C.I.M.E. Session on "Viscosity Solutions and Applications"

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FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

"Mathematics Inspired by Biology"

is the subject of the First 1997 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R), the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.), and European Community will take place, under the scientific directions of Professors VINCENZO CAPASSO (Università di Milano) and ODO DIEKMANN (University of Utrecht) in Martina Franca (Taranto), **from 13 to 20 June, 1997.**

Courses

a) **Dynamics of physiologically structured populations** (6 lectures in English)
Prof. Odo DIEKMANN (University of Utrecht)

Outline:

1. Formulation and analysis of general linear models. The connection with multi-type branching processes. The definition of the basic reproduction ration R_0 and the Malthusian parameter r. Spectral analysis and asymptotic large time behaviour.
2. Nonlinear models: density dependence through feedback via environmental interaction variables. Stability boundaries in parameter space. Numerical bifurcation methods.
3. Evolutionary considerations. Invasibility and ESS (Evolutionarily Stable Strategies). Relationship with fitness measures.
4. Case studies:
   - cannibalism
   - Daphnia feeding on algae
   - reproduction strategy of Salmon

References

- J.A.J. Metz & O. Diekmann (eds.), Dynamics of Physiologically Structured Populations, Lect.Notes in Biomath. 68, 1986, Springer.
- O. Diekmann, M. Gyllenberg, J. A. J. Metz & H. R. Thieme, On the formulation and analysis of general deterministic structured population models, I. Linear theory, preprint.
- P. Jagers, The growth and stabilization of populations, Statistical Science 6 (1991), 269-283.
- P. Jagers, The deterministic evolution of general branching populations, preprint.
- O. Diekmann, M. A. Kirkilonis, B. Lisser, M. Louter-Nool, A. M. de Roos & B. P. Sommeijer, Numerical continuation of equilibria of physiologically structured population models, preprint.
- S. D. Mylius & O. Diekmann, On evolutionarily stable life histories, optimization and the need to be specific about density dependence, OIKOS 74 (1995), 218-224.
  J. A. J. Meetz, S. M. Mylius & O. Diekmann, When does evolution optimise? preprint.
- A. M. de Roos, A gentle introduction to physiologically structured population models, preprint.
- F. van den Bosch, A. M. de Roos & W. Gabriel, Cannibalism as a life boat mechanism, J. Math. Biol. 26 (1988), 619-633.
- V. Kaitala & W. W. Getz, Population dynamics and harvesting of semel-parous species with phenotypic and genotypic variability in reproductive age, J. Math. Biol. 33 (1995), 521-556.

b) When is space important in modelling biological systems? (6 lectures in English).
Prof. Rick DURRETT (Cornell University)

Outline:

In these lectures I will give an introduction to stochastic spatial models (also called cellular automata or interacting particle systems) and relate their properties to those of the ordinary differential equations that result if one ignores space and assumes instead that all individuals interact equally. In brief one finds that if the ODE has an attracting fixed point then both approaches (spatial and non-spatial) agree that coexistence will occur. How if the ODE has two or more locally stable equilibria or periodic orbits? Then the two approaches can come to radically different predictions. We will illustrate these principles by the study of a number of different examples

- competitionof Daphnia species in rock pools
- allelopathy in E. coli
- spatial versions of Prisoner's dilemma in Maynard Smith's evolutionary games framework
- a species competition model of Silvertown et al. and how it contrasts with a three species ODE system of May and Leonard

References

- R. Durrett (1955) Ten lectures on Particle Systems, pages 97-201 in Springer Lecture Notes in Math. 1608.
- R. Durrett and S. Levin (1994) Stochastic spatial models: a user's guide to ecological applications. Phil. Trans. Roy. Soc. B 343, 329-350.
- R. Durrett and S. Levin (1994) The importance of being discrete and spatial. Theor. Pop. Biol. 46 (1994), 363-394.
- R. Durrett and S. Levin (1996) Allelopathy in spatially distributed populations. Preprint
- R. Durrett and C. Neuhauser (1996) Coexistence results for some competition models.

c) Random walk systems modeling spread and interaction (6 lectures in English).
Prof. K. P. HADELER (University of Tübingen)

Outline:

Random walk systems are semilinear systems of hyperbolic equations that describe spatial spread and interaction of species. From a modeling point of view they are similar to reaction diffusion equations but they do not show infinitely fast propagation. These systems comprise correlated random walks, certain types of Boltzmann equations, the Cattaneo system, and others. Mathematically, they are closely related to damped wave equations. The aim of the course is the derivation of such systems from biological modeling assumptions, exploration of the connections to other approaches to spatial spread (parabolic equations, stochastic processes), application to biological problems and presentation of a qualitative theory, as far as it is available

1. The problems and their history
   Reaction diffusion equations; velocity jump processes and Boltzmann type equations; correlated random walks; nonlinear interactions; Cattaneo problems; reaction telegraph equations; boundary conditions.
2. Linear theory
   Explicit representations; operator semigroup theory; positivity properties; spectral properties; compactness properties.
3. Semilinear systems
   Invariance and monotonicity; Lyapunov functions; gradient systems; stationary points; global attractors.
4. Spatial spread
   Travelling fronts and pulses; spread of epidemics; bifurcation and patterns; Turing phenomenon; free boundary value problems.
5. Comparison of hyperbolic and parabolic problems.

References

- Goldstein, S, On diffusion by discontinuous movements and the telegraph equation. Quart. J. Mech. Appl. Math. 4 (1951), 129-156.
- Kac, M., A stochastic model related to the telegrapher's equation. (1956), reprinted Rocky Math. J. 4 (1974), 497-509.
- Dunbar, S., A branching random evolution and a nonlinear hyperbolic equation. SIAM J. Appl. Math. 48 (1988), 1510-1526.
- Othmer, H. G., Dunbar, S., Alt, W., Models of dispersal in biological systems. J. Math. Biol. 26 (1988), 263-298.
- Hale, J. K., Asymptotic behavior of dissipative systems. Amer. Math. Soc., Providence, R.I. 1988.
- Temam, R., Infinite dimensional dynamical systems in mechanics and physics. Appl. Math. Series 68, Springer 1988.

- Hadeler, K. P., Reaction telegraph equations and random walks. Canad. Appl. Math. Quart. 2 (1994), 27-43.
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d) Mathematical Modelling in Morphogenesis (6 lectures in English).
Prof. Philip MAINI (Oxford University)

Outline:

A central issue in developmental biology is the formation of spatial and spatio temporal patterns in the early embryon, a process known as morphogenesis. In 1952, Turing published a seminal paper in which he showed that a system of chemicals, reacting and diffusing, could spontaneously generate spatial patterns in chemical concentrations, and he termed this process "diffusion-driven instability". This paper has generated a great deal of research into more general reaction diffusion systems, consisting of nonlinear coupled parabolic partial differential equations (Turing's model was linear), both from the viewpoint of mathematical theory, and from the application to diverse patterning phenomena in development. In these lectures I shall discuss the analysis of these equations and show how this analysis carries over to other models, for example, mechanochemical models. Specifically, I shall focus on linear analysis and nonlinear bifurcation analysis. This analysis will be extended to non-standard problems.

References

- G. C. Cruywagen, P.K. Maini, J. D. Murray, Sequential pattern formation in a model for skin morphogenesis, IMA J. Math. Appl. Med. & Biol., 9 (1992), 227-248.
- R. Dillon, P.K. Maini, H. G. Othmer, Pattern formation in generalised Turing systems. I. Steady-state patterns in systems with mixed boundary conditions, J. Math. Biol., 32 (1994), 345-393.
- P. K. Maini, D. L. Benson, J. A. Sherratt, Pattern formation in reaction diffusion models with spatially inhomogeneous diffusion coefficients, IMA J. Math. Appl. Med. & Biol.,) (1992), 197-213.
- J. D. Murray, Mathematical biology, Springer Verlag, 1989.
- A. M. Turing, The chemical basis of morphogenesis, Phil. Trans. Roy. Soc. Lond., B 237 (1952), 37-72.

e) The Dynamics of Competition (6 lectures in English).
Prof. Hal L. SMITH (Arizona State University, Tempe)

Outline:

Competitive relations among populations of organisms are among the most studied by ecologists and there is a vast literature on mathematical modeling of competition. Quite recently, there have been a number of breakthroughs in the mathematical understanding of the dynamics of competitive systems, that is, of those systems of ordinary, delay, difference and partial differential equations arising in the modeling of competition. In these lectures I intend to describe some of the new results and consider their applications to microbial competition for nutrients in a chemostat.

References

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- Hess, P. and Lazer, A. C., On an abstract competition model and applications, Nonlinear Analysis T.M.A., 16 (1991), 917-940.
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- Hsu, S. B., Smith, H. L. and Waltmanm P., Dynamics of Competition in the unstirred chemostat, Canadian Applied Math. Quart., 2 (1994), 461-483.

FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

"Advanced Numerical Approximation of Nonlinear Hyperbolic Equations"

is the subject of the Second 1997 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R), the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.), and European Community will take place, under the scientific direction of Professors ALFIO QUARTERONI (Politecnico di Milano) at Grand Hotel San Michele, Cetraro (Cosenza), **from 23 to 28 June, 1997.**

Courses

a) **Discontinuous Galerkin Methods for Nonlinear Conservation Laws** (5 lectures in English).
Prof. Bernardo COCKBURN (University of Minnesota, Minneapolis)

Outline:
Lecture 1:
The nonlinear scalar conservation law as paradigm. The continuous dependence structure associated with the vanishing viscosity method for nonlinear scalar conservation laws. The entropy inequality and the entropy solution. The DG method as a vanishing viscosity method.

Lecture 2:
Features of the DG method. The use of two-point monotone numerical fluxes as an artificial viscosity associated with interelement edges: Relation with monotone schemes. Slope limiting as an artificial viscosity associate with the interior of the elements: Relation with the streamline-diffusion method.

Lecture 3:
Theoretical properties of the DG method. Stability properties and a priori and a posteriori error estimates.

Lecture 4:
Computational results for the scalar conservation law.

Lecture 5:
Extension to general multidimensional hyperbolic systems and computational results for the Euler equations of gas dynamics.

b) **Adaptive methods for differential equations and application to compressible flow problems** (6 lectures in English)
Prof. Claes JOHNSON (Chalmers University of Technology, Göteborg)

Outline:
We present a general methodology for adaptive error control for Galerkin methods for differential equations based on a posteriori error estimates involving the residual of the computed solution. The methodology is developed in the monograph Computational Differential Equations by Eriksson, Estep, Hansbo and Johnson, Cambridge University Press, 1996, and the companion volume Advanced Computational Differential Equations, by Eriksson, Estep, Hansbo, Johnson and Levenstam (in preparation), and is realized in the software femlab available on Internet `http://www.math.chalmers.se/femlab`. The a posteriori error estimates involve stability factors which are

estimated through auxiliary computation solving dual linearized problems. The size of the stability factor determines if the the specific problem considered is computable in the sense that the error in some norm can be made sufficiently small with the available computational resources. We present applications to a variety of problems including heat conduction, compressible and incompressible fluid flow, reaction-diffusion problems, wave propagation, elasticity/plasticity, and systems of ordinary differential equations.

Lecture 1: Introduction. A posteriori error estimates for Galerkin methods
Lecture 2: Applications to diffusion problems
Lecture 3: Applications to compressible and incompressible flow
Lecture 4: Applications to reaction-diffusion problems
Lecture 5: Applications to wave propagation, and elasticity/plasticity.
Lecture 6: Applications to dynamical systems.

c) Essentially Non-Oscillatory (ENO) And Weighted, Essentially Non-Oscillatory (WENO) Schemes For Hyperbolic Conservation Laws (6 lectures in English)
Prof. Chi-Wang SHU (Brown University, Providence)

Outline:
In this mini-course we will describe the construction, analysis, and application of ENO (Essentially Non-Oscillatory) and WENO (Weighted Essentially Non-Oscillatory) schemes for hyperbolic conservation laws. ENO and WENO schemes are high order accurate finite difference schemes designed for problems with piecewise smooth solutions with discontinuities. The key idea lies at the approximation level, where a nonlinear adaptive procedure is used to automatically choose the smoothest local stencil, hence avoiding crossing discontinuities in the interpolation as much as possible. The talk will be basically self-contained, assuming only the background of hyperbolic conservation laws which will be provided by the first lectures of Professor Tadmor.

Lecture 1: ENO and WENO interpolation
We will describe the basic idea of ENO interpolation, starting from the Newton form of one dimensional polynomial interpolations. We will show how the local smoothness of a function can be effectively represented by its divided or undivided differences, and how this information can help in choosing the stencils in ENO or the weights in WENO. Approximation results will be presented.

Lecture 2: More on interpolation
It will be a continuation of the first. We will discuss some advanced topics in the ENO and WENO interpolation, such as different building blocks, multi dimensions including general triangulations, sub-cell resolutions, etc.

Lecture 3: Two formulations of schemes for conservation laws
We will describe the conservative formulations for numerical schemes approximating a scalar, one dimensional conservation law. Both the cell averaged (finite volume) and point value (finite difference) formulations will be provided, and their similarities, differences, and relative advantages will be discussed. ENO and WENO interpolation procedure developed in the first two lectures will then be applied to both formulations. Total variation stable time discretization will be discussed.

Lecture 4: Two dimensions and systems
Is a continuation of the third. We will discuss the generalization of both formulations of the scheme to two and higher dimensions and to systems. Again the difference and relative advantages of both formulations will be discussed. For systems, the necessity of using local characteristic decompositions will be illustrated, together with some recent attempts to make this part of the algorithm, which accounts for most of the CPU time, cheaper.

Lecture 5: Practical issues
Practical issues of the ENO and WENO algorithms, such as implementation for workstations, vector and parallel supercomputers, how to treat various boundary conditions, curvilinear coordinates, how to use artificial compression to sharpen contact discontinuities, will be discussed. ENO schemes applied to Hamilton-Jacobi equations will also be discussed.

Lecture 6: Applications to computational physics
Application of ENO and WENO schemes to computational physics, including compressible Euler and Navier-Stokes equations, incompressible flow, and semiconductor device simulations, will be discussed.

d) **High Resolution Methods for the Approximate Solution of Nonlinear Conservation Laws and Related Equations** (5 lectures in English)
Prof. Eitan TADMOR (UCLA and Tel Aviv University)

Outline:
The following issues will be addressed.
Conservation laws: Scalar conservation laws, One dimensional systems; Riemann's problem, Godunov, Lax-Friedrichs and Glimm schemes, Multidimensional systems
Finite Difference Methods: TVD Schemes, three- and five-points stencil scheme, Upwind vs. central schemes, TVB approximations. Quasimonotone schemes; Time discretizations
Godunov-Type Methods and schemes
Finite Volume Schemes and error estimates
Streamline Diffusion Finite Element Schemes; The entropy variables
Spectral Viscosity and Hyper Viscosity Approximations; Kinetic Approximations

Lecture 1: Approximate solutions of nonlinear conservation laws - a general overview
During the recent decade there was an enormous amount of activity related to the construction and analysis of modern algorithms for the approximate solution of nonlinear hyperbolic conservation laws and related problems.
To present the successful achievements of this activity we discuss some of the analytical tools which are used in the development of the convergence theories associated with these achievements. In particular, we highlight the issues of compactness, compensated compactness, measure-valued solutions, averaging lemma,...while we motivate our overview of finite-difference approximations (e.g. TVD schemes), finite-volume approximations (e.g. convergence to measure-valued solutions on general grids), finite-element schemes (e.g., the streamline-dissusion method), spectral schemes (e.g. spectral viscosity method), and kinetic schemes (e.g., BGK-like and relaxation schemes).

Lecture 2: Approximate solutions of nonlinear conservation law - nonoscillatory central schemes
We discuss high-resolution approximations of hyperbolic conservation laws which are based on t *centraldifferencing*. The building block of such schemes is the use of staggeredgrids. The main advantage is simplicity, since no Riemann problems are involved. In particular, we avoid the time-consuming field-by-field decompositions required by (approximate) Riemann solvers of upwind difference schemes.
Typically, staggering suffers from excessive numerical dissipation. Here, excessive dissipation is compensated by using modern, high-resolution, non-oscillatory reconstructions.
We prove the non-oscillatory behavior of our central procedure in the scalar framework: For both the second- and third-order schemes we provide total-variation bounds, one-sided Lipschitz bounds (which in turn yield precise error estimates), as well as entropy and multidimensional L^∞ stability estimates.
Finally, we report on a variety of numerical experiments, including second -and third-order approximations of one-dimensional problems (Euler and MHD equations), as well as two-dimensional systems (including compressible and incompressible equations). The numerical experiments demonstrate that these central schemes offer simple, robust, Riemann-solver-free approximations for the solution of one and two-dimensional systems. At the same time, these central schemes achieve the same quality results as the high-resolution upwind schemes.

Lecture 3: Approximate solutions of nonlinear conservation laws - convergence rate estimates
Convergence analysis of approximate solutions to nonlinear conservation laws is often accomplished by BV or compensated-compactness arguments, which lack convergence rate estimates. An L^1 -error estimate is available for monotone approximations.
We present an alternative convergence rate analysis. As a stability condition we assume Lip^+-stability in agreement with Oleinik's E-condition.
We show that a family of approximate solutions, v^{eps}, which is Lip^+ -stable,satisfies

$$\| v^{eps}(.,t) - u(.,t) \|_{Lip} \leq C \| v^{eps}(.,0) - u(.,0) \|_{Lip}$$

Consequently, familiar L^p and new pointwise error estimates are derived. We demonstrate these estimates for viscous and kinetic approximations, finite-difference schemes, spectral methods, coupled systems with relaxation...

Lecture 4: Approximate solutions of nonlinear conservation laws - the spectral viscosity method
Numerical tests indicate that the convergence of spectral approximations to nonlinear conservation laws may --- and in some cases we prove it must --- fail, with or without post-processing the numerical solution. This failure is related to the global nature of spectral methods. Since nonlinear conservation laws exhibit spontaneous shock discontinuities, the spectral approximation pollutes unstable Gibbs oscillations overall the computational domain, and the lack of entropy dissipation prevents convergence in these cases.

The Spectral Viscosity (SV) method attempts to stabilize the spectral approximation by augmenting the latter with high frequency viscosity regularization, which could be efficiently implemented in the spectral, rather than the physical space. The additional SV is small enough to retain the formal spectral accuracy of the underlying approximation; yet, the SV is shown to be large enough to enforce a sufficient amount of entropy dissipation, and hence, by compensated compactness arguments, to prevent unstable spurious oscillations. Recent convergence results for the SV approximations of initial- and initial-boundary value problems will be surveyed. Numerical experiments will be presented to confirm that by post-processing the SV solution, one recovers the exact entropy solution within spectral accuracy.

Lecture 5: Approximate solutions of nonlinear conservation laws: entropy, kinetic formulations and regularizing effects

We present a new formulation of multidimensional scalar conservation laws and certain 2x2 one-dimensional systems (including the isentropic equations), which includes both the equation and the entropy criterion. This formulation is a kinetic one involving an additional variable called velocity by analogy. We also give some applications of this formulation to new compactness and regularity results for entropy solutions based upon the velocity averaging lemmas. Finally, we show that this kinetic formulation is in fact valid and meaningful for more general classes of equations like equations involing nonlinear second-order terms.

Seminars

A number of seminars will be offered during the Session.

Applications

Those who want to attend the Session should fill in an application to the Director of C.I.M.E at the address below, **not later than April 15, 1997.**

An important consideration in the acceptance of applications is the scientific relevance of the Session to the field of interest of the applicant.

Applicants are invited to submit, along with their application, a letter of recommendation.

Participation will only be allowed to persons who have applied in due time and have had their application accepted.

CIME will be able to partially support some of the youngest participants. Those who plan to apply for support have to mention explicitely in the application form.

Attendance

No registration fee is requested.
Lectures will be held at Grand Hotel San Michele, Cetraro (Cosenza) on June 23, 24, 25, 26, 27, 28.
Participants are requested to register at the Grand Hotel San Michele on June 22, 1997.

Site and lodging

The Session will be held at Grand Hotel San Michele, Cetraro (Cosenza, Italy).

Prices for full pensions (bed, meals) are roughly 140.000 lires p.p. a day in a single room, 120.000 lires in a double room. Cheaper arrangements for multiple lodging in a residence are available. More detailed information may be obtained from the Direction of the Hotel (tel +39-982-91012; telefax +39-982-91430)

Lecture Notes

Lecture notes will be published as soon as possible after the Session.

ROBERTO CONTI
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FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
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"Arithmetical Theory of Elliptic Curves"

is the subject of the Third 1997 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R), the European Community under the "Training and Mobilty of Researchers" Programme and the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.), will take place, under the scientific direction of Professor CARLO VIOLA (Università di Pisa) at Grand Hotel San Michele, Cetraro (Cosenza), **from 12 to 19 July, 1997**.

Courses

a) Iwasawa Theory for Elliptic Curves Without Complex Multiplication. (6 lectures in English).
Prof. John COATES (Cambridge University)

Outline:
Let E be an elliptic curve over $\mathbf{Q}$ without complex multiplication, and let p be a prime number. A considerable amount is now known about the Iwasawa theory of E over the cyclotomic $\mathbf{Z}_p$ extension of $\mathbf{Q}$, and some of this material will be discussed in the courses by Greenberg and Rubin. On the other hand, very little is known about the Iwasawa theory of E over the field F_∞ which is obtained by adjoining all p-power division points on E to $\mathbf{Q}$. If G_∞ denotes the Galois group of F_∞ over Q, then G_∞ is an open subgroup of $GL_2(\mathbf{Z}_p)$ by a theorem of Serre, since E is assumed not to have complex multiplication. The course will begin by discussing some of the basic properties of the non-abelian Iwasawa algebra $\Lambda = \mathbf{Z}_p[[G_\infty]]$. It will then give our present fragmentary knowledge of various basic Iwasawa modules over Λ, which arise when one studies the arithmetic of E over F_∞ . Much of the course will be concerned with posing open questions which seem to merit further study.

References

The course will only assume basic results about elliptic curves and their Galois cohomology, most of which are contained in

- J. Silverman, "*The Arithmetic of Elliptic Curves*", GTM 106, (1986), Springer.

b) Iwasawa Theory for Elliptic Curves. (6 lectures in English)
Prof. Ralph GREENBERG (University of Washington, Seattle)

Outline:
This course will present some of the basic results and conjectures concerning the behavior of the Selmer group of an elliptic curve defined over a number field F in a tower of cyclotomic extensions of F. We will mostly discuss the case where the elliptic curve

has good ordinary reduction at the primes of F dividing a rational prime p and the cyclotomic extensions are obtained by adjoin p-power roots of unity of F. One of the main results we will prove is Mazur's "Control Theorem". The proof will depend on a rather simple description of the Selmer group which also will allow us to study specific examples. We will also discuss the "Main Conjecture" which was formulated by Mazur and its relationship to the Birch and Swinnerton-Dyer Conjecture. If time permits, we will discuss generalizations of this Main Conjecture.

References

1) J. Silverman, The Arithmetic of Elliptic Curves, GTM 106, Springer.
2) S. Lang, Cyclotomic Fields I and II, GTM 121, Springer.
3) L. Washington, Introduction to Cyclotomic Fields, GTM 83, Springer.
4) J. P. Serre, Cohomologie galoisienne, LNM 5, Springer.
5) J. Tate, Duality Theorems in Galois Cohomology over Number Fields, Proceedings of the ICM, Stockholm, 1962, pp. 288-295.

In reference 1, one should become familiar with the Tate module of an elliptic curve and reduction modulo a prime p. In references 2 or 3 one can find an introduction to the structure of finitely generated modules over the Iwasawa algebra. It may be useful to consult references 4 and 5 to become familiar with Galois cohomology.

c) Two-dimensional representations of Gal $(\overline{\mathbf{Q}}/\mathbf{Q})$. (6 lectures in English)
Prof. Kenneth A. RIBET (University of California, Berkeley)

Outline:

The general theme of the course is that two-dimensional representations of *Gal* $(\overline{\mathbf{Q}}/\mathbf{Q})$ can be shown to have large images in appropriate circumstances. The lectures will touch on topics to be selected from the following list:

1. *Recent work by Darmon and Merel on modular curves* associated to normalizers of non-split Cartan Subgroups of $\mathbf{GL}(2,\mathbf{Z}/p\mathbf{Z})$. A preprint by these authors proves, among other things, that the Fermat-like equations $x^p + y^p = 2z^p$ has only the trivial non-zero solutions in integers x, y and z when p is an odd prime number! Technically, the main theorem of the paper is an analogue of a result pertaining to split Cartan subgroups of $\mathbf{GL}(2,\mathbf{Z}/p\mathbf{Z})$ which appears in B. Mazur's 1978 paper "*Rational isogenies of prime degree*". The connection with two-dimensional representations of $Gal(\overline{\mathbf{Q}}/\mathbf{Q})$ comes about because Merel has used the Darmon-Merel theorem to prove that the *mod p* representations of $Gal(\overline{\mathbf{Q}}/\mathbf{Q})$ arising from non-CM Frey elliptic curves have large images for $p > 2$. [To read about this work prior to the conference, download Darmon-Merel paper (`http://www.math.mcgill.ca/darmon/pub/Winding/paper.html`) and follow the references given in that article]

.2. *Semistable representations.* In his 1972 article on Galois properties of torsion points of elliptic curves, J.-P. Serre proved that the *mod p* representation of $Gal(\overline{\mathbf{Q}}/\mathbf{Q})$ associated to a semistable elliptic curve over $\mathbf{Q}$ is either reducible or surjective, provided that the prime p is at least 7. In his 1995 Bourbaki seminar on the work of Wiles and Taylor-Wiles, Serre extended his theorem to cover the primes $p = 3,5$. Using group-theoretic results of L. E. Dickson, one may replace $\mathbf{Z}/p\mathbf{Z}$ by a finite etale $\mathbf{Z}/p\mathbf{Z}$ algebra. [See (`ftp://math.berkeley.edu/pub/Preprints/Ken-Ribet/semistable.tex`).].

3. *Adelic representations.* A recent theorem of R. Coleman, B. Kaskel and K. Ribet concerns point P on the modular curve $X_0(37)$ for which the class of the divisor $(P) - (P_\infty)$ has finite order. (Here P_∞ is the "cusp at infinity" on $X_0(37)$.) The theorem states that the only such points are the two cusps on $X_0(37)$. Our proof relies heavily on results of B. Kaskel, who calculated the image of the "adelic" representation $Gal(\overline{\mathbf{Q}}/\mathbf{Q}) - GL(2, R)$ which is defined by the action of $Gal(\overline{\mathbf{Q}}/\mathbf{Q})$ on the torsion points on the Jacobian of $X_0(37)$. Here R is the ring of pairs $(a, b) \in \widehat{Z} \times \widehat{Z}$ for which a and b have the same

parity. In my lectures, I would like to explain how Kaskel arrives at this results. [Although neither Kaskel's work nor the Coleman-Kaskel-Ribert is available at this time, one might look at 6 of Part I of the Lang-Trotter book on "Frobenius distributions" (Lecture Notes in Math, **504**) to get a feel for this kind of study].

4. *l-adic representation attached to modular forms.* Let $f_1, \dots, f_p$ be newforms of weights $k_1, \dots, k_p$ possibly of different levels and characters. Let $E = E_1 \times \cdots \times E_t$ be the product of the number fields generated by the coefficients of the different forms. For each prime , the l-adic representations attached to the different f_i furnish a product representation $\rho_l : Gal(\overline{\mathbf{Q}}/\mathbf{Q}) \to \mathbf{GL}\,(2, E \otimes Q_l)$. In principle, we know how to calculate the image of ρ "up to finite groups", i.e., the Lie algebras of $\rho Gal(\overline{\mathbf{Q}}/\mathbf{Q})$. I hope to explain the answer - and how we know that it is correct. [One might consult my article in Lecture Notes in Math. **601** and my article in Math. Annalen **253** (1980)].

d) Elliptic curves with complex multiplication and the conjecture of Birch and Swinnerton-Dyer. (6 lecture in English)
Prof. Carl RUBIN (Ohio State University)

Outline:

The conjecture of Birch and Swinnerton-Dyer relates the arithmetic of an elliptic curve with the behavior of its L-function. These lectures will give a survey of the state of our knowledge of this conjecture in the case of elliptic curves with complex multiplication, where the results are strongest. Specific topics will include:

1) Elliptic curves with complex multiplication.
2) Descent (following Coates and Wiles).
3) Elliptic units.
4) Euler systems and ideal class groups.
5) Iwasawa theory and the "main conjecture".

References

- Shimura, G., Introduction to the arithmetic theory of automorphic functions, Princeton: Princeton Univ. Press (1971), Chapter 5.
- Silverman, J., Advanced topics in the arithmetic of elliptic curves, Graduate Texts in Math. 151, New York: Springer-Verlag (1994), Chapter II.
- de Shalit, E., Iwasawa theory of elliptic curves with complex multiplication, Perspectives in Math. 3, Orlando: Academic Press (1987).
- Coates, J., Wiles, A., On the conjecture of Birch and Swinnerton-Dyer, Invent. math. 39 (1977) 223-251.
- Rubin, K., The main conjecture. Appendix to: Cyclotomic fields I and II, S. Lang, Graduate Texts in Math. 121, New York: Springer-Verlag (1990) 397-419.
- Rubin, K., The 'main conjectures' of Iwasawa theory for imaginary quadratic fields, Invent. Math. 103 (1991) 25-68.

LIST OF C.I.M.E. SEMINARS			Publisher
1954 -	1.	Analisi funzionale	C.I.M.E.
	2.	Quadratura delle superficie e questioni connesse	"
	3.	Equazioni differenziali non lineari	"
1955 -	4.	Teorema di Riemann-Roch e questioni connesse	"
	5.	Teoria dei numeri	"
	6.	Topologia	"
	7.	Teorie non linearizzate in elasticità, idrodinamica,aerodinamica	"
	8.	Geometria proiettivo-differenziale	"
1956 -	9.	Equazioni alle derivate parziali a caratteristiche reali	"
	10.	Propagazione delle onde elettromagnetiche	"
	11.	Teoria della funzioni di più variabili complesse e delle funzioni automorfe	"
1957 -	12.	Geometria aritmetica e algebrica (2 vol.)	"
	13.	Integrali singolari e questioni connesse	"
	14.	Teoria della turbolenza (2 vol.)	"
1958 -	15.	Vedute e problemi attuali in relatività generale	"
	16.	Problemi di geometria differenziale in grande	"
	17.	Il principio di minimo e le sue applicazioni alle equazioni funzionali	"
1959 -	18.	Induzione e statistica	"
	19.	Teoria algebrica dei meccanismi automatici (2 vol.)	"
	20.	Gruppi, anelli di Lie e teoria della coomologia	"
1960 -	21.	Sistemi dinamici e teoremi ergodici	"
	22.	Forme differenziali e loro integrali	"
1961 -	23.	Geometria del calcolo delle variazioni (2 vol.)	"
	24.	Teoria delle distribuzioni	"
	25.	Onde superficiali	"
1962 -	26.	Topologia differenziale	"
	27.	Autovalori e autosoluzioni	"
	28.	Magnetofluidodinamica	"

1963 -	29. Equazioni differenziali astratte	"
	30. Funzioni e varietà complesse	"
	31. Proprietà di media e teoremi di confronto in Fisica Matematica	"
1964 -	32. Relatività generale	"
	33. Dinamica dei gas rarefatti	"
	34. Alcune questioni di analisi numerica	"
	35. Equazioni differenziali non lineari	"
1965 -	36. Non-linear continuum theories	"
	37. Some aspects of ring theory	"
	38. Mathematical optimization in economics	"
1966 -	39. Calculus of variations	Ed. Cremonese, Firenze
	40. Economia matematica	"
	41. Classi caratteristiche e questioni connesse	"
	42. Some aspects of diffusion theory	"
1967 -	43. Modern questions of celestial mechanics	"
	44. Numerical analysis of partial differential equations	"
	45. Geometry of homogeneous bounded domains	"
1968 -	46. Controllability and observability	"
	47. Pseudo-differential operators	"
	48. Aspects of mathematical logic	"
1969 -	49. Potential theory	"
	50. Non-linear continuum theories in mechanics and physics and their applications	"
	51. Questions of algebraic varieties	"
1970 -	52. Relativistic fluid dynamics	"
	53. Theory of group representations and Fourier analysis	"
	54. Functional equations and inequalities	"
	55. Problems in non-linear analysis	"
1971 -	56. Stereodynamics	"
	57. Constructive aspects of functional analysis (2 vol.)	"
	58. Categories and commutative algebra	"

1972 -	59. Non-linear mechanics		"
	60. Finite geometric structures and their applications		"
	61. Geometric measure theory and minimal surfaces		"
1973 -	62. Complex analysis		"
	63. New variational techniques in mathematical physics		"
	64. Spectral analysis		"
1974 -	65. Stability problems		"
	66. Singularities of analytic spaces		"
	67. Eigenvalues of non linear problems		"
1975 -	68. Theoretical computer sciences		"
	69. Model theory and applications		"
	70. Differential operators and manifolds		"
1976 -	71. Statistical Mechanics		Ed Liguori, Napoli
	72. Hyperbolicity		"
	73. Differential topology		"
1977 -	74. Materials with memory		"
	75. Pseudodifferential operators with applications		"
	76. Algebraic surfaces		"
1978 -	77. Stochastic differential equations		"
	78. Dynamical systems		Ed Liguori, Napoli and Birhäuser Verlag
1979 -	79. Recursion theory and computational complexity		"
	80. Mathematics of biology		"
1980 -	81. Wave propagation		"
	82. Harmonic analysis and group representations		"
	83. Matroid theory and its applications		"
1981 -	84. Kinetic Theories and the Boltzmann Equation	(LNM 1048)	Springer-Verlag
	85. Algebraic Threefolds	(LNM 947)	"
	86. Nonlinear Filtering and Stochastic Control	(LNM 972)	"
1982 -	87. Invariant Theory	(LNM 996)	"
	88. Thermodynamics and Constitutive Equations	(LN Physics 228)	"
	89. Fluid Dynamics	(LNM 1047)	"

1983 -	90. Complete Intersections	(LNM 1092)	Springer-Verlag
	91. Bifurcation Theory and Applications	(LNM 1057)	"
	92. Numerical Methods in Fluid Dynamics	(LNM 1127)	"
1984 -	93. Harmonic Mappings and Minimal Immersions	(LNM 1161)	"
	94. Schrödinger Operators	(LNM 1159)	"
	95. Buildings and the Geometry of Diagrams	(LNM 1181)	"
1985 -	96. Probability and Analysis	(LNM 1206)	"
	97. Some Problems in Nonlinear Diffusion	(LNM 1224)	"
	98. Theory of Moduli	(LNM 1337)	"
1986 -	99. Inverse Problems	(LNM 1225)	"
	100. Mathematical Economics	(LNM 1330)	"
	101. Combinatorial Optimization	(LNM 1403)	"
1987 -	102. Relativistic Fluid Dynamics	(LNM 1385)	"
	103. Topics in Calculus of Variations	(LNM 1365)	"
1988 -	104. Logic and Computer Science	(LNM 1429)	"
	105. Global Geometry and Mathematical Physics	(LNM 1451)	"
1989 -	106. Methods of nonconvex analysis	(LNM 1446)	"
	107. Microlocal Analysis and Applications	(LNM 1495)	"
1990 -	108. Geometric Topology: Recent Developments	(LNM 1504)	"
	109. H_∞ Control Theory	(LNM 1496)	"
	110. Mathematical Modelling of Industrial Processes	(LNM 1521)	"
1991 -	111. Topological Methods for Ordinary Differential Equations	(LNM 1537)	"
	112. Arithmetic Algebraic Geometry	(LNM 1553)	"
	113. Transition to Chaos in Classical and Quantum Mechanics	(LNM 1589)	"
1992 -	114. Dirichlet Forms	(LNM 1563)	"
	115. D-Modules, Representation Theory, and Quantum Groups	(LNM 1565)	"
	116. Nonequilibrium Problems in Many-Particle Systems	(LNM 1551)	"

1993 -	117. Integrable Systems and Quantum Groups	(LNM 1620)	Springer-Verlag
	118. Algebraic Cycles and Hodge Theory	(LNM 1594)	
	119. Phase Transitions and Hysteresis	(LNM 1584)	"
1994 -	120. Recent Mathematical Methods in Nonlinear Wave Propagation	(LNM 1640)	"
	121. Dynamical Systems	(LNM 1609)	"
	122. Transcendental Methods in Algebraic Geometry	(LNM 1646)	"
1995 -	123. Probabilistic Models for Nonlinear PDE's	(LNM 1627)	"
	124. Viscosity Solutions and Applications	(LNM 1660)	"
	125. Vector Bundles on Curves. New Directions	(LNM 1649)	"
1996 -	126. Integral Geometry, Radon Transforms and Complex Analysis	to appear	"
	127. Calculus of Variations and Geometric Evolution Problems	to appear	"
	128. Financial Mathematics	(LNM 1656)	"
1997 -	129. Mathematics Inspired by Biology	to appear	"
	130. Advanced Numerical Approximation of Nonlinear Hyperbolic Equations	to appear	"
	131. Arithmetic Theory of Elliptic Curves	to appear	"
	132. Quantum Cohomology	to appear	"