let T be a locally noetherian topological space and let $T = \bigcup_{i \in I} T_i$ $(T_i \cap T_j = \emptyset$ for $i \neq j)$ be the decomposition into the connected components of T. Then we claim that T_i is open. Let O be any noetherian open subset of T. Set $I' := \{i \in I \mid O \cap T_i \neq \emptyset\}$. Then $O = \bigcup_{i \in I'} (O \cap T_i)$. We claim that I' is finite. Indeed, if not, we have a union $O = O_1 \cup \cdots \cup O_r$ $(r \in \mathbb{Z}_{>0})$ of the

irreducible components of O since O is noetherian. Then there exist a positive integer s ($s \le r$) and $i \ne i'$ ($i, i' \in I'$) such that $O_s \cap T_i \ne \emptyset$ and $O_s \cap T_{i'} \ne \emptyset$. Since O_s is irreducible, O_s is connected. Hence $O_s \cup T_i \cup T_{i'}$ is also connected. However, this union contains both T_i and $T_{i'}$; this contradicts the definition of T_i. Thus we see that I' is finite and that $T_i \cap O = O \setminus (\bigcup_{i' \in I' \setminus \{i\}} T_{i'})$ is open in T. Now we see that T_i is open since T is locally noetherian. In conclusion, the disjoint sum $T = \bigcup_{i \in I} T_i$ is the disjoint sum of clopen subspaces of T.

Take a decomposition $\Delta = \{D_\lambda\}_{\lambda \in \Lambda}$ of D by smooth components and let $D_\lambda = \bigcup_{\sigma \in \Sigma_\lambda} D_{\lambda,\sigma}$ be the decomposition of D_λ into the connected components. Since X is locally noetherian, so is D_λ. Hence, by the previous paragraph, we have the decomposition $D_\lambda = \coprod_{\sigma \in \Sigma_\lambda} D_{\lambda,\sigma}$ into clopen subschemes. In particular, each $D_{\lambda,\sigma}$ is smooth over S and we have an equality $D_\lambda = \sum_{\sigma \in \Sigma_\lambda} D_{\lambda,\sigma}$ in $\mathrm{Div}(X/S)_{\ge 0}$. Therefore $\Delta_0 := \{D_{\lambda,\sigma}\}_{\lambda \in \Lambda, \sigma \in \Sigma_\lambda}$ is a refinement of Δ.

We say that a decomposition of D by smooth components is *very fine* if each member of the decomposition is connected as a topological space.

It is clear that Δ_0 is very fine. Hence we have shown that any decomposition of D by smooth components admits a refinement by a very fine one. To prove (A.0.3), it suffices to prove that there exists only one very fine decomposition of D by smooth components.

Let $\Delta = \{D_\lambda\}_{\lambda \in \Lambda}$ and $\Delta' = \{D'_{\lambda'}\}_{\lambda' \in \Lambda'}$ be very fine decompositions of D by smooth components. Fix an index $\lambda \in \Lambda$. For each point z in D_λ, take an open neighborhood U_z of z in X such that $\Delta_{U_z} = \Delta'_{U_z}$ ((A.0.1)). Then there exists a unique $\lambda'(\lambda, z) \in \Lambda'$ such that $D_\lambda \cap U_z = D'_{\lambda'(\lambda,z)} \cap U_z$.

We claim that $\lambda'(\lambda, z)$ does not depend on z. Let V be a subset of D_λ defined as follows:

$$V := \left\{ w \in D_\lambda \,\middle|\, \begin{array}{l} \exists \text{ sequence of points } z = z_0, z_1, \ldots, z_n = w \text{ in } D_\lambda \\ \text{satisfying } D_\lambda \cap U_{z_j} \cap U_{z_{j+1}} \ne \emptyset \ (0 \le j \le n-1) \end{array} \right\}.$$

Then, if w is a point of V, $D_\lambda \cap U_w$ is contained in V; V is an open set of D_λ. On the other hand, if $w \in D_\lambda$ is not contained in V, the points in U_w are not contained in V; V is a closed set of D_λ. Since D_λ is connected and V is non-empty (since $z \in V$), $D_\lambda = V$. Let w be a point of D_λ. Then there exists a sequence of points $z = z_0, z_1, \ldots, z_n = w$ in D_λ satisfying $D_\lambda \cap U_{z_j} \cap U_{z_{j+1}} \ne \emptyset$ for all $0 \le j \le n-1$. Since $D_\lambda \cap U_{z_j} = D'_{\lambda'(\lambda,z_j)} \cap U_{z_j}$ and $D_\lambda \cap U_{z_{j+1}} = D'_{\lambda'(\lambda,z_{j+1})} \cap U_{z_{j+1}}$,

$$D_\lambda \cap U_{z_j} \cap U_{z_{j+1}} = D'_{\lambda'(\lambda,z_j)} \cap U_{z_j} \cap U_{z_{j+1}} = D'_{\lambda'(\lambda,z_{j+1})} \cap U_{z_j} \cap U_{z_{j+1}}.$$

This implies the equality $\lambda'(\lambda, z_j) = \lambda'(\lambda, z_{j+1})$ since $U_{z_j} \cap U_{z_{j+1}}$ is non-empty and that $\Delta_{U_{z_j} \cap U_{z_{j+1}}} = \Delta'_{U_{z_j} \cap U_{z_{j+1}}}$. Hence we have

$$\lambda'(\lambda, z) = \lambda'(\lambda, z_0) = \cdots = \lambda'(\lambda, z_n) = \lambda'(\lambda, w).$$

Thus we have proved the claim. We denote this index by $\lambda'(\lambda)$. Then we have

$$D_\lambda = \bigcup_{z \in D_\lambda} (D_\lambda \cap U_z) = \bigcup_{z \in D_\lambda} (D'_{\lambda'(\lambda)} \cap U_z) \subset D'_{\lambda'(\lambda)},$$

and we see that D_λ is an open subscheme of $D'_{\lambda'(\lambda)}$. On the other hand, D_λ is a closed subscheme of $D'_{\lambda'(\lambda)}$ since it is closed in X. Since $D'_{\lambda'(\lambda)}$ is connected, $D_\lambda = D'_{\lambda'(\lambda)}$.

It is clear that the map $\lambda \mapsto \lambda'(\lambda)$ is injective. This fact and the equality $\sum_{\lambda \in \Lambda} D_\lambda = D = \sum_{\lambda' \in \Lambda'} D'_{\lambda'}$ imply the bijectivity of the correspondence $\lambda \mapsto \lambda'(\lambda)$. □

In the case where X is not locally noetherian as a topological space, there exists an example of a relative SNCD which does not have the finest decomposition by smooth components:

Example A.0.4. Let k be a field and let $A := \prod_{n \in \mathbb{N}} k$ be the countable product of k. Set $S := \mathrm{Spec}(A), X := \mathrm{Spec}(A[x])$ and let D be a relative SNCD on X over S defined by the ideal $(x) \subset A[x]$. Then $D \xrightarrow{\sim} S$. Let $\Delta := \{D_\lambda\}_{\lambda \in \Lambda}$ be any decomposition of D by smooth components. Then each D_λ is closed in D and it is open in D because the composite morphism $D_\lambda \xrightarrow{\subset} D \xrightarrow{\sim} S$ is smooth. Since D_λ is closed in $D \simeq \mathrm{Spec}(A)$, D_λ is quasi-compact.

For a subset T of $\mathbb{N}$, define $e_T := (e_{T,n})_{n \in \mathbb{N}} \in A$ by $e_{T,n} = 1$ (resp. 0) if $n \in T$ (resp. $n \notin T$). Then the sets $U_T := \{\mathfrak{p} \in \mathrm{Spec}(A) \mid e_T \notin \mathfrak{p}\}$ $(T \subset \mathbb{N})$ forms an open basis of $\mathrm{Spec}(A)(= S \simeq D)$. It is easy to see that, for a finite number of subsets $T_1, T_2, \ldots, T_l$ of $\mathbb{N}$, we have $\bigcup_{i=1}^{l} U_{T_i} = U_{\bigcup_{i=1}^{l} T_i}$; if $T_1 \cap T_2 = \emptyset$, then $U_{T_1} \cap U_{T_2} = \emptyset$.

Because D_λ is open in D and quasi-compact, D_λ is a union of a finite numbers of open sets of the form U_T. By the fact in the previous paragraph, $D_\lambda \xrightarrow{\sim} U_{T_\lambda}$ for a non-empty subset T_λ of $\mathbb{N}$.

If $\sharp\ T_\lambda = 1$ $(\forall \lambda \in \Lambda)$, then we can deduce an equality $\mathrm{Spec}(A) = \bigcup_{n \in \mathbb{N}} U_{\{n\}}$. However, a maximal ideal of A containing the ideal

$$\{(x_n)_{n \in \mathbb{N}} \in A \mid x_n = 0 \text{ for except finite numbers of } n\text{'s}\}$$

does not belong to $\bigcup_{n \in \mathbb{N}} U_{\{n\}}$ and it is a contradiction. Thus, for some $\lambda \in \Lambda$, the cardinality of T_λ is greater than 1. Then we have a disjoint sum $T_\lambda = T' \coprod T''$ of T_λ with $T' \neq \emptyset$ and $T'' \neq \emptyset$. Hence we have a refinement of Δ by factorizing D_λ as $D_\lambda = U_{T'} + U_{T''}$. Thus Δ is not the finest decomposition. Hence, in this case, there does not exist the finest decomposition of D by smooth components.

As we have seen in the above, we cannot have the finest decomposition of D in general. However, we can prove that any two decompositions admit a common refinement in the following way.

Let $\leq$ be a partial order in $\mathrm{Div}(X/S)_{\geq 0}$ such that $E_1 \leq E_2$ if and only if there exists an effective Cartier divisor F on X over S such that $E_2 = E_1 + F$. Then we have the following lemma:

Lemma A.0.5. *Let $f\colon X \longrightarrow S$ be as in the beginning of this section and let $\Delta = \{D_\lambda\}_{\lambda \in \Lambda}$ be a family of smooth effective Cartier divisors on X/S. For nonnegative integers $n_{1,\lambda}$ and $n_{2,\lambda}$ ($\lambda \in \Lambda$), $\sum_{\lambda \in \Lambda} n_{1,\lambda} D_\lambda \leq \sum_{\lambda \in \Lambda} n_{2,\lambda} D_\lambda$ if and only if $n_{1,\lambda} \leq n_{2,\lambda}$ for all $\lambda \in \Lambda$.*

Proof. It suffices to prove the 'only if' part. For a point $t \in S$ and for a scheme T over S, set $T_t := T \times_S t$. Then $\sum_{\lambda \in \Lambda_t} n_{1,\lambda} D_{\lambda,t} \leq \sum_{\lambda \in \Lambda_t} n_{2,\lambda} D_{\lambda,t}$, where $\Lambda_t := \{\lambda \in \Lambda \mid D_{\lambda,t} \neq \emptyset\}$. Since the assertion is well-known in the case where S is the spectrum of a field, $n_{1,\lambda} \leq n_{2,\lambda}$ for $\lambda \in \Lambda_t$. Since any $\lambda \in \Lambda$ belongs to Λ_t for some point $t \in S$, $n_{1,\lambda} \leq n_{2,\lambda}$ for all $\lambda \in \Lambda$. □

Using (A.0.5), we define the operation $\wedge$ for the elements in $\mathrm{Div}_D(X/S)_{\geq 0}$ as follows:

Proposition A.0.6. *Let the notations be as above. Then the following hold:*

(1) *For two elements E_1 and E_2 in $\mathrm{Div}_D(X/S)_{\geq 0}$, there exists a unique element $E_1 \wedge E_2 \in \mathrm{Div}_D(X/S)_{\geq 0}$ satisfying the following equality:*

$$E_1 \wedge E_2 = \max\{F \in \mathrm{Div}_D(X/S)_{\geq 0} \mid F \leq E_1 \text{ and } F \leq E_2\}.$$

(2) *Let E and F be elements in $\mathrm{Div}_D(X/S)_{\geq 0}$. If E is smooth over S, then $E \wedge F$ is clopen in E. In particular, $E \wedge F$ is smooth over S.*

(3) *Let E be an element in $\mathrm{Div}_D(X/S)_{\geq 0}$. Let $\{F_\sigma\}_{\sigma \in \Sigma}$ be a family of elements in $\mathrm{Div}_D(X/S)_{\geq 0}$ of locally finite intersection such that $F_\sigma \wedge F_{\sigma'} = 0$ for any $\sigma, \sigma' \in \Sigma$ with $\sigma \neq \sigma'$. Then*

$$E \wedge (\sum_{\sigma \in \Sigma} F_\sigma) = \sum_{\sigma \in \Sigma} (E \wedge F_\sigma).$$

Proof. (1): Take a decomposition $\{D_\lambda\}_{\lambda \in \Lambda}$ of D by smooth components. Then, by the definition of $\mathrm{Div}_D(X/S)_{\geq 0}$, there exists an open covering $X = \bigcup_{j \in J} X_j$ of X such that $E_i|_{X_j} = \sum_{\lambda \in \Lambda_{X_j}} n_{i,j,\lambda}(D_\lambda|_{X_j})$ ($i = 1, 2$) for some nonnegative integers $n_{i,j,\lambda}$, where Λ_{X_j} is the set $\{\lambda \in \Lambda \mid D_\lambda|_{X_j} \neq \emptyset\}$. Define $F_j \in \mathrm{Div}_{D|_{X_j}}(X_j/S)_{\geq 0}$ by $F_j := \sum_{\lambda \in \Lambda_{X_j}} \min\{n_{1,j,\lambda}, n_{2,j,\lambda}\}(D_\lambda|_{X_j})$. Then it is easy to see (by using (A.0.5)) that $F_j \leq E_i|_{X_j}$ ($i = 1, 2$) and that F_j is the maximum element among the elements F's in $\mathrm{Div}_{D|_{X_j}}(X_j/S)_{\geq 0}$ satisfying $F \leq E_i|_{X_j}$ ($i = 1, 2$). By using this characterization, one can see that there exists a unique element $F \in \mathrm{Div}_D(X/S)_{\geq 0}$ such that $F|_{X_j} = F_j$ for any $j \in J$. Set $E_1 \wedge E_2 := F$. Then $E_1 \wedge E_2$ satisfies the equality in (1).

(2): Since $E \wedge F$ is closed in E, it suffices to prove that $E \wedge F$ is open in E. Since the problem is local, we may assume that $F = \sum_{\lambda \in \Lambda} n_\lambda D_\lambda$ ($n_\lambda \in \mathbb{N}$). Moreover, we may assume, by the smoothness of E, that there exists an

element $\lambda_0 \in \Lambda$ satisfying $E = D_{\lambda_0}$. Then, $E \wedge F = E$ (resp. 0) if $n_{\lambda_0} \geq 1$ (resp. $n_{\lambda_0} = 0$). In both cases, $E \wedge F$ is open in E.

(3): Since the problem is local, we may assume that Σ is a finite set and that

$$E = \sum_{\lambda \in \Lambda} n_\lambda D_\lambda, \quad F_\sigma = \sum_{\lambda \in \Lambda_\sigma} m_{\sigma,\lambda} D_\lambda,$$

where $n_\lambda, m_{\sigma,\lambda}$ are nonnegative integers and Λ_σ is a subset of Λ satisfying $\Lambda_\sigma \cap \Lambda_{\sigma'} = \emptyset$ for $\sigma \neq \sigma'$. Then

$$E \wedge (\sum_{\sigma \in \Sigma} F_\sigma) = \sum_{\sigma \in \Sigma} \sum_{\lambda \in \Lambda_\sigma} \min\{n_\lambda, m_{\sigma,\lambda}\} D_\lambda = \sum_{\sigma \in \Sigma} (E \wedge F_\sigma).$$

□

Using (A.0.6), we can prove that any two decompositions of a relative SNCD by smooth components have a common refinement:

Proposition A.0.7. *Let $f\colon (X,D) \longrightarrow S$ be as in the beginning of this section. Let $\Delta = \{D_\lambda\}_{\lambda \in \Lambda}$ and $\Delta' = \{D'_{\lambda'}\}_{\lambda' \in \Lambda'}$ be two decompositions of D by smooth components. Then there exists a decomposition Δ'' of D by smooth components which is a refinement of Δ and Δ'.*

Proof. By (A.0.6), $D_\lambda \wedge D'_{\lambda'}$ is smooth over S (possibly empty) and we have the following equalities

$$D_\lambda = \sum_{\lambda' \in \Lambda', D_\lambda \wedge D'_{\lambda'} \neq 0} D_\lambda \wedge D'_{\lambda'}, \qquad D'_{\lambda'} = \sum_{\lambda \in \Lambda, D_\lambda \wedge D'_{\lambda'} \neq 0} D_\lambda \wedge D'_{\lambda'}.$$

Set $\Lambda'' := \{(\lambda, \lambda') \in \Lambda \times \Lambda' \mid D_\lambda \wedge D'_{\lambda'} \neq 0\}$. Then it is easy to see from the above equalities that $\Delta'' := \{D_\lambda \wedge D'_{\lambda'}\}_{(\lambda,\lambda') \in \Lambda''}$ has a desired property. □

Lastly we discuss log structures on X associated to D. Though we always consider log structures in the Zariski topos of X, (A.0.8) and (A.0.9) below remain valid if we consider log structures in the etale topos of X.

By (2.1.9), $(X, M(D))$ is a fine log scheme; in fact, it is log smooth over S.

Let us recall $N(D) := \mathcal{O}_X \cap j_*(\mathcal{O}_U^*)$ with structural morphism $N(D) \xrightarrow{\subset} \mathcal{O}_X$. From the local expression of $M(D)$ given in (2.1.9), there exists a natural inclusion $M(D) \xrightarrow{\subset} N(D)$. Both $M(D)$ and $N(D)$ have U as the maximal open subscheme where they are trivial. By [55, (8.2), (11.6)], they coincide in the case where S is regular. However we prove $M(D) \subsetneq N(D)$ in general, in fact, we give an equivalent condition for the equality $M(D) = N(D)$ ((A.0.8) below) and show that $N(D)$ is not a fine log structure in general ((A.0.9) below):

Proposition A.0.8. *Let $f\colon (X,D) \longrightarrow S$ be in the beginning of this section. Then the following hold*:

(1) *If S is reduced at the points of $f(D)$, then $M(D) = N(D)$.*

(2) *If S is not reduced at some point of $f(D)$, then $M(D) \neq N(D)$.*

Consequently, the log structures $M(D)$ and $N(D)$ coincide if and only if S is reduced at the points of $f(D)$.

Proof. Let z be a point of X. Set $B := \mathcal{O}_{X,z}$ and $X_z := \mathrm{Spec}(B)$. Take an open neighborhood V of z which admits the diagram (2.1.7.2).

(1): It suffices to prove the surjectivity of the homomorphism $M(D) \xrightarrow{\subset} N(D)$. To prove this, we may work locally. By (2.1.9), it suffices to prove $\Gamma(X_z, N(D)) = B^* y_1^{\mathbb{N}} \cdots y_s^{\mathbb{N}}$. Here we denote the pull-back of $N(D)$ to X_z by the same symbol. We may assume that S is noetherian because we can reduce the general case to the noetherian case by the well-known technique similar to that of Case IV in the proof of (A.0.1). Furthermore, we may assume that S is the spectrum of a noetherian local ring A and that $t = f(z)$ is the closed point of S. By the assumption, A is reduced.

Let g be an element of $\Gamma(X_z, N(D)) = B \cap B[(y_1 \cdots y_s)^{-1}]^*$. Because y_i is a non-zero divisor of B $(1 \leq i \leq s)$, there exist an element $h \in B$ and a nonnegative integer a satisfying $gh = (y_1 \cdots y_s)^a$ in B.

For a prime ideal $\mathfrak{p}$ of A, consider the equation $gh = (y_1 \cdots y_s)^a$ in the ring $B/\mathfrak{p}B$. Then, since $y_i(B/\mathfrak{p}B)$'s $(1 \leq i \leq s)$ are different prime ideals of an integral domain $B/\mathfrak{p}B$, there exist unique integers $b_{\mathfrak{p},i}$ $(1 \leq i \leq s)$ with $0 \leq b_{\mathfrak{p},i} \leq a$ satisfying $g \in y_i^{b_{\mathfrak{p},i}} B + \mathfrak{p}B$ and $h \in y_i^{a-b_{\mathfrak{p},i}} B + \mathfrak{p}B$ for any prime ideal $\mathfrak{p}$ of A. Let $\mathfrak{m}$ be the maximal ideal of A. Then $g \in y_i^{b_{\mathfrak{p},i}} B + \mathfrak{m}B$ and $h \in y_i^{a-b_{\mathfrak{p},i}} B + \mathfrak{m}B$. By the uniqueness of $b_{\mathfrak{m},i}$, we can conclude that $b_{\mathfrak{p},i}$ is independent of $\mathfrak{p}$. Set $b_i := b_{\mathfrak{p},i}$. Then $g \in \bigcap_{\mathfrak{p} \in \mathrm{Spec}(A)} (y_i^{b_i} B + \mathfrak{p}B)$. Because $B/y_i^{b_i} B$ is flat over A and because A is a reduced noetherian ring, $\bigcap_{\mathfrak{p} \in \mathrm{Spec}(A)} (y_i^{b_i} B + \mathfrak{p}B)$ is equal to $y_i^{b_i} B$. Because B is flat over $A[y_1, \ldots, y_s]$, $\bigcap_{i=1}^s (y_i^{b_i} B) = (\prod_{i=1}^s y_i^{b_i}) B$. Hence $g \in (\prod_{i=1}^s y_i^{b_i}) B$. Similarly, $h \in (\prod_{i=1}^s y_i^{a-b_i}) B$. Define $g' := g/(\prod_{i=1}^s y_i^{b_i})$ and $h' := h/(\prod_{i=1}^s y_i^{a-b_i})$. Then $g'h' = 1$. Consequently $\Gamma(X_z, N(D)) = B^* y_1^{\mathbb{N}} \cdots y_s^{\mathbb{N}}$.

(2): Let t be a point of $f(D)$ where S is not reduced at t. To prove (2), we may assume that S is the spectrum of a nonreduced local ring A with the closed point t. Denote by $\mathfrak{m}$ the maximal ideal of A and by κ the residue field $A/\mathfrak{m}$.

Consider a point z in D_t such that D is smooth at z. Then there exists an open neighborhood V of z which admits the cartesian diagram (2.1.7.2) for the case where $s = 1$, $S_0 = S = \mathrm{Spec}(A)$ and $g(z)$ is equal to the maximal ideal $\mathfrak{m}A[y_1, \ldots, y_d] + (y_1, \ldots, y_d)$. The induced morphism $g^* \colon A[y_1, \ldots, y_d] \longrightarrow B$ by g is ind-etale. In the following, we denote the image $g^*(a)$ of an element $a \in A[y_1, \ldots, y_d]$ by a by abuse of notation.

To prove (2), it suffices to prove that $\Gamma(X_z, N(D)) \neq B^* y_1^{\mathbb{N}}$. Set $y := y_1$ for simplicity of notation. Take an element ϵ of A satisfying $\epsilon \neq 0$ but $\epsilon^2 = 0$. Then we have $(y + \epsilon)(y - \epsilon) = y^2$ in B. Hence $y + \epsilon \in \Gamma(X_z, N(D))$. Assume that $y + \epsilon \in B^* y^{\mathbb{N}}$. Then there exists a nonnegative integer n and an element $u \in B^*$ such that

$$y + \epsilon = uy^n. \tag{A.0.8.1}$$

Apply the projection $B \longrightarrow B/(y) = \mathcal{O}_{D,z}$ to the equality (A.0.8.1). If $n = 0$, $\epsilon = u$ in $B/(y)$; this is a contradiction. If $n \geq 1$, $\epsilon = 0$ in $B/(y)$; this contradicts the choice of ϵ since the natural ring homomorphism $A \longrightarrow B/(y)$ is faithfully flat. In conclusion, $y + \epsilon \in \Gamma(X_z, N(D))$ does not belong to $\Gamma(X_z, M(D))$. □

Proposition A.0.9. *Let the notations be as in the proof of* (A.0.8) *above. Then* $\Gamma(X_z, N(D))/\mathcal{O}^*_{X,z}$ *is not a finitely generated monoid.*

Proof. Keep the notation in the proof of (A.0.8) (2). Assume that $\Gamma(X_z, N(D))/B^*$ were finitely generated. Since $(y^n + \epsilon)(y^n - \epsilon) = y^{2n}$ ($n \in \mathbb{Z}_{>0}$), $y^n + \epsilon$ is an element in $\Gamma(X_z, N(D))$. Fix a surjective homomorphism $\varphi : \mathbb{N}^r \longrightarrow \Gamma(X_z, N(D))/B^*$ of monoids for some r and for each positive integer n, take an element $h_n \in \mathbb{N}^r$ satisfying $\varphi(h_n) = y^n + \epsilon$. Then there exist elements $a_n \in \mathbb{Q}$ $(1 \leq n \leq r+1)$ with $(a_1, a_2, \ldots, a_{r+1}) \neq (0, 0, \ldots, 0)$ such that $\sum_{n=1}^{r+1} a_n h_n = 0$ in $\mathbb{Q}^r$. By multiplying a_n's by some integers, we may assume that a_n's are integers with $\gcd\{a_i \mid a_i \neq 0, (1 \leq i \leq r+1)\} = 1$. Set

$$\{n \mid 1 \leq n \leq r+1, a_n > 0\} =: \{n_1, n_2, \ldots, n_q\},$$
$$\{n \mid 1 \leq n \leq r+1, a_n < 0\} =: \{n_{q+1}, n_{q+2}, \ldots, n_{q+s}\},$$
$$m_i := \begin{cases} a_{n_i}, & (1 \leq i \leq q), \\ -a_{n_i}, & (q+1 \leq i \leq q+s). \end{cases}$$

Then we obtain the equality $\sum_{i=1}^{q} m_i h_{n_i} = \sum_{i=q+1}^{q+s} m_i h_{n_i}$ in $\mathbb{N}^r$. By applying φ to this equality, we see that there exists an element $v \in B^*$ such that

$$v \prod_{i=1}^{q} (y^{n_i} + \epsilon)^{m_i} = \prod_{i=q+1}^{q+s} (y^{n_i} + \epsilon)^{m_i} \tag{A.0.9.1}$$

in B.

Define integers P and Q by $P := \sum_{i=1}^{q} m_i n_i$ and $Q := \sum_{i=q+1}^{q+s} m_i n_i$, respectively. Since $\epsilon^2 = 0$, we obtain the following equality

$$v\{y^P + (\sum_{i=1}^{q} m_i y^{P-n_i})\epsilon\} = y^Q + (\sum_{i=q+1}^{q+s} m_i y^{Q-n_i})\epsilon. \tag{A.0.9.2}$$

Let $\kappa[y]^{\mathrm{loc}}$ be the localization of the polynomial ring $\kappa[y]$ at the prime ideal (y). Let β' be the composite morphism $A[y, y_2, \ldots, y_d] \longrightarrow \kappa[y] \longrightarrow \kappa[y]^{\mathrm{loc}}$, where the first homomorphism sends $a \in A$ (resp. y, y_i $(2 \leq i \leq d)$) to a mod $\mathfrak{m}$ (resp. y,0) and the second homomorphism is the localization. Then the ring $C := B \otimes_{A[y,y_2,\ldots,y_d],\beta'} \kappa[y]^{\mathrm{loc}}$ is a local ring which is ind-etale over $\kappa[y]^{\mathrm{loc}}$. Hence C is a discrete valuation ring with uniformizer y. Denote the natural homomorphism $B \longrightarrow C$ by β and apply β to the equality (A.0.9.2). Then we obtain $P = Q$ since $\beta(v) \in C^*$. Hence we obtain the equality

$$v\{y^P + (\sum_{i=1}^{q} m_i y^{P-n_i})\epsilon\} = y^P + (\sum_{i=q+1}^{q+s} m_i y^{P-n_i})\epsilon. \tag{A.0.9.3}$$

Let $p \geq 0$ be the characteristic of κ. For a positive integer L, let $(\mathrm{C})_L$ be the claim that m_i is a multiple of p for i's with $n_i \geq L$. We prove that $(\mathrm{C})_L$ holds for any L by descending induction. If $L > n_i$ for any $1 \leq i \leq q+s$, $(\mathrm{C})_L$ obviously holds. Assume that $(\mathrm{C})_{L+1}$ is true. Then, if there is no integer i with $n_i = L$, $(\mathrm{C})_L$ obviously holds since $(\mathrm{C})_L$ is equivalent to $(\mathrm{C})_{L+1}$. Assume that there exists an integer i_0 with $n_{i_0} = L$. We may assume that $1 \leq i_0 \leq q$. Let γ' be the homomorphism $A[y, y_2, \ldots, y_d] \longrightarrow (A/p\epsilon A)[y]/(y^{P-L+1})$ which sends $a \in A$ (resp. y, y_i $(2 \leq i \leq d)$) to the class of a (resp. y, 0). Then the ring $R := B \otimes_{A[y,y_2,\ldots,y_d],\gamma'} (A/p\epsilon A)[y]/(y^{P-L+1})$ is a local ring which is ind-etale over the local ring $(A/p\epsilon A)[y]/(y^{P-L+1})$. Denote the homomorphism $B \longrightarrow R$ by γ and apply γ to the equality (A.0.9.3). Then we obtain an equality $m_{i_0} y^{P-L}\epsilon = 0$ in R by the induction hypothesis. Since R is faithfully flat over $(A/p\epsilon A)[y]/(y^{P-L+1})$, the equality $m_{i_0} y^{P-L}\epsilon = 0$ in R implies that in $(A/p\epsilon A)[y]/(y^{P-L+1})$. Hence we have $m_{i_0}\epsilon = 0$ in $A/p\epsilon A$. This implies that there exists an element $a \in A$ such that $(m_{i_0} - pa)\epsilon = 0$ in A. Hence m_{i_0} is equal to the zero in κ, and consequently m_{i_0} is a multiple of p. Therefore the claim $(\mathrm{C})_L$ holds. Now, by the descending induction, $(\mathrm{C})_1$ holds and all m_i's $(1 \leq i \leq r)$ are multiples of p. This contradicts $\gcd\{m_i \mid m_i \neq 0, (1 \leq i \leq r)\} = 1$. In conclusion, $\Gamma(X_z, N(D))/\mathcal{O}^*_{X,z}$ is not a finitely generated monoid. □

The proposition (A.0.9) tells us that $N(D)$ is far from nice in general. This is the reason why we have considered only $M(D)$ in the text.

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Index

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Addresses:

Professor J.-M. Morel, CMLA,
École Normale Supérieure de Cachan,
61 Avenue du Président Wilson, 94235 Cachan Cedex, France
E-mail: Jean-Michel.Morel@cmla.ens-cachan.fr

Professor F. Takens, Mathematisch Instituut,
Rijksuniversiteit Groningen, Postbus 800,
9700 AV Groningen, The Netherlands
E-mail: F.Takens@math.rug.nl

Professor B. Teissier, Institut Mathématique de Jussieu,
UMR 7586 du CNRS, Équipe "Géométrie et Dynamique",
175 rue du Chevaleret
75013 Paris, France
E-mail: teissier@math.jussieu.fr

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