

C.I.M.E. Session on "Probabilistic Models for Nonlinear PDE's and Numerical Applications"

List of participants

- O. ARCUDI, Dipartimento di Matematica, Via Belzoni 7, 35131 Padova, Italy
- M. BEN ALAYA, CERMICS-ENPC, La Courtine, 93167 Noisy le Grand cedex, France
- P. BERNARD, Laboratoire de Math. Appl., Les Cezeaux, 63177 Aubière cedex, France
- M. BOSSY, INRIA, 2004 route de Lucioles, BP 63, 06902 Sophia Antipolis cedex, France
- PH. BRIAND, Laboratoire de Math. Appl., Univ. Blaise Pascal, 63177 Aubière cedex, France
- B. CADRE, IRMAR, Univ. de Rennes I, Campus de Beaulieu, 35042 Rennes cedex, France
- L. CAMELLINO, Dipartimento di Matematica, Univ. di Tor Vergata, Via della Ricerca Scientifica, 00133 Roma, Italy
- D. CHEVANCE, INRIA, 2004 route des Lucioles, BP 93, 06902 Sophia Antipolis cedex, France
- J.M.C. CLARK, Dept. of Electrical and Electronic Eng., Imperial College, Exhibition Road, London SW7 2BT, UK
- F. COQUET, IRMAR, Univ. de Rennes I, Campus de Beaulieu, 35042 Rennes cedex, France
- R. DI LISIO, Dipartimento di Matematica, Univ. La Sapienza, P.le Aldo Moro 2, 00185 Roma, Italy
- T. FUJIWARA, Dept. of Math., Hyogo Univ. of Teacher Education, Yashiro, Hyogo, 673-14 Japan
- G. GIACOMIN, Inst. f. Ang. Math., Univ. Zurich, Winterthurer Str. 190, CH-8057 Zurich 7
- A. GRORUD, C.M.I. Univ. de Provence, 39 rue Joliot-Curie, 13453 Marseille cedex 13, France
- E.I. HAUSENBLAS ERIKA, Institute of Mathematics, Hellbrunnerstr. 34, A-5020 Salzburg
- A. JAKUBOWSKI, Faculty of Math. and Info., Nicholas Copernicus Univ., ul. Chopina 12/18, 87-100 Torun, Poland
- V. KATSOUROS, Dept. of Electrical and Electronic Eng., Imperial College, Exhibition Road, London SW7 2BT, UK
- D. LEPINGLE, Mathématiques, Univ. d'Orléans, B.P. 6759, 45067 Orléans cedex, France
- Z. LOZANOV CRVENKOVIC, Inst. of Math., Univ. of Novi Sad, Trg D. Obradovica 4, 21000 Novi Sad, Yugoslavia
- A. MORO, Dipartimento Statistico, Viale Morgagni 59, 50134 Firenze, Italy
- B. PACCHIAROTTI, Dip.to di Matematica, Univ. La Sapienza, P.le Aldo Moro 2, 00185 Roma, Italy
- L. PARESCHI, Dipartimento di Matematica, Piazza di Porta S. Donato 5, 40127 Bologna, Italy
- A. RAMPONI, Dipartimento di Matematica, Univ. di Tor Vergata, Via della Ricerca Scientifica, 00133 Roma, Italy
- H. REGNIER, INRIA, 2004 route des Lucioles, BP 63, 06902 Sophia Antipolis cedex, France
- V. RICCI, Dipartimento di Matematica, Univ. La Sapienza, Piazzale A. Moro 2, 00185 Roma, Italy

A. ROZKOSZ, Faculty of Math. and Info., Nicholas Copernicus Univ., ul. Chopina 12/18,  
87-100 Torun, Poland

P. SEUMEN TONOU, INRIA, 2004 route des Lucioles, BP 63, 06902 Sophia Antipolis cedex, France

L. SLOMINSKI, Faculty of Math. and Info., Nicholas Copernicus Univ., ul. Chopina 12/18,  
87-100 Torun, Poland

M.B. ZAVELANI ROSSI, Dipartimento di Matematica, Univ. La Sapienza, P.le A. Moro 2,  
00185 Roma, Italy

M. ZERVOS, Dept. of Electrical and Electronic Eng., Imperial College, Exhibition Road,  
London SW7 2BT, UK

FONDAZIONE C.I.M.E.  
CENTRO INTERNAZIONALE MATEMATICO ESTIVO  
INTERNATIONAL MATHEMATICAL SUMMER CENTER

**“Integral Geometry, Radon Transforms  
and Complex Analysis”**

is the subject of the first 1996 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R.) and the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.), will take place, under the scientific direction of Professor ENRICO CASADIO TARABUSI (Università di Roma “La Sapienza”), Professor MASSIMO PICARDELLO (Università di Roma “Tor Vergata”) and Professor GIUSEPPE ZAMPIERI (Università di Padova), in Venezia, **from 3 to 12 June 1996**.

**Courses**

**a) Radon Transforms, Wavelets and Applications.** (6 lectures in English).

Prof. C. A. BERENSTEIN (University of Maryland, College Park, USA)

- 1) The use of wavelets to localize the 2-dimensional Radon Transform. Application to Computerized Axial Tomography. Possible extensions to Nuclear Magnetic Resonance and comparison with  $\Lambda$ -tomography. Open problems for the exterior Radon Transforms.
- 2) Edge detections and singularities of functions in terms of their Radon transforms. Applications to the work of Boman-Quinto (Theorem of Hörmander- Kashiwara-Kawai).
- 3) The inverse conductivity problem and the hyperbolic Radon transform. Inversion, range, relation with the spherical transform.

**b) Holomorphic mappings between real analytic submanifolds in  $C^N$**

Prof. P. EBENFELT (University of California, San Diego)

*Summary:* Let  $M$  and  $M'$  be real analytic submanifolds in  $C^N$ , and let  $p_0 \in M$  and  $p'_0 \in M'$ . We shall study the class  $C$  of holomorphic mappings  $H$  from the neighborhood of  $p_0$  (in  $C^N$ ) into  $C^N$  such that  $H(p_0) = p'_0$ ,  $H(M) \subset M'$  and such that  $\text{Jac}(H) \neq 0$  on  $M$ . Properties of this class depend intimately on the geometric properties of  $M$  and  $M'$  near the points  $p_0$  and  $p'_0$  respectively. We shall define the notion of CR (Cauchy-Riemann), finite type and minimality, holomorphic nondegeneracy, etc. We shall also introduce a new invariant sequence of sets, called *the Segre sets*, attached to  $M$  at  $p_0$  that is useful in analyzing the class  $C$  and that provides a new characterization of finite type. One of the main results presented here gives essentially necessary and sufficient conditions on  $M$ , assuming that  $M$  and  $M'$  are *real algebraic*, such that every holomorphic mapping in  $C$  is *algebraic*. We shall also give results and state open problems in the general case (i.e.  $M$  and  $M'$  are real analytic) as well as consider some variations of the situation described above.

**c) Complex Integral Geometry, d-bar Cohomology, Representations.** (6 lectures in English)

Prof. S. GINDIKIN (Rutgers University, New Brunswick, USA)

- 1) Admissible complexes of lines in  $C^3$  and  $SL(2, C)$ .
- 2) The Gelfand-Graev-Shapiro operator  $k$ .
- 3) Integral Geometry on  $SL(n, C)$ .
- 4) Rational Curves and twistors.

- 5) Integral geometry and Plancherel formula on complex semi-simple Lie groups.
- 6) Real integral geometry and  $\bar{\partial}$ -cohomology on tubes. Connection with hyperfunctions.
- 7) The Penrose transform and holomorphic language for  $\bar{\partial}$ -cohomology.

The course will be concentrated around two fundamental concepts of integral geometry:

- universal structure of local inversion formulas and the operator  $\mathcal{K}$ ;
- integral geometrical language for  $\bar{\partial}$ -cohomology and hyperfunctions.

Both these concepts have essential connections with the theory of representations.

**d) Analytic and Group-theoretic Methods in Integral Geometry.** (6 lectures in English)

Prof. S. HELGASON (MIT, Cambridge, USA)

Radon transforms for double fibrations. Inversion and range problems. Radon transform on symmetric spaces. Various applications to differential equations, solvability results, wave equations on symmetric spaces, including systems of multi-temporal wave equations. The Poisson integral as a Radon transform with integral geometric interpretation of the Hua equations on bounded symmetric domains. Bundle-valued Radon transforms.

**e) Singular Radon Transforms** (6 lectures in English)

Prof. E. M. STEIN (Princeton University, Princeton, USA)

- 1) The background: Euclidean case.
- 2) Singular Radon transforms and the Heisenberg group.
- 3) Relations with the  $\bar{\partial}$ -Neumann problem.
- 4) Other directions: nilpotent groups and discrete analogues.

**f) Analytic discs and the theory of CR functions** (6 lectures in English)

Prof. A. TUMANOV (University of Illinois, Urbana-Champaign, USA)

- 1) Introduction to the theory of CR functions: the classical theorems on the holomorphic extendibility of CR functions on hypersurfaces in complex space by Hartogs and Bochner and H. Lewy. The approximation theorem by Baouendi and Treves. Constructions of analytic discs with boundaries in a prescribed generic manifold in complex space. Bishop's equation. Extending CR functions to analytic discs.
- 2) Geometry of analytic discs with boundaries in a generic manifold in complex space. The defect of a disc. Minimal CR manifolds. Extending CR functions to wedges. Extending CR functions from manifolds with boundaries and edges.
- 3) Propagation of extendibility of CR functions: the conormal bundle of a generic manifold. The CR structure in the conormal bundle. Propagation of extendibility of CR functions along CR orbits. Application: Trepreau's example of a CR function that cannot be represented as a sum of boundary values of holomorphic functions.
- 4) Other applications of analytic discs: Extending CR function from globally minimal manifolds. A Morera theorem for CR functions. Regularity of CR mappings.

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## “Calculus of Variations and Geometric Evolution Problems”

is the subject of the second 1996 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R.) and the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.), will take place, under the scientific direction of Professors STEFAN HILDEBRANDT (Universität Bonn) and MICHAEL STRUWE (ETH-Zentrum, Zürich) at Grand Hotel San Michele, Cetraro (Cosenza), from 15 to 22 June, 1996.

### Courses

- a) **Variational methods for Ginzburg-Landau equations** (6 lectures in English)  
Prof. Fabrice BETHUEL (E.N.S., Cachan)

Ginzburg-Landau models arise in various areas of physics. In their simplest form, they involve a functional of the type

$$E_{\varepsilon}(u) = \frac{1}{2} \int_{\Omega} |\nabla u|^2 + \frac{1}{2\varepsilon^2} (1 - |u|^2)^2$$

where  $\varepsilon$  is some positive parameter,  $\Omega$  a domain in  $\mathbb{R}^2$  and  $u$  a complex valued map from  $\Omega$  to  $\mathbb{C}$ . We will be interested in the case  $\varepsilon$  is small, so that in the limit  $\varepsilon \rightarrow 0$ , we have to deal with  $S^1$ -valued maps, and corresponding topological defects. The aim of the course is to present recent developments concerning stationary maps for  $E_{\varepsilon}$ , as well as some dynamical properties. The emphasis will be put on the notion of renormalized energies, defined on a space of configuration of particles of positive and negative charges.

1. Different models from physics: superconductivity, abelian-Higgs models, Gross-Pitaevskii equations
2. The Dirichlet problem: asymptotic behavior of minimizers, renormalized energies
3. Non minimizing solutions: Morse theory and topological methods
4. Problems with symmetries: use of an  $S^1$ -index
5. Dynamics

### References

- [1] For 1) one may usefully consult  
D. Saint James, C. Sarma and E. J. Thomas, Type II Superconductivity, Pergamon Press, New York and Oxford (1969).  
A. Jaffe and C. Taubes, Vortices and Monopoles, Birkhäuser, Boston, (1980).
- [2] 2) relies most on results in  
F. Bethuel, H. Brézis et F. Hélein, Ginzburg-Landau vortices, Birkhauser 1994. See also  
M. Struwe, On the asymptotic behavior of minimizers of the Ginzburg-Landau model in 2-dimensions, J. Differential Integral Equations 7 (1994) Erratum 8 (1995)
- [3] For 3) a good introduction to Morse theory can be found in  
M. Struwe, Variational methods, Springer (1990) or  
P. Rabinowitz, Minimax methods in critical point theory with applications to differential equations, CBMS Regional Conference Series Math. 65, Amer. Math. Soc., Providence 1986.

The results presented are taken mainly from

L. Almeida and F. Béthuel, Topological methods for the Ginzburg-Landau equation, C.R. Acad. Sci. Paris **320** (1995), 935-938 and detailed paper to appear.

[4] For 4) Morse theory with symmetries is presented for instance in

T. Bartsch, Topological methods for variational problems with symmetries, Springer (1993).

The results presented are taken from

L. Almeida and F. Béthuel, Multiplicity for the Ginzburg-Landau equation in presence of symmetries, to appear in Houston J. of Math.

[5] Finally for 5) we will present results of

L. Pismen and J. Rubinstein, Notion of vortex lines in the Ginzburg-Landau model, Physica D47 (1991), 353-360 and

F. H. Lin, Some dynamical properties of Ginzburg-Landau vortices, to appear in CPAM

**b) Diffusion in science and geometry** (6 lectures in English)

Prof. R. HAMILTON (University of California, San Diego)

The course will discuss various diffusion processes which arise in science and geometry, and the parabolic partial differential equations describing them, including the Heat Equation, Diffusion Reaction Equations, the Porous Medium Equation and the Motion of a Surface by its Mean Curvature or by its Gauss Curvature.

The course will also cover the interesting phenomena which arise in these processes, including speed of propagation, travelling wave solutions, formation of singularities and continuation through singularities.

**c) Variational models for microstructure and phase transitions** (6 lectures in English)

Prof. Stefan MÜLLER (ETH-Zentrum, Zürich)

Elastic crystals and other physical systems that undergo phase transitions often exhibit a complex microgeometry of different phases. In the last ten years important progress has been made to understand such microstructures as minimizers or almost minimizers of suitable energy functionals. The analysis of these variational problems leads to easily stated but rather deep mathematical problems that involve an interplay of different mathematical areas such as the calculus of variations, nonlinear partial differential equations, differential geometry and functional analysis.

In the lectures I will first describe the setting and the basic questions and then discuss some recent progress in the area. The prerequisites are some familiarity with Sobolev spaces and weak convergence. No knowledge of continuum mechanics is assumed.

Since the subject is rather young there is no textbook presentation available. References [1] and [2] below are two research papers which fundamentally influenced the subject. Their reading is tough at times (and not required as prerequisite) but certainly very rewarding. Sverak's ICM article is a very compact survey of current results and interconnections between questions in different areas of mathematics. References [4] and [5] address, in the context of simple model problems, the special topic of regularizing perturbations and their effect on the geometry of the microstructure. All of the above contain plenty of references for further study.

**References**

- [1] J. M. Ball and R. D. James, Fine phase mixtures as minimizers of energy, Arch. Rat. Mech. Anal. **100** (1987), pp. 13-52.
- [2] J. M. Ball and R. D. James, Proposed tests of a theory of fine microstructure and the two-well problem, Phil. Trans. R. Soc. London A **338** (1992), pp. 389-450.
- [3] V. Sverak, Lower semicontinuity of variational integrals and compensated compactness, to appear in: Proc. Int. Congress Math. (Zürich 1994), Birkhäuser.
- [4] S. Müller, Singular perturbations as a selection criterion for periodic minimizing sequences, Calculus of Variations **1** (1993), pp. 169-204.
- [5] R. V. Kohn and S. Müller, Surface energy and microstructure in coherent phase transitions, Comm. Pure Appl. Math. **47** (1994), pp. 405-435.

**d) Parametric surfaces of prescribed mean curvature** (6 lectures in English)

Prof. Klaus STEFFEN (Universität Düsseldorf)

In these lectures we treat the following existence problem. For a given function  $H : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$  and a given  $(n-1)$ -dimensional boundary contour  $\Gamma$  in  $\mathbb{R}^{n+1}$  to find an oriented  $n$ -dimensional surface in  $\mathbb{R}^{n+1}$  bordered by  $\Gamma$  with

prescribed mean curvature  $H$ , i.e. the mean curvature of the surface at each point  $x$  of its support is  $H(x)$ . The problem will be considered first in the setting of 2-dimensional parametric surfaces  $x: R^2 \supset D \rightarrow R^3$ . For  $H \equiv 0$  this is the classical Plateau problem. For  $H \neq 0$  it is known that the problem does not always have a solution and various sufficient conditions on  $H$  and  $\Gamma$  have been given in the literature. For example if  $\sup |H|$  is sufficiently small in terms of geometric quantities associated with  $\Gamma$ , then a solution exists.

Using the direct method of the Calculus of Variations one finds that a sufficient condition for existence (and also a necessary one for the method) is an isoperimetric condition of the type  $\left| \int_A H dx \right| \leq cP(A)$

with  $c < 1/n$  for all sets  $A \subset R^{n+1}$  with finite perimeter  $P(A)$ . Such isoperimetric conditions can be deduced in various ways from the isoperimetric inequality and they can be employed to give a unified treatment of the existence theory.

The method of isoperimetric conditions is also suitable to treat the existence problem in the setting of Geometric Measure Theory. One then obtains hypersurfaces of prescribed mean curvature in  $R^{n+1}$  with certain possible singularities if  $n \geq 7$ . Moreover, one can study the problem in an ambient Riemannian manifold  $M$  instead of  $R^{n+1}$ . Here various isoperimetric inequalities enter and new phenomena then arise because there are boundaries  $\Gamma \subset M$  which do not border a hypersurface of constant mean curvature  $H \neq 0$  in  $M$ . It is thus interesting to exhibit conditions on which ensure the solvability of the problem at least for sufficiently small  $|H|$  and then to find reasonable geometric bounds on the range of the functions  $H$  which can be admitted. Some recent results in this direction will be discussed and applied also to the setting of 2-dimensional parametric surfaces in a 3-dimensional Riemannian manifold.

#### References

- Dierkes, U., Hildebrandt, S., Kuster, A., Wohlrab, O.: Minimal Surfaces Vol. 1, Vol. 2. Grundlehren math. Wiss. 295, 296. Springer-Verlag, Berlin-Heidelberg-New York, 1992.
- Duzaar, F.: On the existence of surfaces with prescribed mean curvature and boundary in higher dimensions. Ann. Inst. Henri Poincaré (Anal. Non Lin.) **10** (1993), 191-214.
- Duzaar, F., Steffen, K.:  $\lambda$ -minimizing currents. Manuscr. Math. **80** (1993), 403-447.
- Federer, H.: Geometric measure theory. Springer Verlag, Berlin-Heidelberg-New York, 1969.
- Steffen, K.: Isoperimetric inequalities and the problem of Plateau. Math. Ann. **222** (1976), 97-144.
- Struwe, M.: Plateau's problem and the calculus of variations. Mathematical Notes 35, Princeton University Press, Princeton 1988.

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**“Financial Mathematics”**

is the subject of the third 1996 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R.) and the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.), will take place, under the scientific direction of Professor WOLFGANG RUNGGALDIER (Università di Padova) in Bressanone, **from 8 to 13 July, 1996.**

**Courses**

- a) **Market microstructure and adverse selection** (5 lectures in English)  
Prof. Bruno BIAIS and Prof. Jean Charles ROCHET (Université de Toulouse)

**1. Outline**

The course will study how private information is incorporated into asset prices. We will start with competitive agents and exogenous market structure. Then we will relax the competitiveness assumption. In the third part we will relax the exogenous market structure assumption and look for optimal trading mechanisms.

- 1.1 Information revelation with competitive agents  
Grossman and Stiglitz, 1980
- 1.2 Information revelation with strategic agents  
Kyle, 1985; Kyle, 1989; Biais, Hillion, 1994
- 1.3 Optimal mechanisms  
Rochet, Vila, 1993; Glosten, 1989; Biais, Bossaerts, Rochet, 1995

**2. References**

Biais, B., 1994, Insider and liquidity trading in stock and options markets, *Review of Financial Studies*, 743--781  
Biais, B., P. Bossaerts, and J.C. Rochet, 1995 An optimal mechanism to sell unseasoned equity, mimeo, Université de Toulouse.  
Glosten, L., 1989, Insider trading, liquidity and the role of the monopolist specialist, *Journal of Business*, 211--235  
Grossman, S., and J. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review*, 393--408  
Kyle, A. S, 1985, Continuous auctions and insider trading, *Econometrica*, 1315--1335  
Kyle, A. S, 1989, Informed speculation with imperfect competition, *Review of Economic Studies*, 317--356  
Rochet, J.C., and J.L. Vila, Insider trading without normality, *Review of Financial Studies*, 131--152

- b) **An introduction to the theory of interest rate derivatives** (5 lectures in English)  
Prof. Tomas BJÖRK (Stockholm School of Economics)

1. The term structure of interest rates. Classical approach to interest rate derivatives via locally riskless portfolios. Incomplete markets. The basic PDE. Market price of risk. Martingale formulation.
2. Interest rate models under Q-measure. Affine term structures. Fitting the yield curve. Hull & White. Heath-Jarrow-Morton.
3. Forward measures. Computation of interest rate options. Forwards, futures and swaps.
4. Extensions to models including jumps. (i.e. Shirakawa and BKR.)

**References**

[1] Artzner, P. & Delbaen, F. (1989) Term Structure of Interest Rates: the Martingale Approach. *Advances in Appl. Math.* 10, 95-129. Björk, T. (1995) in The term structure of discontinuous interest rates. Working Paper, Royal Institute of Technology, Stockholm. Forthcoming.



- [2] Björk, T. & Kabanov, Y. & Runggaldier, W. (1995) Bond market structure in the presence of marked point processes. Working Paper, Stockholm School of Economics.
- [3] Cox, J. & Ingersoll, J. & Ross, S. (1985) A Theory of the Term Structure of Interest Rates. *Econometrica* 53, 385-408.
- [4] Geman, H. & El Karoui, N. & Rochet, J.-C. (1995) Changes of numeraire, changes of probability measure and option pricing. *J. Appl. Prob.* 32, 443-458.
- [5] Heath, D. & Jarrow, R. & Morton, A. (1987) Bond Pricing and the Term Structure of Interest Rates. *Econometrica* 60:1, 77-106.
- [6] Ho, T. & Lee, S. (1986) Term Structure Movements and Pricing Interest Rate Contingent Claims. *Journal of Finance* 41, 1011-1029.
- [7] Hull, J. & White, A. (1990) Pricing Interest-Rate Derivative Securities. *The Review of Financial Studies* 3:4, 573-592.
- [8] Jamshidian, F. (1989) An Exact Bond Option Formula. *Journal of Finance*. 44, 205-209.
- [9] Shirakawa, H. (1991) Interest rate options pricing with Poisson-Gaussian forward rate curve processes. *Mathematical finance*. 1, 77-94.
- [10] Vasicek, O. (1977) An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics*. 5, 177-188.

**c) Optimal trading under constraints (6 lectures in English)**  
 Prof. Jaksá CVITANIC (Columbia University, New York)

- continuous-time, Ito diffusions model for the financial market: linear Stochastic Differential Equations; the problem of hedging (super-replication) of contingent claims; the fair price of a claim in a complete market; martingale approach to Black-Scholes; martingale representation theorem, Feynman-Kac Partial Differential Equation.
- incomplete markets and markets with convex portfolio constraints: no short-selling, no borrowing, etc.; no-arbitrage upper and lower bounds for the price; convex duality, support functions, dynamic programming, variational inequalities.
- the problem of utility maximization from consumption and terminal wealth in a complete market and markets with constraints; a dual problem; existence of optimal portfolios, examples with explicit solutions; Hamilton-Jacobi-Bellman Partial Differential Equations.
- the problem of "drawdown constraints": how to invest optimally while always having more than a given fraction of your wealth's running maximum.
- hedging for a "large investor": nonlinear price equations, prices influenced by investor's strategy; Forward-Backward Stochastic Differential Equations, nonlinear Feynman-Kac Partial Differential Equations.
- markets with transaction costs: hedging and utility maximization; non-existence of nontrivial hedging portfolios.

**Bibliography**

- CVITANIC, J. & KARATZAS, I. (1992) Convex duality in constrained portfolio optimization. *Ann Appl Probab* 2, 767-818.
- CVITANIC, J. & KARATZAS, I. (1993) Hedging contingent claims with constrained portfolios. *Ann Appl Probab* 3, 652-681.
- CVITANIC, J. & KARATZAS, I. (1995) On portfolio optimization under drawdown constraints. *IMA Volumes in Mathematics and its Applications* 65, 35-46.
- DAVIS, M.H.A & NORMAN, A. (1990) Portfolio selection with transaction costs. *Math Operations Research* 15, 676-713.
- GROSSMAN, S.J. & ZHOU, Z. (1993) Optimal investment strategies for controlling drawdowns. *Math Finance* 3, 241-276.
- KARATZAS, I. (1989) Optimization problems in the theory of continuous trading. *SIAM J. Control Optim.* 27, 1221-1259.
- KARATZAS, I. & SHREVE, S. (1991) Brownian Motion and Stochastic Calculus (2nd edition). Springer-Verlag, New York.
- MA, J., PROTTER, P. & YONG, J. (1994) Solving Forward-Backward Stochastic Differential Equations explicitly - a four step scheme. *Probab Theory and Related Fields* 98, 339-359.
- SONER, H.M., SHREVE, S. & CVITANIC, J. (1995) There is no nontrivial hedging portfolio for option pricing with transaction costs. *Ann. Appl. Probab.* 5, 327-355.

**d) Backwards stochastic differential equations. Finance and optimization. (6 lectures in English)**  
 Prof. Nicole EL KAROUTI (Université de Paris VI)

- 1) Backwards stochastic differential equations (BSDE)
  - Existence and uniqueness, a priori estimates, linear BSDE
  - A comparison theorem
  - Dependence upon parameters; continuity, differentiability
- 2) Concave BSDE's and associated control problem
  - BSDE as minimum or minimax
  - Concave BSDE's as value function of a control problem
  - European option pricing with convex bounded portfolio constraints
  - Recursive utility : classical properties and variational

- Consol rate, swap rates, and credit quality
  - 3) The Markovian case
    - Markov properties of solutions of BSDE's
    - BSDE's and non-linear partial differential equations
    - Applications to non-linear pricing
    - Simulations of BSDE's
  - 4) The problem of recursive utility maximization from consumption and terminal wealth
    - a maximum principle
  - 5) Forward-Backward Stochastic Differential Equations
    - Existence and uniqueness
    - nonlinear Feynman-Kac partial differential equations
    - optimal dynamics and dual process
    - hedging for a "large investor" (Cvitanic)
  - 6) Reflected BSDE's and non-linear pricing of American options.
- e) Market imperfections and non-linear pricing rules** (6 lectures in English)  
 Prof. Elyés JOUINI (ENSAE, Malakoff, France)
- 1) Martingales, arbitrage and equilibrium in securities markets with imperfections (sub-linear pricing rules).
  - 2) Arbitrage prices in securities markets with shortsales constraints and different borrowing and lending rate (the discrete and continuous cases, numerical results).
  - 3) Stochastic dominance, portfolio choices and efficient strategies with market frictions.
  - 4) Stationarity and arbitrage, characterization of the martingale measures.
  - 5) Incomplete markets and transaction costs : optimal portfolio choice.
  - 6) Fixed costs and absence of arbitrage opportunities.

### Reference

- Bensaid,B., H. Pages, J.-P. Lesne and J. Scheinkman, "Derivative asset pricing with transaction costs" *Mathematical Finance*, 1992, 2, 63-86.
- Carassus, L. and E. Jouini, "Investment opportunities, short-sales constraints and arbitrage opportunities", Working Paper, CREST, 1995.
- Cvitanic, J. and I. Karatzas, "Hedging contingent claims with constraint portfolios", *Ann. Appl. Probab.* 1993, 3, 652-681.
- Harrison, J. and D. Kreps, "Martingales and arbitrage in multi-period securities markets", *J. Economic Th.*, 1979, 20, 381-408.
- Jouini, E. "Imperfections de marché et arbitrage", to appear in Y.Simon (Ed.), *Encyclopédie des marchés financiers*, Economica.
- Jouini,E. and H. Kallal, "Martingales and arbitrage in securities markets with transaction costs", *J. Economic Th.*, 1995, 66(1), 178-197.
- Jouini,E. and H. Kallal, "Arbitrage in securities markets with short-sales constraints", *Mathematical Finance*, 1995, 5(3), 197-232.
- Jouini,E. and H. Kallal, "Efficient trading strategies in the presence of market frictions", Working Paper, CREST, 1995.
- Jouini,E. and H. Kallal, "Equilibrium in securities markets with bid-ask spreads", Working Paper, CREST, 1995.
- Jouini,E., P.-F. Koehl and N. Touzi, "Incomplete markets, transaction costs and liquidity effects", Working Paper, CREST, 1995.

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