A

Distributions and Fourier Transformation

In this appendix we give a summary of the facts of Distributions and Fourier transformation that we need in Chapter 3. We follow Hörmander [37], Chapter I., and Yosida [69], pp. 46–52. The proofs of the results of this section can be found in these books. In the appendix we consider complex-valued functions in general, i.e. $C^k(\mathbb{R}^n)$, $C^\infty(\mathbb{R}^n)$ denotes $C^k(\mathbb{R}^n, \mathbb{C})$, $C^\infty(\mathbb{R}^n, \mathbb{C})$.

A.1 Distributions

Definition A.1 (supp). *The support of $\varphi \in C^k(\mathbb{R}^n)$, $k \geq 0$, is defined as* $\mathrm{supp}(\varphi) = \overline{\{x \in \mathbb{R}^n \mid \varphi(x) \neq 0\}}$.

Let $\Omega \subset \mathbb{R}^n$ be a set. $C_0^\infty(\Omega)$ denotes the space of functions in $C^\infty(\mathbb{R}^n)$ with compact support which is contained in Ω.

We define the space $\mathcal{D}'(\mathbb{R}^n)$ of distributions. The condition (A.1) is equivalent to the continuity of T with respect to a certain norm of $C_0^\infty(\mathbb{R}^n)$.

Definition A.2 (Distribution). *A linear operator $T\colon C_0^\infty(\mathbb{R}^n) \to \mathbb{C}$ is called a distribution, if for each compact set $K \subset \mathbb{R}^n$ there are constants C and k, such that*

$$|T(\varphi)| \leq C \sum_{|\alpha| \leq k} \sup_{x \in K} |D^\alpha \varphi(x)| \tag{A.1}$$

holds for all $\varphi \in C_0^\infty(K)$.

Here we define for the multiindex $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in \mathbb{N}_0^n$ the following expressions: $|\alpha| := \sum_{j=1}^n \alpha_j$ and $D^\alpha := \partial_{x_1}^{\alpha_1} \ldots \partial_{x_n}^{\alpha_n}$. The space of all distributions is denoted by $\mathcal{D}'(\mathbb{R}^n)$ since it is the dual of the space $\mathcal{D}(\mathbb{R}^n) := C_0^\infty(\mathbb{R}^n)$ of test functions. We also write $T(\varphi) = \langle T, \varphi \rangle$.

The last notation is derived from the L^2 scalar product. The next example shows that all locally integrable functions define a distribution.

Example A.3 *Every function $f \in L^1_{loc}(\mathbb{R}^n)$, i.e. f is locally integrable, defines a distribution T through*

$$T(\varphi) := \int_{\mathbb{R}^n} f(x)\varphi(x)\,dx = \langle f(x), \varphi(x)\rangle.$$

Indeed, we have $\left|\int_{\mathbb{R}^n} f(x)\varphi(x)\,dx\right| \leq \sup_{x\in K}|\varphi(x)|\int_K |f(x)|\,dx$ for a test function $\varphi \in C_0^\infty(K)$.

Thus, all L^1_{loc}-functions belong to $\mathcal{D}'(\mathbb{R}^n)$. But the space of distributions is larger than L^1_{loc}; e.g. Dirac's delta-distribution is not an L^1_{loc}-function.

Example A.4 *Dirac's δ-distribution δ is defined by $\langle\delta,\varphi\rangle := \varphi(0)$ for all $\varphi \in C_0^\infty(\mathbb{R}^n)$.*

δ is a distribution, but no L^1_{loc} function.

We show that δ is a distribution: For $0 \notin K$ this is clear. Now let $0 \in K$. For $x \in K$ and $\varphi \in C_0^\infty(K)$ we have $|\langle\delta,\varphi\rangle| = |\varphi(0)| \leq \sup_{x\in K}|\varphi(x)|$.

We show that δ is no L^1_{loc}-function. Let χ be a function with $\chi \in C_0^\infty(\mathbb{R}^n;[0,1])$, $\operatorname{supp}(\chi) \subset \overline{B_1(0)}$ and $\chi(x) = 1$ for $\|x\| \leq \frac{1}{2}$. Set $\varphi_\epsilon(x) := \chi\left(\frac{x}{\epsilon}\right)$. Hence, $\operatorname{supp}\varphi_\epsilon \subset \overline{B_\epsilon(0)}$. We assume that there is a function $f \in L^1_{loc}(\mathbb{R}^n)$ such that

$$1 = |\varphi_\epsilon(0)| = |\langle\delta,\varphi_\epsilon\rangle| = \left|\int_{\mathbb{R}^n} f(x)\varphi_\epsilon(x)\,dx\right| \leq \int_{\mathbb{R}^n} 1_{\overline{B_\epsilon(0)}}(x)|f(x)|\,dx.$$

By Lebesgue's Theorem this term tends to zero for $\epsilon \to 0$, since $1_{\overline{B_\epsilon(0)}}$ converges to zero almost everywhere and $\int_{\|x\|\leq 1}|f(x)|\,dx < \infty$ if $f \in L^1_{loc}(\mathbb{R}^n)$, contradiction. Thus, δ is no L^1_{loc}-function.

Definition A.5 (Support of a distribution). *Let $T \in \mathcal{D}'(\mathbb{R}^n)$, and let $\Omega \subset \mathbb{R}^n$ be an open set.*

1. *We say that $T = 0$ on Ω, if*

$$\langle T, \varphi\rangle = 0$$

   holds for all $\varphi \in C_0^\infty(\Omega)$.
2. *The* support of a distribution *T is the complement of the largest open set, where $T = 0$ holds.*
3. *The space of all distributions with compact support is denoted by $\mathcal{E}'(\mathbb{R}^n)$. It is the dual of the space $\mathcal{E}(\mathbb{R}^n) = C^\infty(\mathbb{R}^n)$.*

Example A.6 *The δ-distribution has the support $\{0\}$. Thus, it belongs to $\mathcal{E}'(\mathbb{R}^n)$ and one can apply it to functions of $C^\infty(\mathbb{R}^n)$.*

We now define operations on distributions. The idea is to apply the respective operation to the smooth test functions. We define multiplication by a smooth function, differentiation etc. Note, that for a function φ we define $\check{\varphi}(x) = \varphi(-x)$.

Definition A.7. *Let $T \in \mathcal{D}'(\mathbb{R}^n)$, $\varphi \in C_0^\infty(\mathbb{R}^n)$.*

1. Multiplication by a function $a \in C^\infty(\mathbb{R}^n)$*; we have* $aT \in \mathcal{D}'(\mathbb{R}^n)$*.*

$$\langle aT, \varphi\rangle := \langle T, a\varphi\rangle.$$

2. Conjugation

$$\langle \overline{T}, \varphi\rangle := \overline{\langle T, \overline{\varphi}\rangle}.$$

3. Check

$$\langle \check{T}, \varphi\rangle := \langle T, \check{\varphi}\rangle.$$

4. Differentiation*; we have* $\frac{\partial}{\partial x_j} T \in \mathcal{D}'(\mathbb{R}^n)$*.*

$$\left\langle \frac{\partial}{\partial x_j} T, \varphi \right\rangle := -\left\langle T, \frac{\partial}{\partial x_j}\varphi \right\rangle.$$

These formulas also hold if T is given by a smooth function f through $T(\varphi) = \int_{\mathbb{R}^n} f(x)\varphi(x)\, dx$. For 3. this is shown by partial integration; note, that the test functions have compact support.

Definition A.8 (Convolution). *For two continuous functions f, g, one of which has compact support, we define $(f * g)(x) := \int_{\mathbb{R}^n} f(y)g(x-y)\, dy$.*

Definition A.9 (Convolution for distributions). *We define the convolution of a distribution $T \in \mathcal{D}'(\mathbb{R}^n)$ with a function $\varphi \in C_0^\infty(\mathbb{R}^n)$ as follows:*

$$(T * \varphi)(x) := T^y(\varphi(x-y)).$$

The superscript y denotes the application of T to φ with respect to y. We have $T * \varphi \in C^\infty(\mathbb{R}^n)$ and $\partial^\alpha(T * \varphi) = (\partial^\alpha T) * \varphi = T * (\partial^\alpha \varphi)$.
Convolution of two distributions: For $T, S \in \mathcal{D}'(\mathbb{R}^n)$, one of which has compact support, we define $T * S \in \mathcal{D}'(\mathbb{R}^n)$ by

$$(T * S) * \varphi = T * (S * \varphi)$$

for all $\varphi \in C_0^\infty(\mathbb{R}^n)$.

Proposition A.10

1. *Let $T \in \mathcal{D}'(\mathbb{R}^n)$, $\varphi \in C_0^\infty(\mathbb{R}^n)$. Then we have $(T * \varphi)\check{} = \check{T} * \check{\varphi}$.*
2. *Let $T, S \in \mathcal{D}'(\mathbb{R}^n)$, one of which has compact support.*
   Then we have $(T * S)\check{} = \check{T} * \check{S}$.

Proof: For 1. we have

$$(T * \varphi)\check{}(x) = (T * \varphi)(-x) = T^y(\varphi(-x-y))$$

$$(\check{T} * \check{\varphi})(x) = \check{T}^y(\check{\varphi}(x-y)) = \check{T}^y(\varphi(-x+y)) = T^y(\varphi(-x-y)).$$

To prove 2. we have for all $\varphi \in C_0^\infty(\mathbb{R}^n)$ by 1.

$$\begin{aligned}(T * S)\check{} * \check{\varphi} &= [(T * S) * \varphi]\check{} \\ &= [T * (S * \varphi)]\check{} \\ &= \check{T} * (S * \varphi)\check{} \\ &= \check{T} * (\check{S} * \check{\varphi}) \\ &= (\check{T} * \check{S}) * \check{\varphi},\end{aligned}$$

which proves 2. □

A.2 Fourier Transformation

We follow [69], Chapter VI., and [37], Chapters I. and II. There, one can also find the proofs of the results stated here. There are different conventions for the definition of the Fourier transform concerning the constant 2π. We use the definition of the Fourier transform given in [37], cf. the following definition.

Definition A.11 (Fourier transformation in L^1). *Let $f \in L^1(\mathbb{R}^n)$. We define the Fourier transform $\hat{f}$ by*

$$\hat{f}(\omega) := \int_{\mathbb{R}^n} f(x) e^{-i\omega^T x}\, dx.$$

If $\hat{f} \in L^1(\mathbb{R}^n)$, then the inversion formula $f(x) = (2\pi)^{-n} \int_{\mathbb{R}^n} \hat{f}(\omega) e^{ix^T\omega}\, d\omega$ holds.

L^1 is not mapped into itself under Fourier transformation. The Schwartz space of rapidly decreasing functions, however, will be mapped into itself by the Fourier transformation, cf. Proposition A.15.

Definition A.12 (The Schwartz space). *We define the following function space $\mathcal{S}(\mathbb{R}^n)$ of rapidly decreasing functions: $\varphi \in \mathcal{S}(\mathbb{R}^n)$ if and only if*

1. *$\varphi \in C^\infty(\mathbb{R}^n)$ and*
2. *for all multiindices α, β there is a constant $C_{\alpha,\beta}$ such that*

$$\sup_{x \in \mathbb{R}^n} \left| x^\beta D^\alpha \varphi(x) \right| \le C_{\alpha,\beta} \qquad \textit{holds.}$$

Proposition A.13 (Properties of $\mathcal{S}(\mathbb{R}^n)$)

1. *$\mathcal{S}(\mathbb{R}^n) \subset L^p(\mathbb{R}^n)$ for all $p \ge 1$.*
2. *$C_0^\infty(\mathbb{R}^n) \subset \mathcal{S}(\mathbb{R}^n) \subset C^\infty(\mathbb{R}^n)$.*
3. *$C_0^\infty(\mathbb{R}^n)$ is dense in $\mathcal{S}(\mathbb{R}^n)$.*

Definition A.14 (Fourier transformation in $\mathcal{S}(\mathbb{R}^n)$).
Let $\varphi \in \mathcal{S}(\mathbb{R}^n)$. Then $\hat{\varphi}(\omega) = \int_{\mathbb{R}^n} \varphi(x) e^{-i\omega^T x}\, dx$.

Proposition A.15 *Let $\varphi \in \mathcal{S}(\mathbb{R}^n)$. Then*

$$\begin{aligned} \hat{\varphi} &\in \mathcal{S}(\mathbb{R}^n) \\ \varphi(x) &= (2\pi)^{-n} \int_{\mathbb{R}^n} \hat{\varphi}(\omega) e^{i\omega^T x}\, d\omega \text{ and} \\ \hat{\hat{\varphi}} &= (2\pi)^n \check{\varphi}. \end{aligned}$$

Moreover, $\check{\hat{\varphi}} = \hat{\check{\varphi}}$.

Proof: We show that last equation:

$$\check{\hat{\varphi}}(\omega) = \hat{\varphi}(-\omega) = \int_{\mathbb{R}^n} \varphi(x) e^{i\omega^T x}\, dx = \int_{\mathbb{R}^n} \varphi(-x) e^{-i\omega^T x}\, dx = \hat{\check{\varphi}}(\omega).$$

□

Proposition A.16 *Let $\varphi, \psi \in \mathcal{S}(\mathbb{R}^n)$. Then we have $\varphi * \psi \in \mathcal{S}(\mathbb{R}^n)$. Moreover, denoting $\langle \cdot, \cdot \rangle = \langle \cdot, \cdot \rangle_{L^2}$, we have*

$$\begin{aligned} \langle \hat{\varphi}, \psi \rangle &= \langle \varphi, \hat{\psi} \rangle \\ \langle \varphi, \overline{\psi} \rangle &= (2\pi)^{-n} \langle \hat{\varphi}, \overline{\hat{\psi}} \rangle \text{ (Parseval's formula)} \\ \widehat{\varphi * \psi} &= \hat{\varphi} \cdot \hat{\psi} \\ \widehat{\varphi \cdot \psi} &= (2\pi)^{-n} \hat{\varphi} * \hat{\psi}. \end{aligned}$$

Definition A.17 ($\mathcal{S}'(\mathbb{R}^n)$, Fourier transformation in $\mathcal{S}'(\mathbb{R}^n)$). *We define $\mathcal{S}'(\mathbb{R}^n)$ as the space of continuous linear operators on $\mathcal{S}(\mathbb{R}^n)$. Then*

$$\mathcal{E}'(\mathbb{R}^n) \subset \mathcal{S}'(\mathbb{R}^n) \subset \mathcal{D}'(\mathbb{R}^n).$$

Moreover, $C_0^\infty(\mathbb{R}^n)$ is dense in $\mathcal{S}'(\mathbb{R}^n)$.

We define the Fourier transformation for $T \in \mathcal{S}'(\mathbb{R}^n)$ by

$$\langle \hat{T}, \varphi \rangle := \langle T, \hat{\varphi} \rangle$$

for $\varphi \in \mathcal{S}(\mathbb{R}^n)$.

Proposition A.18 (Fourier transformation in $\mathcal{S}'(\mathbb{R}^n)$ – properties)

1. *For $T \in \mathcal{S}'(\mathbb{R}^n)$ we have $\hat{\hat{T}} = (2\pi)^n \check{T}$ and $\check{\hat{T}} = \hat{\check{T}}$.*
2. *For $T \in \mathcal{E}'(\mathbb{R}^n)$ we have $\hat{T}(\omega) = T^x(e^{-ix^T\omega})$; this is an analytic function with respect to ω which is polynomially bounded for $\omega \in \mathbb{R}^n$ (Theorem of Paley-Wiener).*
3. *For $T \in \mathcal{S}'(\mathbb{R}^n)$ we have $\overline{\hat{\check{T}}} = \hat{\overline{T}}$.*

4. For $T_1 \in \mathcal{S}'(\mathbb{R}^n)$ and $T_2 \in \mathcal{E}'(\mathbb{R}^n)$ we have $T_1 * T_2 \in \mathcal{S}'(\mathbb{R}^n)$ and

$$\widehat{T_1 * T_2} = \hat{T}_1 \cdot \hat{T}_2,$$
$$\widehat{\hat{T}_1 \cdot \hat{T}_2} = (2\pi)^n \, \check{T}_1 * \check{T}_2.$$

Note that the product $\hat{T}_1 \cdot \hat{T}_2$ is defined, since $\hat{T}_2 \in C^\infty(\mathbb{R}^n)$.

PROOF: We show 3.: For $\varphi \in \mathcal{S}(\mathbb{R}^n)$ we have $\overline{\hat{\varphi}(\omega)} = \int_{\mathbb{R}^n} e^{ix^T\omega} \overline{\varphi(x)} \, dx = \check{\hat{\overline{\varphi}}}(\omega)$. Hence,

$$\langle \overline{\hat{\hat{T}}}, \varphi \rangle = \overline{\langle \hat{\hat{T}}, \overline{\varphi} \rangle} = \overline{\langle T, \check{\hat{\overline{\varphi}}} \rangle} = \overline{\langle T, \overline{\hat{\varphi}} \rangle} = \langle \hat{\overline{T}}, \varphi \rangle.$$

4. The second formula follows from the first by Fourier transformation in $\mathcal{S}'(\mathbb{R}^n)$, 1. and Proposition A.10. □

Definition A.19 (Sobolev space). *We define for $s \in \mathbb{R}$ the* Sobolev space

$$H^s(\mathbb{R}^n) := \{ u \in \mathcal{S}'(\mathbb{R}^n) \mid (1 + \|\omega\|^2)^{\frac{s}{2}} \, \hat{u}(\omega) \in L^2(\mathbb{R}^n) \}$$

equipped with the scalar product

$$\langle u, v \rangle := \int_{\mathbb{R}^n} (1 + \|\omega\|^2)^s \hat{u}(\omega) \overline{\hat{v}(\omega)} \, d\omega.$$

$H^s(\mathbb{R}^n)$ is a Hilbert space. For $s \in \mathbb{N}_0$, the Sobolev space $H^s(\mathbb{R}^n)$ coincides with the space

$$\{ u \in L^2(\mathbb{R}^n) \mid D^\alpha u \in L^2(\mathbb{R}^n), |\alpha| \le s \}$$

with scalar product $\langle u, v \rangle = \sum_{|\alpha| \le s} \int_{\mathbb{R}^n} D^\alpha u(x) \, \overline{D^\alpha v(x)} \, dx$. The induced norms are equivalent to each other.

B

Data

B.1 Wendland Functions

In the following table we present the functions ψ_1 and ψ_2 for the Wendland functions $\psi_{3,1}(cr)$, $\psi_{4,2}(cr)$ and $\psi_{5,3}(cr)$. Note that these are the Wendland functions defined in Definition 3.9 up to a constant, cf. also Table 3.1 in Section 3.1.4. Note that $x_+ = x$ for $x \geq 0$ and $x_+ = 0$ for $x < 0$.

	$\psi_{3,1}(cr)$
$\psi(r)$	$(1-cr)_+^4[4cr+1]$
$\psi_1(r)$	$-20c^2(1-cr)_+^3$
$\psi_2(r)$	$60c^3\frac{1}{r}(1-cr)_+^2$

	$\psi_{4,2}(cr)$
$\psi(r)$	$(1-cr)_+^6[35(cr)^2+18cr+3]$
$\psi_1(r)$	$-56c^2(1-cr)_+^5[1+5cr]$
$\psi_2(r)$	$1680c^4(1-cr)_+^4$

	$\psi_{5,3}(cr)$
$\psi(r)$	$(1-cr)_+^8[32(cr)^3+25(cr)^2+8cr+1]$
$\psi_1(r)$	$-22c^2(1-cr)_+^7[16(cr)^2+7cr+1]$
$\psi_2(r)$	$528c^4(1-cr)_+^6[6cr+1]$

B.2 Figures

The parameters for the figures. The grid points are $x_0 + \alpha\left(i + \frac{j}{2}, j\frac{\sqrt{3}}{2}\right)$ where $i, j \in \mathbb{Z}$ without the equilibrium ($i = j = 0$) plus some additional points (add.), thus altogether N points. For the three-dimensional example we used the points $x_0 + \alpha\left(i + \frac{j}{2} + \frac{l}{2}, j\frac{\sqrt{3}}{2} + \frac{l}{2\sqrt{3}}, l\sqrt{\frac{2}{3}}\right)$ with $i, j, l \in \mathbb{Z}$, excluding the equilibrium ($i = j = l = 0$); α is proportional to the fill distance. This hexagonal grid and its generalization have been discussed at the beginning of Chapter 6.

The local Lyapunov function $\mathfrak{q} = \mathfrak{v}$ or $\mathfrak{q} = \mathfrak{d}$ was used, the sets $\{x \mid \mathfrak{q}(x) \leq L\}$ and $\{x \mid q(x) \leq R\}$ were calculated, where the Lyapunov function Q was approximated. Note that we did not add a suitable constant to q so that sometimes $R < 0$. For interpolation of V via W we always used a Taylor polynomial of order $P = 5$. The mixed interpolations in Figures 5.4 and 6.11 used a second grid X_M^0 with M points. The scaled Wendland function $\psi_{l,k}(cr)$ with $l = \left\lfloor \frac{n}{2} \right\rfloor + k + 1$ was used as radial basis function.

Chemostat (1.1)

Fig.	k	c	α	add.	N	Q'	R	local	L
1.1 to 1.4	1	$\frac{1}{6}$	$\frac{1}{16}$	0	153	$-\|x\|^2$	-1.7	$\mathfrak{v}$	0.025

Example throughout the book (2.11)

Fig.	k	c	α	add.	N	M	Q'	R	local	L
2.1, 4.1, 4.2, 5.1l, 6.1r	2	$\frac{2}{3}$	0.4	2	24	–	$-\|x\|^2$	-0.95	$\mathfrak{v}$	0.09
4.3l, 4.4l, 5.1r	2	$\frac{2}{3}$	0.2	0	76	–	$-\|x\|^2$	-1.5	$\mathfrak{v}$	0.09
4.3r	2	$\frac{2}{3}$	0.2	0	76	–	$-\|x\|^2$	-1.5	$\mathfrak{v}$	0.045
4.4r	2	$\frac{2}{3}$	0.2	0	76	–	via W	0.2	–	–
5.2l	2	$\frac{2}{3}$	0.15	4	140	–	$-\|x\|^2$	-1.8	$\mathfrak{v}$	0.09
5.2r	2	$\frac{2}{3}$	0.1	0	312	–	$-\|x\|^2$	-3	$\mathfrak{v}$	0.09
5.3	2	$\frac{2}{3}$	0.075	0	484	–	$-\|x\|^2$	-3.4	$\mathfrak{v}$	0.09
5.4	2	$\frac{2}{3}$	0.2	0	70	10	$-\|x\|^2$	1.1	$\mathfrak{v}$	0.09

Speed-control (6.1)

Fig.	k	c	α	add.	N	Q'	R	local	L
6.2, 6.3m	–	–	–	–	–	–	–	$\mathfrak{d}$	0.035
6.3l, 6.4, 6.5	1	$\frac{5}{6}$	0.05	0	223	-1	-6	$\mathfrak{v}$	0.032

Toy example (6.2)

Fig.	k	c	α	add.	N	Q'	R	local	L
6.6r, 6.7l, 6.8l	1	$\frac{1}{2}$	0.3	4	126	$-\|x\|^2$	-2.9	$\mathfrak{v}$	0.35
6.7m	1	$\frac{1}{2}$	0.3	0	122	$-\|f(x)\|^2$	-0.65	$\mathfrak{v}$	0.35
6.7r	1	$\frac{1}{2}$	0.3	0	122	-1	-0.8	$\mathfrak{v}$	0.35
6.8m	1	$\frac{1}{2}$	0.3	0	122	via W	0.85	–	–

Three-dimensional Example (6.3)

Fig.	k	c	α	add.	N	Q'	R	local	L
6.9	1	$\frac{9}{20}$	0.35	0	137	$-\|x\|^2$	-0.595	$\mathfrak{v}$	0.05
6.10	1	$\frac{9}{20}$	0.35	0	137	via W	0.0654	–	–

van-der-Pol (6.4)

Fig.	k	c	α	add.	N	M	Q'	R	local	L
6.11	1	$\frac{10}{21}$	0.22	0	236	16	$-\|x\|^2$	1.6	$\mathfrak{v}$	1.15
6.12l	1	$\frac{10}{21}$	0.22	0	236	–	-2.5	-0.25	$\mathfrak{v}$	1.15

C

Notations

$'$	orbital derivative: $Q'(x) = \langle \nabla Q(x), f(x) \rangle$, cf. Definition 2.18
$\dot{}$	temporal derivative $\dot{x}(t) = \frac{d}{dt}x(t)$
$A(x_0)$	basin of attraction of x_0, cf. Definition 2.9
$\tilde{B}_r^Q(x_0)$	the set $\{x \in \mathbb{R}^n \mid Q(x) < r^2\}$ where Q is a Lyapunov function, cf. Definition 2.23
$\delta_{\tilde{x}}$	Dirac's delta-distribution at $\tilde{x} \in \mathbb{R}^n$, i.e. $\delta_{\tilde{x}} f(x) = f(\tilde{x})$, cf. Example A.4
$\mathfrak{d}(x)$	local Lyapunov function $\mathfrak{d}(x) = \|S(x-x_0)\|^2$ satisfying $\mathfrak{d}'(x) \leq 2(-\nu + \epsilon)\mathfrak{d}(x)$, cf. Lemma 2.28
$\mathcal{D}'(\mathbb{R}^n)$	space of distributions, cf. Definition A.2
$\mathcal{E}'(\mathbb{R}^n)$	space of distributions with compact support, cf. Definition A.5
f	$f \in C^\sigma(\mathbb{R}^n, \mathbb{R}^n)$, right-hand side of the ordinary differential equation $\dot{x} = f(x)$, cf. (2.1)
$\mathcal{F}$	native space of functions, cf. Definition 3.12
$\mathcal{F}^*$	dual of the native space, cf. Definition 3.12
$\mathfrak{h}(x)$	Taylor polynomial of $V(x)$ with $V'(x) = -\|x - x_0\|^2$, cf. Definition 2.52
$H^s(\mathbb{R}^n)$	Sobolev space, cf. Definition A.19
I	identity matrix $I = \mathrm{diag}(1, 1, \ldots, 1)$
$\tilde{K}_r^Q(x_0)$	the set $\{x \in \mathbb{R}^n \mid Q(x) \leq r^2\}$ where Q is a Lyapunov function, cf. Definition 2.23
L	Lyapunov function with $L'(x) = -\bar{c}\, L(x)$ for $x \in A(x_0)$, cf. Corollary 2.40
$\mathfrak{n}(x)$	$\mathfrak{n}(x) = \mathfrak{h}(x) + M\|x - x_0\|^{2H}$, such that $\mathfrak{n}(x) > 0$ holds for $x \neq x_0$, cf. Definition 2.56
$\omega(x)$	ω-limit set of x, cf. Definition 2.12
Ω	non-characteristic hypersurface: $(n-1)$-dimensional manifold, often level set of a (local) Lyapunov function, cf. Definition 2.36
$\psi_{l,k}(r)$	Wendland function, cf. Definition 3.9

$\Psi(x)$	radial basis function, here $\Psi(x) = \psi_{l,k}(c\|x\|)$ with $c > 0$ and a Wendland function $\psi_{l,k}(r)$
$\mathcal{S}(\mathbb{R}^n)$	Schwartz space of rapidly decreasing functions, cf. Definition A.12
$\mathcal{S}'(\mathbb{R}^n)$	dual of the Schwartz space, cf. Definition A.17
$S_t\xi$	flow, solution $x(t)$ of $\dot{x} = f(x)$, $x(0) = \xi$, cf. Definition 2.1
T	Lyapunov function with $T'(x) = -\bar{c} < 0$ for $x \in A(x_0) \setminus \{x_0\}$ and $T(x) = H(x)$ for $x \in \Omega$, cf. Theorem 2.38
t	function which approximates T
V	Lyapunov function with $V'(x) = -p(x)$ for $x \in A(x_0)$, cf. Theorem 2.46
v	function which approximates V
$\mathfrak{v}$	local Lyapunov function satisfying $\langle \nabla \mathfrak{v}(x), Df(x_0)(x - x_0)\rangle = -\|x - x_0\|^2$, cf. Remark 2.34
V^*	Lyapunov function with $(V^*)'(x) = -p(x)$ for $x \in A(x_0) \setminus \{x_0\}$ and $V^*(x) = H(x)$ for $x \in \Omega$, cf. Proposition 2.51
x_0	equilibrium point of $\dot{x} = f(x)$, i.e. $f(x_0) = 0$, cf. Definition 2.6

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