A Skorokhod Theorem

In this whole appendix (K, d) is a precompact metric space equipped with the $\sigma-$algebra of its Borel sets. Our aim here is to prove two theorems due to Skorokhod. The first one asserts that we can parameterize any borelian probability measure μ on K as the push-forward $\mu = \chi_\sharp\lambda$ of the Lebesgue measure λ on $[0, 1]$ by some borelian application $\chi : [0, 1] \to K$. The same construction can be extended to prove that the weak convergence of a sequence of probability measures μ_n to a probability measure μ on K is equivalent to the pointwise convergence of suitable parametrizations χ_n of μ_n towards a parameterization χ of μ.

Definition A.1. *Let (K, d) be a precompact metric space and μ be a probability measure on K. We call parameterization of μ a measurable application $\chi : \omega \in [0, 1] \to K$ such that $\mu = \chi_\sharp\lambda$ where λ is the Lebesgue measure on $[0, 1]$. That is to say $\mu(A) = \lambda(\chi^{-1}(A))$.*

Remark A.2. As a direct consequence of a parameterization χ of μ, if $\phi : K \to \mathbb{R}^+$ is a $\mu-$measurable function, then $\int_K \phi(\gamma)d\mu(\gamma) = \int_\Omega \phi(\chi(\omega))d\omega$ (see [5], Def. 1.70, p. 32). As an illustrative example the Dirac mass at 0 is parameterized by the null constant application on $[0, 1]$. In the same way, an atomic measure $\sum_1^n a_i\delta_{x_i}$ can be parameterized by the piecewise constant function $\chi(\omega) = x_1$ on $[0, a_1]$, $\chi(\omega) = x_2$ on $]a_1, a_2]$ and so on.

Theorem A.3. *(Skorokhod Theorem) Let (K, d) be a precompact metric space and μ be a probability measure on K. Then there exists a parameterization χ of μ.*

Theorem A.3 follows directly from Lemma A.6 where a more specific construction is achieved.

Lemma A.4. *There exists a filtration of K made of finite partitions $\mathcal{F}_l = \{F_j^l : 1 \leq j \leq J_l\}$, where $J_l \in \mathbb{N}^*$, such that the diameters of the sets F_j^l are less than 2^{-l}.*

Proof. We construct this filtration recursively. In order to construct $\mathcal{F}_1$, we cover K with a finite number of balls of radii $1/4$. Let us denote by B_i, where $1 \leq i \leq n$, the intersection of these balls with K. Let us find a partition of

$K = \cup_i B_i$ with at most n elements. To do this, we denote $\tilde{F}_1^1 := B_1$ and, in a recursive way, we define $\tilde{F}_{i+1}^1 := B_{i+1} \setminus \cup_{j\leq i} B_j$. If any of the $\tilde{F}_i^1$ is empty, we do not take it into account, so that we obtain a family of non empty elements F_i^1 where $i \leq J_1$. Since the F_i^1 are precompact, we can iterate the above process by covering them with balls of radius $1/8$. Proceeding iteratively we construct the desired filtration.

Lemma A.5. *Let μ be a probability measure on K. There exists a filtration made of finite partitions $\mathcal{F}_l = \{F_j^l : 1 \leq j \leq J_l\}$, $J_l \in \mathbb{N}^*$, such that the diameters of F_j^l are less than 2^{-l+1} and $\mu(\partial F_j^l) = 0$ for all l and $j \leq J_l$.*

Proof. To obtain this filtration, we slightly modify the construction of Lemma A.4. We only need to request in addition that $\mu(\partial F_j^l) = 0$ for all l and $j \in J_l$. For that, it is enough to perturb the radii $r_l = 2^{-l}(1+\epsilon_l)$, with $\epsilon_l \leq 1$ so that μ does not charge the boundaries of the balls with radius r_l used to construct $\mathcal{F}_l$.

The filtration obtained in Lemma A.5 allows us to define a canonical parameterization of μ. The idea is to group together the ω's whose images are close.

Lemma A.6. *Let μ be a probability measure on K and $\mathcal{F}$ be the filtration constructed in Lemma A.5. There exists a parameterization χ of μ such that for all l, the sets*

$$\Omega_{j,l} = \{\omega : \chi(\omega) \in F_j^l\}$$

are intervals ordered in an increasing way with j.

Proof. We construct χ by successive approximations χ_n using the filtration of Lemma A.5.

Step 1: Definition of χ_n. Let $t_0^n := 0$ and $t_j^n := \sum_{i\leq j} \mu(F_i^n)$ where $1 \leq j \leq J_n$. The application χ_n is defined as a piecewise constant function sending each interval $[t_{j-1}^n, t_j^n[$ onto an arbitrary element of F_j^n. By construction, $\Omega_{j,l} := \{\omega : \chi_n(\omega) \in F_j^l\} = [t_{j-1}^l, t_j^l[$ for all $j \leq J_l$. We notice that the intervals $[t_{j-1}^l, t_j^l[$ where $1 \leq j \leq J_l$, are intervals ordered in an increasing way when j goes from 1 to J_l, so that its union is $[0, 1[$. Notice also that $\mu(F_j^l) = |\Omega_{j,l}|$.

Step 2: The sequence $\chi_n(\omega)$ converges for all ω. Let us prove that χ_n is a Cauchy sequence. Let us first observe that $\chi_n(\Omega_j^m) \subset F_j^m$ for any $n \geq m$. Indeed, let us fix m and $n \geq m$. By the definition of filtration, Ω_j^m is the union of Ω_k^n where k describes the set of indices such that $F_k^n \subset F_j^m$. Thus, χ_n sends every element of Ω_k^n to an element of $F_k^n \subset F_j^m$. A fortiori, the image of Ω_j^m under χ_n is in F_j^m. Now, since the sets F_j^m have diameter less than 2^{-m}, we deduce that $d(\chi_n(\omega), \chi_m(\omega)) < 2^{-m}$ for all $m \leq n$. Thus, $\chi_n(\omega)$ is a Cauchy sequence.

Let χ be the pointwise limit of χ_n. Observe that χ is measurable as a pointwise limit of measurable functions.

Step 3: The measure $\chi_\sharp\lambda$ is exactly μ. We have to show that $\chi_\sharp\lambda(F_j^l) = \mu(F_j^l)$ for all (j,l). The measures μ and $\chi_\sharp\lambda$ will then be equal on the sets F_j^l which form a Π-system. Then the extension Theorem of Π-systems (Lemma 1.6, p.19 [93]) shows that $\mu = \chi_\sharp\lambda$ on the σ-algebra generated by this Π-system, that is, on the σ-algebra of Borel sets of K.

Let us fix l, $j \leq J_l$, and let us define

$$G_p := \{\gamma \in F_j^l : d(\gamma, \partial F_j^l) \geq 1/p\}.$$

This is a non decreasing sequence of sets such that $\cup_p G_p = F_j^l \setminus \partial F_j^l$. Fix $\epsilon > 0$. For a sufficiently large p, we have

$$\mu(G_p) \geq \mu(F_j^l) - \epsilon. \tag{A.1}$$

Now, consider an l' such that $2^{-l'} < \frac{1}{2p}$. For any $y \in G_p$, there exists k so that $y \in F_k^{l'}$. Since the diameter of $F_k^{l'}$ is less than $\frac{1}{2p}$, $F_k^{l'} \subset G_{2p}$ so that $\bar{F}_k^{l'} \subset F_j^l$. For $n \geq l'$, the construction of χ_n ensures that $\chi_n(\Omega_k^{l'}) \subset F_k^{l'}$. Since χ is the pointwise limit of χ_n,

$$\chi(\Omega_k^{l'}) \subset \bar{F}_k^{l'} \subset F_j^l. \tag{A.2}$$

We obtain a covering of G_p with sets of the form $F_k^{l'}$ satisfying (A.2), and, using (A.1), we have $\chi_\sharp\lambda(F_j^l) \geq \mu(F_j^l) - \epsilon$. This being true for all $\epsilon > 0$, we deduce that $\chi_\sharp\lambda(F_j^l) \geq \mu(F_j^l)$. Since these sets form a partition for $1 \leq j \leq J_l$, and $\chi_\sharp\lambda$ is a probability measure, the inequality is indeed an equality, that is : $\chi_\sharp\lambda(F_j^l) = \mu(F_j^l)$. As a consequence, we have $\chi^{-1}(F_j^l) = \Omega_{j,l}$ modulo a null set.

Definition A.7. *Let $(\mu_n)_n$ and μ be probability measures on (K,d). We say that μ_n tends to μ "pointwise" whenever there exist parameterizations χ_n and χ of μ_n and of μ, respectively, such that $d(\chi_n(\omega), \chi(\omega)) \to 0$ almost everywhere in $[0,1]$.*

Theorem A.8 (Skorokhod convergence Theorem). *Let $(\mu_n)_n$ be a sequence of probability measures on (K,d). The sequence μ_n weakly-* converges to μ if and only if μ_n to μ tends to μ "pointwise".*

Proof. Assume that μ_n converges to μ "pointwise", and let χ_n, χ denote the parameterizations of μ_n and μ, respectively. Since $\chi_n(\omega)$ converges to $\chi(\omega)$ for almost every ω, using Lebesgue's theorem, for all $\phi \in C(K)$, we have

$$\begin{aligned} < \mu_n, \phi > &= \int_K \phi(\gamma) d\mu_n(\gamma) = \int_{[0,1]} \phi(\chi_n(\omega)) d\omega \\ &\to \int_{[0,1]} \phi(\chi(\omega)) d\omega = \int_K \phi(\gamma) d\mu(\gamma) =< \mu, \phi > . \end{aligned}$$

Conversely, let μ_n be weakly-* converging to μ. Let us consider the filtration associated with μ constructed in Lemma A.5. Since $\mu(\partial F_j^l) = 0$, we deduce that $\mu_n(F_j^l)$ converges to $\mu(F_j^l)$. Next, applying Lemma A.6 to measures μ_n and μ, we get applications χ_n and χ such that $\chi_{n\sharp}\lambda = \mu_n$ and $\chi_\sharp\lambda = \mu$. The fact that $\mu_n(F_j^l)$ converges to $\mu(F_j^l)$ implies that $|\Omega_{j,l}^n|$ converges to $|\Omega_{j,l}|$, where $\Omega_{j,l}^n := \{\omega : \chi_n(\omega) \in F_j^l\}$ and $\Omega_{j,l} := \{\omega : \chi(\omega) \in F_j^l\}$. This convergence of measures implies the convergence of intervals $\Omega_{j,l}^n$ to some intervals $\Omega_{j,l}$, ordered in an increasing way with j.

We are now in a position to prove that for almost all ω the sequence $\chi_n(\omega)$ converges to $\chi(\omega)$. Notice that for almost all ω and for any $l \in \mathbb{N}$, there exists a $j \leq J_l$ such that ω is in the interior of $\Omega_{j,l}$. Indeed, there is a finite number of such intervals at each rank of the filtration, and thus, the set of its endpoints is countable, hence of measure zero. Thus, for n large enough, we have that $\omega \in \Omega_{j,l}^n$, i.e., $\chi_n(\omega) \in F_j^l$. This yields $d(\chi_n(\omega), \chi(\omega)) < 2^{-l}$.

B Flows in Tubes

In this appendix, we shall consider a fluid with laminar flow in a tube. We recall how Poiseuille's law can be derived from Navier-Stokes equation. Next, we discuss the optimality of the circular section.

B.1 Poiseuille's Law

Let us consider a tube of constant circular section with a straight axis. We take (x, y, l) as coordinates in the tube, where $l \in [0, L]$ is the distance along the axis and $(x, y) \in D(0, r)$ are orthogonal cartesian coordinates.

We assume a stationary regime and that the flow is laminar, that is to say the velocity is oriented by the axis and is constant on all trajectories, so that $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$. The velocity v at a point of a tube along the z-axis is given by Navier-Stokes equation

$$-\triangle\, v(l)(x, y) = \frac{1}{\eta}\frac{\partial p}{\partial z}, \text{ where } \triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Hence, $\frac{\partial p}{\partial z} = \mathit{constant}$ (where η denotes the viscosity coefficient). Thus, the gradient of pressure has the form $\frac{[p]}{L}$ where $[p]$ denotes the pressure difference at the ends of the tube, and we shall denote it by $\bigtriangledown p$. In other words, p is a linear interpolation of the initial and final pressures in the tube. We assume that the pressure is constant on the initial and ending sections of the tube, so that the pressure is constant on each section of the tube. For simplicity, let us take $\eta = 1$.

Under these hypotheses, we can calculate the velocity and the corresponding flow through the whole tube

$$v(x, y, l) = \frac{(r^2 - (x^2 + y^2))}{4} \bigtriangledown p$$

$$f = \int_{D(0,r)} v(x, y, l) = \frac{1}{4} r^4 \bigtriangledown p = r^2 v_{max}$$

The power dissipated by the steady flow is $W = fL \bigtriangledown p$. This is to be identified with $W = Lf^2 R$ where by definition R stands for the resistivity of the tube. Thus we obtain $R = 4/r^4$: Poiseuille's law says that the resistivity of a tube scales as the inverse fourth power of the radius.

B.2 Optimality of the Circular Section

What is the optimal form of the section of a tube? If we prescribe the pressure at both ends of a tube of constant section, the circular form ensures the maximal flow. We briefly present the result obtained in [83] and [1].

Let us recall the definition of the rearrangement of a set (see [56]). If $A \subset \mathbb{R}^d$, we denote by A^* the ball $B(0,r) = \{x \mid |x| < r\}$ such that $|B(0,r)| = |A|$. If $f : \mathbb{R}^d \to \mathbb{R}$ is a Borel measurable function vanishing at infinity, we define the symmetric decreasing rearrangement of f by $f^*(x) = \int_0^\infty \chi^*_{\{|f|>t\}}(x)dt$. It results from the definition that $|\{x \mid |f(x)| > t\}| = |\{x \mid f^*(x) > t\}|$ and $||f||_p = ||f^*||_p$.

Let u be such that $-\triangle u(x,y) = \bigtriangledown p$ in the domain Ω. Let v be such that $-\triangle v(x,y) = (\bigtriangledown p)^* = \bigtriangledown p$ in Ω^*. Then, it can be shown that $u^* \leq v$ [83]. As a consequence, the flow in a tube of section Ω is such that $\int_\Omega u = \int_{\Omega^*} u^* \leq \int_{\Omega^*} v$. Then a circular section is always more advantageous from the point of view of the flow.

In [1], the authors prove the uniqueness of the optimal form : if $\max u = \max v$, then there is x_0 such that $\Omega = x_0 + \Omega^*$ and $u = v(\cdot + x_0)$. Then, if Ω is an optimal form, we have $\int_\Omega u = \int_{\Omega^*} v$ and $u^* \leq v$, hence $\max u = \max u^* = \max v$ necessarily. Then there is x_0 such that $\Omega = x_0 + \Omega^*$, and, therefore, the circular form is the unique optimum.

C Notations

$\mathcal{C}(X,Y)$ set of continuous maps from X to Y endowed with sup-norm
X convex compact set of $\mathbb{R}^N$
$\mathcal{P}(X)$ metric space of probabilities on X
K set of 1-Lipschitz curves or paths $\mathbb{R}^+ \to X$, page 25
$L(\gamma)$ length of path, page 31
$T(\gamma)$ stopping time of a path, page 25
$\mathcal{C}([a,b],\mathbb{R}^N)$ set of continuous curves from $[a,b]$ to $\mathbb{R}^N$, endowed with the sup-norm $||\cdot||$
$\mathcal{C}_b(\mathbb{R}^N,\mathbb{R}^N)$ set of continuous bounded functions from $\mathbb{R}^N$ to $\mathbb{R}^N$
δ_x, Dirac mass at x
$|A|$ Lebesgue measure of a measurable set $A \subset \mathbb{R}$
$\lambda(A)$ Lebesgue measure of a measurable set $A \subset \mathbb{R}$
$conv(E)$ the convex hull of E
$E\Delta F = (E \setminus F) \cup (F \setminus E)$ symmetric difference of sets
$\mathcal{H}^1$ one-dimensional Hausdorff measure (length)
$M^\alpha(G) = \sum_{e\in E(G)} f(e)^\alpha \mathcal{H}^1(e)$ Gilbert energy, page 13
$M^\alpha(\mu^+,\mu^-)$ minimal Gilbert-Xia transport energy from μ^+ to μ^-, page 13
$\tilde{E}^\alpha(\chi)$ energy of a pattern page, 14
$\Omega = [0,1]$ or a fixed subset of $\mathbb{R}$ with finite measure, set of fiber indices
$\omega \in \Omega$ a fiber index
$[\omega]_t$ branch of ω at time t, page 14
μ^+, μ^- positive Borel measures in X with equal mass
π a probability measure on $X \times X$ or "transference plan"
$\boldsymbol{P}$ traffic plan, page 26
$\pi_{\boldsymbol{P}}$ transference plan associated with a traffic plan $\boldsymbol{P}$, page 26
π_0, page 26
π_∞, page 26
$\pi_{0\sharp}\boldsymbol{P}$, $\pi_{\infty\sharp}\boldsymbol{P}$, page 26
$t \mapsto \chi(\omega,t)$, fiber indexed by ω
$(\omega,t) \mapsto \chi(\omega,t)$, parameterized traffic plan or pattern, pages 27, 28
$\mathbf{P}_\chi$ law in K of a parameterized traffic plan, page 28
$|\chi| := |\Omega|$ the total mass transported by χ
$T_\chi(\omega) := \inf\{t : \chi(\omega) \text{ is constant on } [t,\infty)\}$, page 28
$T(\omega)$ abbreviation for $T_\chi(\omega)$, page 28

$\tau(\omega) = \chi(\omega, 0)$ initial point of a fiber page, 29
$\sigma(\omega) = \chi(\omega, T(\omega))$ final point of a fiber, page 29
$\pi = (\tau, \sigma)_\sharp \lambda$ transference plan of χ
$\mu^+(\chi)(A) := |\{\omega : \chi(\omega, 0) \in A\}|$, irrigating measure of χ
$\mu^+_\chi = \tau_\sharp \lambda$, irrigating measure of χ
$\mu^-(\chi)(A) := |\{\omega : \chi(\omega, T_\chi(\omega)) \in A\}|$, measure irrigated by χ
$\mu^-_\chi = \sigma_\sharp \lambda$, measure irrigated by χ
$\mathrm{TP}(\mu^+, \mu^-)$ set of traffic plans χ such that $\mu^-_\chi = \mu^-$ and $\mu^+_\chi = \mu^+$, page 143
TP_C set of traffic plans such that $\int_\Omega T_\chi(\omega)\, d\omega \le C$, page 26
$\mathrm{TP}_C(\mu^+, \mu^-) := \mathrm{TP}(\mu^+, \mu^-) \cap \mathrm{TP}_C$.
$[\omega]_t \subset \Omega$ branch of the fiber ω at time t in a pattern, page 14
$|[\omega]_t|$ measure of the branch containing ω at time t, page 14
$\tilde{E}^\alpha(\chi) = \int_\Omega \int_0^{T(\omega)} |[\omega]_t|^{\alpha-1} d\omega dt$ energy of a pattern, page 14
$\mathcal{D}(\chi) = \cup_{\omega \in \Omega} \{\omega\} \times [S(\omega), T(\omega)]$ domain of a traffic plan, page 48
χ_D restriction of χ to a sub-domain of $\mathcal{D}(\chi)$, page 47
$\Omega^\chi_x := \{\omega : x \in \chi(\omega, \mathbb{R})\}$ path class of x in χ, page 31
$\Omega_x := \Omega^\chi_x$ abbreviation
$|x|_\chi = |\Omega^\chi_x|$ multiplicity of χ at x, page 31
$|x|_{\boldsymbol{P}}$ multiplicity of $\boldsymbol{P}$ at x, page 31
S_χ, $S_{\boldsymbol{P}}$ support of $\boldsymbol{P}$ or of its parameterized traffic plan χ, set of points x such that $|x|_\chi > 0$, page 31
$E^\alpha(\chi) = \int_\Omega \int_{\mathbb{R}^+} |\chi(\omega, t)|^{\alpha-1}_\chi |\dot{\chi}(\omega, t)| dt d\omega$ energy of a traffic plan $\boldsymbol{P}$ parameterized by χ, page 35
$E^\alpha(\mu^+, \mu^-) := \min_{\mathrm{TP}(\mu^+, \mu^-)} E^\alpha(\chi)$ minimal energy between μ^+ and μ^-, page 55
$\Omega_{\overrightarrow{xy}}$ set of fibers passing through x and then through y, page 65
χ_{xy} traffic plan made of all pieces of fibers of χ joining x to y, page 65
$\Gamma^{xy} = S_{\chi_{xy}}$ support of χ_{xy}, page 65
$\Omega_{xy} := \Omega_{\overrightarrow{xy}} \cup \Omega_{\overrightarrow{yx}}$, page 70
Γ_{xy} the only arc with positive multiplicity between x and y, page 70

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Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

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