Appendix A
Keynotes

A.1 Cluster Analysis Reader's Digest

Clustering methods have been and are still the object of applied and theoretical research in many different fields such as statistical pattern recognition, data mining, image processing, biomedical sciences, etc. It is not the aim of this section to present a complete overview of clustering techniques, but rather to provide enough information to justify why a particular technique may be preferred (there is no universal best clustering algorithm, and choices and compromises have to be made). A good review of clustering techniques by Jain *et al.*, from a statistical pattern recognition viewpoint, can be found in [93]. The main concepts can also be found in Duda and Hart [60], Hastie *et al.* [82] and Kaufman and Rousseeuw [96] textbooks.

Most of the clustering algorithms are either partitional, or hierarchical methods. While partitional methods produce a single partition, hierarchical methods produce a nested series of partitions. In this sense, they provide a totally different data description and should not be considered as two competing techniques. However, as shall be seen, because of their different nature, the corresponding strategies for cluster validity assessment may be quite different.

A.1.1 Partitional Clustering Methods

Let us denote by $\mathcal{T} = \{T_k,\ k \in \{1, \ldots, M\}\}$ the data set where each pattern T_k is a D-dimensional feature vector, and by $d_T : \mathcal{T} \times \mathcal{T} \rightarrow \mathbb{R}^+$ the dissimilarity measure. Assuming for the moment that the partition size c is given, the goal of a partitional clustering algorithm is to identify the partition $\mathcal{P}(\mathcal{T}) = \{\mathcal{T}_1, \ldots, \mathcal{T}_c\}$ on $\mathcal{T}$ that optimizes a criterion function. Parametric methods as mixture decomposition will not be addressed here since there is no a *a priori* knowledge on the underlying probability distribution. (In these methods, the data set is assumed to be drawn from a mixture of c underlying parametric distributions, and the goal is to determine

the involved parameters. The standard algorithm is the Expectation-Maximization algorithm [46].) Hence, since there are approximately $c^M/c!$ ways of partitioning a set of M elements into c subsets (a Stirling number of the second kind), optimizing the criterion function by exhaustive search is intractable and iterative optimization procedures are needed.

The simplest and most widely used family of criteria function is the one of related minimum variance criteria [60, 96]. The energy to be minimized here is

$$E = \frac{1}{2} \sum_{m=1}^{c} n_m \langle d_m \rangle,$$

where n_m is the number of points in the m-th cluster, and

$$\langle d_m \rangle = \frac{1}{n_m^2} \sum_{T_i \in \mathcal{T}_m} \sum_{T_j \in \mathcal{T}_m} d_T(T_i, T_j)$$

is the average dissimilarity measure between points in the m-th cluster. If $\mathcal{T}$ was a subset of a vector space, and d_T was the squared Euclidean distance, the resulting criteria would be the sum of variances of each cluster,

$$\sum_{m=1}^{c} \sum_{T \in \mathcal{T}_m} \|T - \langle T_m \rangle\|_2^2, \quad \text{where } \langle T_m \rangle = \frac{1}{n_m} \sum_{T \in \mathcal{T}_m} T.$$

Strictly speaking, this criterion only makes sense when clusters are isotropic, multivariate normally distributed. Moreover, the solution is not invariant to linear transformations of the data. Many variations on this method exists, taking any Minkowski metric or the squared Mahalanobis distance instead of the squared Euclidean distance [93]. Notice however that all of these methods are based on the notions of medoid or centroid (barycenter) of a set of points and this does not make sense unless patterns live in a vector space.

Related minimum variance criteria suffer from the problem that partitions that split large clusters may be favored over ones that maintain the integrity of natural clusters [60]. When natural clusters have very different number of points, the partition minimizing this criteria may not reveal the intrinsic structure of the data. Another weakness of these methods is the lack of ability to extract a very dense cluster embedded in the center of a diffuse cluster. Besides, the partition solution has to be found by iterative optimization procedures. These iterative procedures are to be initialized by a reasonable initial partition and solution can be trapped in local minima [93].

Other popular criterion functions, also defined only when patterns live in Euclidean (or Hermitian) spaces, and closely related to the previous ones, can be derived based on the within cluster scatter matrix $W(\mathcal{P}(\mathcal{T}))$, and the between cluster scatter matrix $B(\mathcal{P}(\mathcal{T}))$ [60],

$$W\left(\mathcal{P}(\mathcal{T})\right) = \sum_{m=1}^{c} \sum_{T \in \mathcal{T}_m} \left(T - \langle T_m \rangle\right) \cdot \left(T - \langle T_m \rangle\right)^{\mathrm{T}},$$

$$B\left(\mathcal{P}(\mathcal{T})\right) = \sum_{m=1}^{c} n_m \left(\langle T_m \rangle - \langle T \rangle\right) \cdot \left(\langle T_m \rangle - \langle T \rangle\right)^{\mathrm{T}},$$

$$S = \sum_{m=1}^{c} \left(T - \langle T \rangle\right) \cdot \left(T - \langle T \rangle\right)^{\mathrm{T}} = W\left(\mathcal{P}(\mathcal{T})\right) + B\left(\mathcal{P}(\mathcal{T})\right),$$

where $\langle T \rangle$ is the barycenter of all patterns in the data set, and S is the total scatter matrix, which is a constant given the data, independent on the partition. One can define optimal partitions as minimizers of $\mathrm{tr}\left[W\left(\mathcal{P}(\mathcal{T})\right)\right]$ (or equivalently maximizers of $\mathrm{tr}\left[B\left(\mathcal{P}(\mathcal{T})\right)\right]$). This turns out to be a minimum variance criterion. Another possibility is to minimize $\det\left[W\left(\mathcal{P}(\mathcal{T})\right)\right]$ whose solution is invariant to linear transformations of the data. In any case, combinatorial optimization is intractable and one has to consider iterative procedures.

A.1.2 Iterative Methods for Partitional Clustering

Most partitional methods are based on the definition of c elements from a pattern space called centrotypes, each defined to be a representative object of one of the clusters. The criterion function to be minimized is usually the sum of the average dissimilarities between each centrotype and all of the other patterns of the same cluster. Typically, iterative methods begin by initialising a set of c centrotypes. Each pattern is then assigned to the cluster corresponding to its closest centrotype (for the considered dissimilarity measure), and centrotypes are re-computed in order to mimimize the criterion function. The iteration ends when centrotypes do not change. The computational efficiency of this approach depends on how easily centrotypes can be computed. The c-means algorithm [115] (also referred in the literature as k-means) runs typically in $O(M)$ [25]. Indeed in this algorithm the dissimilarity measure is the squared Euclidean distance and centrotypes are the clusters' barycenters, which can be easily computed using an update equation. A similar algorithm can be obtained by using the ℓ_1-norm as dissimilarity measure. The centrotypes for this measure (which is more robust to outliers than the squared Euclidean distance) are the cluster medians.

When the dissimilarity measure does not lead to a closed form representation for the centrotypes, a method known as k-medoid which allows clustering with respect to any specified dissimilarity measure can be used [96]. In this method, centrotypes (the so-called medoids) are restricted to be patterns from the data set, and as before patterns are assigned to the cluster corresponding to its closest centrotype. The goal is then to select, among all M patterns the c centrotypes which minimize the sum of the average dissimilarities between each centrotype and all of the other patterns of the same cluster. A widely used implementation for the k-medoid method is the

Partitioning Around Medoids algorithm (PAM), by Kaufman and Rousseeuw [96]. PAM consists of two phases. In the first one, a method for selecting the initial set of c centrotypes or medoids is applied. The second phase is an iterative procedure where in each iteration the set of centrotypes is updated by analyzing all possible pairs of patterns such that one pattern is a centrotype and the other is not, and by swapping the pair which most reduces the value of the criterion function. The cost of a single iteration is $O(c(M-c)^2)$.

A.1.3 Hierarchical Clustering Methods

While partitional clustering algorithms construct a single partition with c clusters (a flat description), hierarchical methods deliver a recursive structure. Since they represent data in different ways, partitional and hierarchical methods do not really compete with one another. Indeed, when data is to be described in terms of classes, subclasses, sub-subclasses (e.g. a biological taxonomy), flat representations do not make sense and hierarchical methods are needed. There are, of course, many applications in which data is not inherently hierarchical, and one has to make a choice among clustering methods from both types. Hierarchical methods are more versatile than partitional methods and can deal with many differently shaped clusters, but generally they are more time consuming.

Depending on the direction they build the hierarchy, these clustering methods can be agglomerative (bottom-up) or divisive (top-down). The former, which are usually computationally simpler, start with each single point as a cluster, and iteratively merge the closest pair of clusters in the sense of a chosen dissimilarity measure. The generic algorithm is as follows [93].

1. Initialization: compute the proximity matrix (the matrix containing the dissimilarity between each pair of patterns).
2. Find the most similar pair of clusters using the proximity matrix. Merge these two clusters.
3. Update the proximity matrix according to this merging.
4. Repeat steps 2 and 3 until all patterns are in one cluster.

At each iteration step, two clusters are merged. The procedure builds up a tree or dendrogram, where leaves are the M elements of $\mathcal{T}$ (step 1). At level l, this tree has $M-l$ nodes, each node being a cluster. At level $l+1$, the closest clusters from level l are merged (step 2). By closest, we mean the pair $\mathcal{T}_i$ and $\mathcal{T}_j$ minimizing a given distance or proximity measure $\delta(\mathcal{T}_i, \mathcal{T}_j)$ between clusters. Different strategies for updating the proximity matrix lead to different hierarchical clustering methods. (Moreover, since all of these algorithms are merging methods, they admit a variational formulation and can be solved as an energy minimization problem; see [137], Chap. 3.) Lance and Williams [103] define a class of methods by specifying a generalized recurrence formula for updating the proximity matrix:

$$\delta(\mathcal{T}_i \cup \mathcal{T}_j, \mathcal{T}_k) = \alpha_i\, \delta(\mathcal{T}_i, \mathcal{T}_k) + \alpha_j\, \delta(\mathcal{T}_j, \mathcal{T}_k) + \beta\, \delta(\mathcal{T}_i, \mathcal{T}_j) + \gamma\, |\delta(\mathcal{T}_i, \mathcal{T}_k) - \delta(\mathcal{T}_j, \mathcal{T}_k)|,$$

where parameter values α_i, α_j, β and γ characterize the particular clustering method. Below we describe the most popular ones.

- Choosing $\alpha_i = \alpha_j = 1/2$, $\beta = 0$ and $\gamma = -1/2$, leads to the following distance between clusters:

$$\delta_{min}(\mathcal{T}_p, \mathcal{T}_q) = \min_{T_i \in \mathcal{T}_p, T_j \in \mathcal{T}_q} d_T(T_i, T_j).$$

  The corresponding algorithm is known as *single-linkage algorithm* [93, 60]. Here the nearest-neighbor points determine the nearest subsets. If elements in $\mathcal{T}$ are viewed as nodes of a graph, merging $\mathcal{T}_p$ and $\mathcal{T}_q$ corresponds to adding an edge between the nearest points in $\mathcal{T}_p$ and $\mathcal{T}_q$. This procedure generates a tree, and if one lets the procedure evolve up to having a single cluster containing all points, the result is a *minimal spanning tree*.
- Taking $\alpha_i = \alpha_j = \gamma = 1/2$, $\beta = 0$, yields

$$\delta_{max}(\mathcal{T}_p, \mathcal{T}_q) = \max_{T_i \in \mathcal{T}_p, T_j \in \mathcal{T}_q} d_T(T_i, T_j).$$

  The resulting algorithm is called *complete-linkage algorithm* [93, 60]. Here distance between two clusters is given by the farthest pair of points in the two clusters. This procedure produces a graph in which edges connect all of the nodes in a cluster. When the nearest clusters are merged, edges between every pair of nodes in the two clusters are added. If the diameter of a partition is defined as the largest diameter for clusters in the partition, then each iteration of the complete-linkage algorithm increases the diameter of the partition as little as possible.
- Taking $\alpha_i = n_i/(n_i + n_j)$, $\alpha_j = n_j/(n_i + n_j)$, and $\beta = \gamma = 0$, leads to a group averaging method, where

$$\delta_{avg}(\mathcal{T}_p, \mathcal{T}_q) = \frac{1}{n_p n_q} \sum_{T_i \in \mathcal{T}_p} \sum_{T_j \in \mathcal{T}_q} d_T(T_i, T_j).$$

- Some clustering methods based on barycenters, such as Ward's minimum variance method [176], can also be represented in terms of Lance and Williams formula. For Ward's method, $\alpha_i = (n_i + n_k)/(n_i + n_j + n_k)$, $\alpha_j = (n_j + n_k)/(n_i + n_j + n_k)$, $\beta = -n_k/(n_i + n_j + n_k)$, $\gamma = 0$, and the corresponding cluster proximity measure is

$$\delta_{ward}(\mathcal{T}_p, \mathcal{T}_q) = \frac{n_p n_q}{n_p + n_q} \|\langle T_p \rangle - \langle T_q \rangle\|_2^2,$$

  where $\langle T_p \rangle$ and $\langle T_q \rangle$ denote the barycenters of $\mathcal{T}_p$ and $\mathcal{T}_q$ respectively.

Time and memory complexity of the algorithms given by the Lance and Williams formula are studied in [44]. Overall, the time required for hierarchical clustering is $O(M^2 \log M)$, and the memory complexity is $O(M^2)$.

In practice, if clusters are compact and well separated, all methods yield the same results. However, when this is not the case, the resulting partitions may be quite different. Depending on the cluster proximity measure, different methods of clustering can be more or less successful with different types of clusters. Single-linkage algorithms suffer from the chaining effect: A single corrupted point somewhere in between two compact clusters can lead to an unwanted merging between them [93, 60]. However, this property is very useful if one wants to detect elongated clusters.

The complete-linkage algorithm tends to produce compact clusters with small diameters. However, patterns assigned to a cluster can be much closer to patterns in other clusters [82, 60].

The single-linkage and the complete-linkage algorithms are both sensitive to outliers since they rely on extremal measures. One way to reduce the influence of outliers is using δ_{avg} as cluster proximity measure though the improvement is often not good enough. Besides, average methods have another drawback compared to single or complete linkage methods: they are not invariant under monotone transformations on the dissimilarity measure d_T (invariance of the former ones is a consequence of being based on extremal values) [82].

To end this section, let us make a few general remarks. In Sect. A.1.1, one of the main assumptions is that the number of clusters c was given, for partitional clustering algorithms. Then, the goal was to find the c-partition on the data optimizing a global criterion (in practice iterative methods are used and the convergence to a global minimum is not ensured). Agglomerative hierarchical clustering methods perform well in making local decisions about cluster merging since they make use of the proximity matrix. As the hierarchy is built by means of local optimization, the level corresponding to a c-partition will not correspond in general to a global optimum (unless clusters are compact and well separated). For instance, Ward's method will not lead to the same c-partition as a c-means method, despite the fact that both attempt to minimize variance. In this sense, one would rather say that partitional methods are better than hierarchical methods. But how to be sure that there are exactly c groups of patterns in the data? Is the criterion function well adapted to the shape of clusters that are present in the data? From this viewpoint, hierarchical clustering may be more appealing than partitional ones. Another argument in favor of hierarchical clustering methods is their versatility and their ability to cope with differently shaped clusters. For instance, the single linkage algorithm can deal with non-isotropic, elongated or concentric clusters while partitional methods like c-means can only deal with isotropic clusters. Since their outputs are nested series of partitions, ranging from M clusters to one single cluster, one can imagine methods to determine the number of clusters as stopping rules in the merging process. If stopping rules are correctly designed, hierarchical methods would also be able to detect clusters having different densities or different number of points.

A.1.4 Cluster Validity Analysis and Stopping Rules

The great variety of clustering methods that have been proposed in the recent past has been followed by an increasing interest in clustering validation methods. In [73], a comprehensive study of these techniques is presented.

Cluster validity analysis deals with assessing the validity of classifications obtained from the application of clustering procedures. There are different validation approaches [58, 73] depending on the amount of prior information on the data. This section deals with *internal validation tests*, which consist in determining if the structure is intrinsically adapted to the data. In other words, internal tests are derived from some *internal criteria* measuring the suitability of the clustering structure for the original data set with no other information than the data themselves.

Classical issues in cluster validity analysis are the assessment of individual cluster validity and the assessment of a whole partition. (In some applications validity of a dendrogram also needs to be assessed. This problem is not addressed here.) These two issues are briefly summarized next.

A.1.4.1 Partition Validity Assessment

A relevant question to address in order to assess the validity of a partition, is deriving the number of clusters [58], denoted by c. Notice that by solving this problem, it cannot be ensured that the c clusters are valid clusters. The most common approach to decide how many clusters are best consists in finding partitions for $c = 1, \ldots, c_{max}$ and optimizing a measure $G(c)$ of partition adequacy, which is usually based on the within-cluster and between-cluster variability. When applied to hierarchical clustering methods these cluster validity assessment techniques are known as *global stopping rules* because the choice of c can be seen as stopping the merging process (in the agglomerative case) at a certain level of the dendrogram.

When dealing with hierarchical classifications, another approach to determine the most appropriate number of clusters are *local stopping rules*. In the agglomerative case, these rules are *merging criteria* to decide whether two clusters should be merged. Usually, the merging process is continued until it is decided, for the first time, that two clusters should not be aggregated.

Milligan and Cooper [125], and Dubes [58], present comparative studies of some stopping rules. Milligan and Cooper's paper provides a particularly comprehensive Monte-Carlo evaluation of these rules, by comparing thirty local and global stopping rules. In their simulation experiment, only strongly clustered data sets (internally cohesive and well separated clusters) were considered. Hence, since clustering this kind of data should not be a challenging problem, techniques that do not perform well on it are also expected to be inefficient when dealing with any data set. The main conclusion of this experiment is that only five or maybe six of the compared rules perform quite well on strongly clustered data. One can also observe that the majority of the stopping rules described in the study are based on heuristics and lack of theoretical foundation. Those derived from rigorous statistical techniques, assume

in general hypotheses on the data which are unrealistic in most real applications (*e.g.* multivariate normal distribution for the patterns). In order to briefly illustrate the considered stopping rules, it is worth describing Calinski and Harabasz's index [26] and Duda and Hart's rule [60], since these methods provided the best results.

- Calinski and Harabasz propose a *global stopping rule* for assessing partitions, by choosing the partition size c that maximizes the index

$$G(c) = \frac{\frac{1}{c-1}\mathrm{tr}\left[B\left(\mathcal{P}(\mathcal{T})\right)\right]}{\frac{1}{M-c}\mathrm{tr}\left[W\left(\mathcal{P}(\mathcal{T})\right)\right]},$$

  where $B\left(\mathcal{P}(\mathcal{T})\right)$ and $W\left(\mathcal{P}(\mathcal{T})\right)$ are respectively the between- and within-cluster scatter matrices of a c-partition $\mathcal{P}$, defined in section A.1.1. The index $G(c)$ is the ratio between the total within-cluster sum of squared distances about the centroids, and the total between-cluster sum of squared distances. This index is only defined for sets of patterns living in an Euclidean space. Moreover, since the index is based on the sum of squares criterion, it has a tendency to partition the data into hyperspherical shaped clusters, having roughly equal numbers of patterns [73] (this is probably the main reason for its first position in Milligan and Cooper's ranking, since their data was strongly clustered, and clusters contained almost the same numbers of points and were pretty isotropic).
- Duda and Hart proposed the $Je(2)/Je(1)$ *local stopping rule* for deciding whether or not a cluster should be split into two subclusters. The rule consists in computing the ratio between the total within sum of squared distances about the centroids of the two clusters for the two-cluster solution ($Je(2)$), and the within sum of squared distances about the centroid when only one cluster is present ($Je(1)$). The method considers a null hypothesis, assuming that all patterns come from a normal distribution, whose mean and variances are empirically estimated over the whole data set. The null hypothesis of one single cluster is rejected if $Je(2)/Je(1)$ is smaller than a specified critical value, fixed by a significance level for the hypothesis testing. While considering a normal distribution as a null hypothesis and using the sum of squared distances may not be well adapted to real clustering problems (particularly when the number of patterns in the data set is not as large to be well represented by an asymptotic distribution), the proposed *a contrario* formulation is appealing from our point of view.

To finish the discussion on partition validity assessment we quote one of Bock's conclusions from his work on significance tests in cluster analysis [24], where a comparison between global and local methods is made.

> *Some care is needed when applying any test for clustering, bearing in mind that different types of clusters may be present simultaneously in the data, and that the number of clusters is, in some sense, dependent on the intended level of information compression. Thus, a global application of a cluster test to a large or high-dimensional data set will not be advisable in most cases. However, a local application (...) to a specific part of the data will often be useful for providing evidence for or against a prospective clustering tendency.*

A.1.4.2 Validity Assessment of Individual Clusters

The problem is now to decide, among the candidate clusters furnished by the clustering procedure, which ones correspond to natural clusters. But what does a natural cluster look like? As pointed out by Gordon [73], it may be difficult to specify a relevant definition of an ideal cluster for a particular data set. However, clusters must reveal structure in the data and can be detected as opposed to a complete absence of structure. Thus, in order to decide whether the candidate clusters are significant, they can be compared to some appropriate random distribution. This leads to a general methodology for cluster validity analysis based on the statistical approach of hypothesis testing [24, 72, 73]. Following Bock [24], this framework consists of these stages:

1. Design a null hypothesis $\mathcal{H}$ for the absence of class structure in the data (a *background model*, or *null model*), meaning that patterns are sampled from a homogeneous population. Then, heterogeneity or clustering structure are involved in the alternative hypothesis $\mathcal{A}$.
2. Define a test statistic, which will be used as a validity index to discriminate between $\mathcal{H}$ and $\mathcal{A}$.
3. If, for a given significance level (error probability) α, the test statistic of the observed data exceeds the corresponding critical value c_α, the null hypothesis $\mathcal{H}$ is rejected, in favor of $\mathcal{A}$.

This general framework can be adapted for assessing the validity of individual clusters. A general approach within this framework is the Monte-Carlo validation, which is described in [73]. Assume one wants to assess the validity of an observed cluster $\mathcal{T}_i$ having n patterns in a data set having M patterns. In the Monte-Carlo validation method, data sets of M patterns are simulated under the background model, and classified using the same clustering procedure that was used to classify the original data. The test statistic is computed for those clusters having n patterns, and the distribution of the test statistic is estimated. Then, using the value of the test statistic of $\mathcal{T}_i$, one can compute the significance level of rejecting $\mathcal{H}$. Two popular test statistics are the maximum F test and the U statistic (see Bock [24] and Gordon [73]).

Appropriate null models for data are the subject of the study presented in [72]. These models, which specify the distribution of patterns in the absence of structure in the data, can be of two types.

– *Standard (data-independent) null models*. Two well known standard null models are the *Poisson model* and the *Unimodal model* [24]. The main problem with the Poisson model is the choice of the region R within which patterns are uniformly distributed (standard choices for normalized data are the unit hypercube and the unit hypersphere). The Unimodal model assumes that the joint distribution of the variables describing the patterns is unimodal, but the choice of the distribution may not be easy.
– *Data-influenced null models*. Here the data is used to influence the specification of the null model. Examples of these null models are the Poisson model where R is chosen to be the convex hull of the data set, or the *Ellipsoidal model*, which is

a multivariate normal distribution, whose mean and covariance matrix are given by the data set.

In [72], Gordon concludes that the results of the tests depend considerably on the choice of the null model and that in general the results based on data-influenced null models are more relevant than those obtained using a standard null model.

A.2 Three classical methods for object detection based on spatial coherence

This section addresses some issues of the generalized Hough transform [14], whose variations are probably the most widely used techniques in object detection. Two frequently used techniques for robust transformation estimation will also be described: geometric hashing[102, 184] and the RANSAC algorithm [64].

A.2.1 The Generalized Hough Transform

In [14] Ballard proposed a generalization of the Hough transform [85] allowing the detection of arbitrary planar shapes undergoing similarity transformations. Most object detection and recognition systems using transformations clustering are based on the generalized Hough transform. The basic idea is to quantize the transformation space into D-dimensional cells. Each transformation point T_i is quantized and then votes for one of these cells. In practice, noise and image quantization induce localization errors in the extracted features and one has to take into account uncertainty in computing T_i. Thus, each pairing of model and image features defines a volume of possible transformations, so it should cast a vote into each cell intersecting this volume (see [75] for an error analysis when using line segments as features).
As with all techniques based on histograms in multidimensional spaces, the generalized Hough method is very sensitive to the choice of quantization precision (this remark also holds for Lamdan and Wolfson's Geometric Hashing [184, 102] described in Sect. A.2). Most of the time, the cell size is chosen by problem specific *ad hoc* arguments (see [113] for an example). However, in the general case, quantization effects may lead to several problems:

- Similar transformation points may vote for different cells. In order to reduce this problem, either votes are counted by adding the votes of neighboring cells (using a sliding window) in the case of no uncertainty in T_i, or, when uncertainty is considered, a vote is cast into each cell intersecting the uncertainty volume.
- In the plane similarity case, for instance, if one wants to do a fine discretization of the 4-D transformation space in order to perform accurate detection, the search space is too large for an exhaustive search. Coarse to fine techniques applied to transformation clustering, first introduced by Stockman [169], can deal with this complexity problem, but there is no reason why the most voted cells at the finer scale should correspond to the most voted ones at coarser scales.
- From the detection viewpoint, the size of the cells is also crucial. Indeed, if quantization is too fine, cells will not have enough votes and correct instances will be missed (false negatives). On the other hand, choosing a very coarse quantization increases the likelihood of large clusters occurring at random (false positives).

These remarks partially motivate our decision to use the clustering techniques described in Chap. 7, along with the validity assessment method proposed in the same

chapter. Indeed, the proposed methodology does not suffer from quantization problems.

The generalized Hough transform is with geometric hashing [102, 182, 184], and the alignment method [89] one of the most popular voting schemes. Given two shapes, the geometric hashing method aims at determining if there is a transformed subset of the features from one shape that matches a subset of the features of the other one. The alignment method is a similar voting method. The generalized Hough transform method, instead of voting over all possible configurations of shapes, consists in voting over all possible transformations mapping a shape to another one. As for all techniques based on histograms in multidimensional spaces, these voting methods are very sensitive to the choice of quantization precision (too large bins may lead to false matches, and too small bins may produce misses). Besides, most of the time, the size of the hash table and the amount of parameters (the size of the bins in the voting stage, the threshold for the amount of votes in each bin, *etc.*) are crippling. The complexity of these voting schemes increases with the invariance degree; affine invariant shape retrieval in large databases is intractable. All these properties make the local features not suitable for shape retrieval in large databases.

A.2.2 Geometric Hashing

In order to illustrate the geometric hashing algorithm, we present the case of similarity or affine transformations.

A query shape $\mathcal{S}$ is searched in a set of shapes.

Preprocessing (off line). For each shape $\mathcal{S}'_i$ in the set of shapes:

1. Extract local invariant features from $\mathcal{S}'_i$. Assume n such features are found.
2. For each local basis b_j (*e.g.* a pair of points for similarity transformations, three non-collinear points for affine transformations) of features:
   a. Compute the quantized coordinates (u, v) of all the remaining features, in the local basis.
   b. Use the couple (u, v) as an index in a hash table, and write the information (i, b_j) in the corresponding bin (i is the index that identifies $\mathcal{S}'_i$).

Recognition stage (on line). For the query shape $\mathcal{S}$:

1. Extract local invariant features from $\mathcal{S}$. Assume n such features are found.
2. Choose arbitrarily a local basis (two or three points, depending on the considered invariance).
3. Compute the quantized coordinates (u, v) of all the remaining features, in the local basis.
4. For each of these coordinates, go to the corresponding bin in the hash table, and cast a vote for each pair (i, b_j) inscribed in the bin.

5. Keep only the pairs (i, b_j) which received more than a certain number of votes. Each of these pairs stands for a potential match.
6. For each potential match, compute the best transformation (in the least squares sense) between all corresponding features, and check if the query features and the features from the corresponding shape are well aligned. If not, go to (2) and choose another basis.

For affine invariant shape recognition, time complexity for the preprocessing stage is $O(n^4)$ for each shape in the set of shapes. If the access time to the hash table is $O(1)$, time complexity for the recognition stage is between $O(m)$ (when the first query basis chosen at random corresponds to a model in the set of shapes) and $O(m^4)$ (when no basis from the query shape corresponds to a model in the set of shapes).

A.2.3 A RANSAC-based Approach

The RANdom SAmple Consensus (RANSAC) algorithm by Fischler and Bolles [64], is certainly one of the most popular robust estimators in computer vision. It has proved very successful in stereo vision tasks, such as estimating homographies and fundamental matrices [81]. The main reason for its success is its ability to deal with large proportions of outliers. Roughly speaking, in its general form, the RANSAC procedure to fit a model consists in randomly selecting a minimal subset of the data (*i.e.* a subset allowing to instantiate the model), then computing the number of inliers consistent with the instantiated model. These two steps are repeated for N minimal subsets of the data. The model having the largest number of inliers is chosen and refined by re-estimating it from the corresponding set of inliers.

Our framework deals with M meaningful matches, and usually M is small enough to test for all corresponding similarity or affine transformations. Hence, using the same ideas, an elementary algorithm would be as follows:

1. For each element in the set of M pairs of local frames corresponding to meaningful matches:
   a. Compute the associated transformation T;
   b. Apply T to all query local frames, and compute their distances to their corresponding scene local frames;
   c. Compute the number of inliers consistent with T, *i.e.* the pairs for which the distance is less than d pixels;
2. Choose the transformation T having the largest number of inliers;
3. Re-estimate T for all pairs of local frames determined as inliers (with a least squares method, for instance).

One can iterate this procedure on the set of outliers in order to find other (less dominant) transformations.

Even for this simple version of the algorithm, two problems arise: the choice of the distance threshold d, and the minimum number of inliers a model should have in order to be valid. The distance threshold d is usually chosen empirically. Otherwise, it can be chosen by considering a significance level α, corresponding to the probability that a point is an inlier [81], which requires hypothesizing a model for the distribution of distances. Concerning the minimum number of inliers to assess model validity, generally it is also fixed by means of arbitrary rules. It seems reasonable to us that this minimum number of inliers depends on the distance threshold, but as far as we know no effort has been done to establish this relation.

A.3 On the Negative Association of Multinomial Distributions

This section presents the notion of *negative association* (a strong notion of negative dependence) and summarizes some relevant consequences first reported by Joag-Dev and Proschan in [94]. Some proofs are also completed when they were just outlined in the original paper. The result is then applied to multinomial distributions.

Definition 20 (Negative association). A set $\mathcal{X} = \{X_1, \ldots, X_n\}$ of real random variables is said to be negatively associated (NA) if for every two disjoint index sets $I, J \subset \{1, \ldots, n\}$,

$$\mathbb{E}\left[f(X_i, i \in I) g(X_j, j \in J)\right] \leqslant \mathbb{E}\left[f(X_i, i \in I)\right] \cdot \mathbb{E}\left[g(X_j, j \in J)\right],$$

for all non-decreasing functions $f : \mathbb{R}^{\#I} \to \mathbb{R}$, $g : \mathbb{R}^{\#J} \to \mathbb{R}$ (a function $h : \mathbb{R}^k \to \mathbb{R}$ is said to be non-decreasing if $h(x_1, \ldots, x_k) \geqslant h(y_1, \ldots, y_k)$ whenever $x_1 \leqslant y_1, \ldots, x_k \leqslant y_k$).

Remark 7. Negative association is a natural generalization of negative correlation.

The negatively associated set $\mathcal{X} = \{X_1, \ldots, X_n\}$ verifies the following properties:

Property 1. For any non-decreasing functions f_i, $i \in \{1, \ldots, n\}$,

$$\mathbb{E}\left[\prod_{i=1}^{n} f_i(X_i)\right] \leqslant \prod_{i=1}^{n} \mathbb{E}\left[f_i(X_i)\right].$$

Proof. Define $f(x_1, \ldots, x_{n-1}) = \prod_{i=1}^{n-1} f_i(x_i)$ and $g(x_n) = f_n(x_n)$ for all $(x_1, \ldots, x_n) \in \mathbb{R}^n$. Since f and g are both non-decreasing, it follows from Definition 20 that

$$\mathbb{E}\left[\prod_{i=1}^{n} f_i(X_i)\right] \leqslant \mathbb{E}\left[\prod_{i=1}^{n-1} f_i(X_i)\right] \mathbb{E}\left[f_n(X_n)\right].$$

Using induction yields the desired result. □

Property 2. For all $(x_1, \ldots, x_n) \in \mathbb{R}^n$,

$$\Pr\left(X_i \geqslant x_i \; \forall \; i \in \{1, \ldots, n\}\right) \leqslant \prod_{i=1}^{n} \Pr\left(X_i \geqslant x_i\right).$$

This follows immediately from Property 1 for $f_i(x) = \chi_{[x \geqslant x_i]}$, the indicator function of event $[x \geqslant x_i]$. The following property is obvious from Definition 20:

Property 3. Non-decreasing functions defined on disjoint subsets of a set of NA random variables are NA.

Property 4. The union of independent sets of NA random variables is NA.

Proof. Let $\mathbf{X}$ and $\mathbf{Y}$ be independent vectors such that for each one its components are sets of NA random variables. Let $(\mathbf{X}_1, \mathbf{X}_2)$ and $(\mathbf{Y}_1, \mathbf{Y}_2)$ denote arbitrary partitions of $\mathbf{X}$ and $\mathbf{Y}$ respectively. Hence, the vector $(\mathbf{X}, \mathbf{Y})$ is NA if and only if $\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)g(\mathbf{X}_2, \mathbf{Y}_2)] \leqslant \mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)]\,\mathbb{E}[g(\mathbf{X}_2, \mathbf{Y}_2)]$. Now,

$$\begin{aligned}\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)g(\mathbf{X}_2, \mathbf{Y}_2)] &= \mathbb{E}\{\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1, \mathbf{Y}_2]\} \\ &= \sum_{(y_1,y_2)} \mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1 = y_1, \mathbf{Y}_2 = y_2] \\ &\qquad \cdot \Pr(\mathbf{Y}_1 = y1, \mathbf{Y}_2 = y2).\end{aligned}$$

Since $(\mathbf{X}_1, \mathbf{X}_2)$ and $(\mathbf{Y}_1, \mathbf{Y}_2)$ are independent, $\{f(\mathbf{X}_1, \mathbf{Y}_1)|\mathbf{Y}_1 = y_1, \mathbf{Y}_2 = y_2\}$ and $\{g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1 = y_1, \mathbf{Y}_2 = y_2\}$ are parametric functions of random vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ respectively. Thus, because of the negative association of $\mathbf{X}$,

$$\begin{aligned}&\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1 = y_1, \mathbf{Y}_2 = y_2] \leqslant \\ &\qquad \mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)|\mathbf{Y}_1 = y_1, \mathbf{Y}_2 = y_2]\,\mathbb{E}[g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1 = y_1, \mathbf{Y}_2 = y_2].\end{aligned}$$

Hence,

$$\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)g(\mathbf{X}_2, \mathbf{Y}_2)] \leqslant \mathbb{E}\{\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)|\mathbf{Y}_1, \mathbf{Y}_2]\,\mathbb{E}[g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1, \mathbf{Y}_2]\}$$

Now, since the conditional expectations

$$\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)|\mathbf{Y}_1, \mathbf{Y}_2] \text{ and } \mathbb{E}[g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1, \mathbf{Y}_2]$$

are respectively $\mathbf{Y}_1$ and $\mathbf{Y}_2$ measurable functions, it follows that

$$\begin{aligned}h_1(\mathbf{Y}_1) \equiv \mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)|\mathbf{Y}_1, \mathbf{Y}_2] = \mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)|\mathbf{Y}_1], \\ h_2(\mathbf{Y}_2) \equiv \mathbb{E}[g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_1, \mathbf{Y}_2] = \mathbb{E}[g(\mathbf{X}_2, \mathbf{Y}_2)|\mathbf{Y}_2].\end{aligned}$$

Finally, using that $\mathbf{Y}$ is NA,

$$\begin{aligned}\mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)g(\mathbf{X}_2, \mathbf{Y}_2)] &\leqslant \mathbb{E}[h_1(\mathbf{Y}_1)h_2(\mathbf{Y}_2)] \\ &\leqslant \mathbb{E}[h_1(\mathbf{Y}_1)]\,\mathbb{E}[h_2(\mathbf{Y}_1)] \\ &= \mathbb{E}[f(\mathbf{X}_1, \mathbf{Y}_1)]\,\mathbb{E}[g(\mathbf{X}_2, \mathbf{Y}_2)]. \qquad \square\end{aligned}$$

These results yield the following proposition.

Proposition 14. *A random vector $\mathbf{X} = (X_1, \ldots, X_n)$ having a multinomial distribution of index M and parameter $\mathbf{p} = (p_1, \ldots, p_n)$ (denoted by $\mathbf{X} \sim Mult(M, \mathbf{p})$), is NA.*

Proof. The variable $\mathbf{X}$ can be decomposed as

$$\mathbf{X} = \sum_{k=1}^{M} \mathbf{Y}_k,$$

where each $\mathbf{Y}_k \sim Mult(1, \mathbf{p})$, and the $\mathbf{Y}_k$'s are mutually independent. Since, for all $k \in \{1, \ldots, M\}$, all elements in $\mathbf{Y}_k$ are zero except for one whose value is 1, vector $\mathbf{Y}_k$ is NA. Indeed, for all I, J disjoint subsets of $\{1, \ldots, n\}$, for all non-decreasing functions $f : \mathbb{R}^{\#I} \to \mathbb{R}$, $g : \mathbb{R}^{\#J} \to \mathbb{R}$,

$$\begin{aligned}
&\mathbb{E}\left[f(\mathbf{Y}_{k,i}, i \in I) g(\mathbf{Y}_{k,j}, j \in J)\right] \leqslant \mathbb{E}\left[f(\mathbf{Y}_{k,i}, i \in I)\right] \cdot \mathbb{E}\left[g(\mathbf{Y}_{k,j}, j \in J)\right] \\
&\quad \Leftrightarrow \mathbb{E}\left[\left(f(\mathbf{Y}_{k,i}, i \in I) - f(0, \ldots, 0)\right)\left(g(\mathbf{Y}_{k,j}, j \in J) - g(0, \ldots, 0)\right)\right] \\
&\qquad \leqslant \mathbb{E}\left[f(\mathbf{Y}_{k,i}, i \in I) - f(0, \ldots, 0)\right] \cdot \mathbb{E}\left[g(\mathbf{Y}_{k,j}, j \in J) - g(0, \ldots, 0)\right].
\end{aligned}$$

The last inequality is true: the right member is non-negative because $f(\mathbf{Y}_{k,i}, i \in I) - f(0, \ldots, 0)$ and $g(\mathbf{Y}_{k,j}, j \in J) - g(0, \ldots, 0)$ are non-negative, and the left member is zero since $(f(\mathbf{Y}_{k,i}, i \in I) - f(0, \ldots, 0)$ and $g(\mathbf{Y}_{k,j}, j \in J) - g(0, \ldots, 0)$ cannot be non-zero at the same time.
Then, using Property 4, it follows that $\{\mathbf{Y}_1, \ldots, \mathbf{Y}_M\}$ is NA. Finally, for all $l \in \{1, \ldots, n\}$, $X_l = \sum_{k=1}^{M} \mathbf{Y}_{k,l}$ are non-decreasing functions defined on disjoint subsets of $\{\mathbf{Y}_1, \ldots, \mathbf{Y}_M\}$. This proves that $\mathbf{X}$ is NA (Property 3). □

Remark 8. Applying Property A.3 to the random vector $\mathbf{X}$ proves Lem. 7, stated in Sect. 7.3.

Appendix B
Algorithms

B.1 LLD Method Summary

Input Image

STEP 1: EXTRACTION

1.1. Level Lines Tree computation (Sect. 2.1)
1.2. Maximal Meaningful Boundaries selection. (Sect. 2.3.2). Active options:
– Clean sub-curves in noise ($\mu = 1$) (Sect. 2.4.1.1)
– Multiscale version (two levels, $N_s = 2$) (Sect. 2.5)
– Local contrast (Sect. 2.6)

Maximal Boundaries

STEP 2: ENCODING

2.1. Curves smoothing (Sect. 3.3)
2.2. Extraction of robust directions: bitangents and flat parts (Sect. 3.1)
2.3. Geometric normalization, similarity and affine invariant: global (Sect. 4.1.3) and local (Sect. 4.2)

Level Line Descriptors (LLDs)

Two sets of LLDs

STEP 3: RECOGNITION

3.1. Background model construction: selection of statistically independent global and local features from LLDs (Sect. 5.2.1) and computation of empirical frequencies of distances between them (Sect. 5.2.1 and Eq. (5.9))
3.2. Given any two LLDs in the databases, compute their distance (Eq. (5.10)) and NFA (Def. 8)
3.3. Use Def. 9 to find ε-meaningful matching pairs

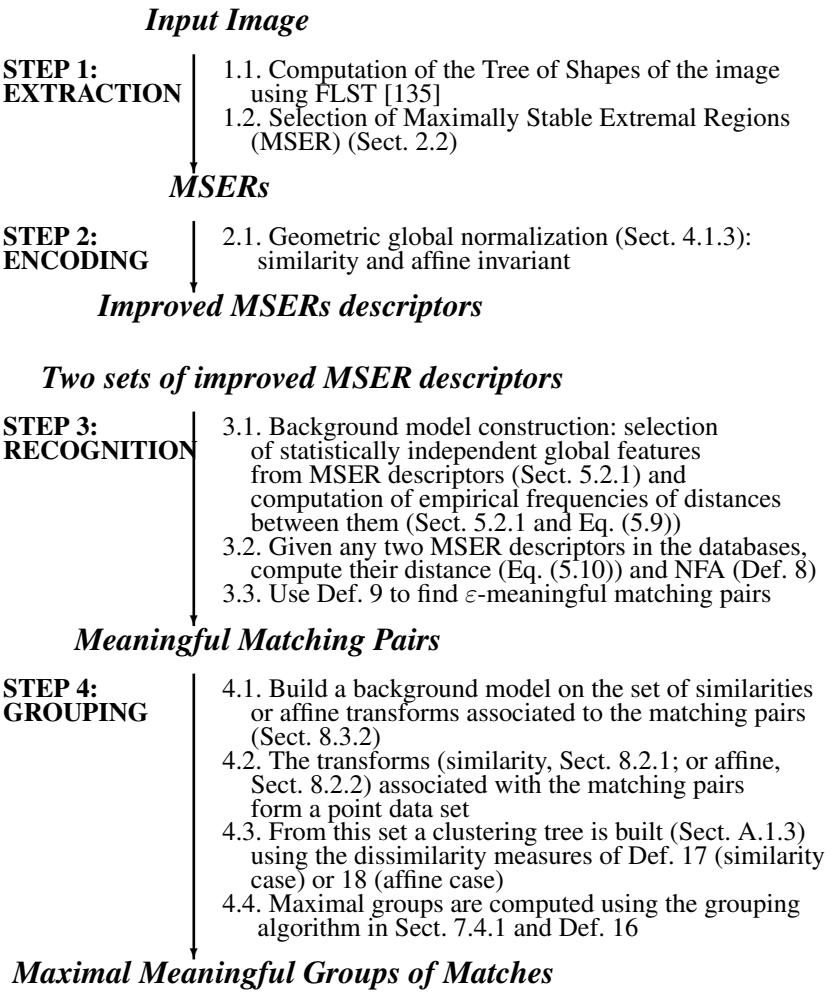
Meaningful Matching Pairs

STEP 4: GROUPING

4.1. Build a background model on the set of similarities or affine transforms associated to the matching pairs (Sect. 8.3.2)
4.2. The transforms (similarity, Sect. 8.2.1; or affine, Sect. 8.2.2) associated with the matching pairs form a point data set
4.3. From this set a clustering tree is built (Sect. A.1.3) using the dissimilarity measures of Def. 17 (similarity case) or 18 (affine case)
4.4. Maximal groups are computed using the grouping algorithm in Sect. 7.4.1 and Def. 16

Maximal Meaningful Groups of Matches

B.2 Improved MSER Method Summary



B.3 Improved SIFT Method Summary

Input Image

STEP 1: EXTRACTION

1.1. Computation of keypoints: extrema of the Laplacian in the image scale space. They are pairs (position, scale). (Sect. 10.1.1, 10.1.2 and [114])

Keypoints

STEP 2: ENCODING

2.1. Associate one or more orientations to each keypoint (dominant gradient directions in the vicinity of the point) (Sect. 10.1.3)
2.2. Associate an image patch to each keypoint (region in the vicinity of the point)
2.3. Subdivide the square patch into 8×8 subsquares
2.4. Keep the orientation histograms inside each square The orientations are weighted by gradient magnitude

Improved SIFT descriptors: keypoint + scale + orientation + image patch

Two sets of improved SIFT descriptors

STEP 3: RECOGNITION

3.1. Background model: the differences of gradient directions between statistically independent image points are uniformly distributed in $(0, \pi)$ (Sect. 11.3.2)
3.2. Given any two keypoints in the databases get their corresponding image patches (up to the similarity transform between the keypoints)(Sect. 11.4)
3.3. Sort a set of statistically independent pixels in the patches and compute the NFA of the matching (Sect. 11.4)
3.4. Keep ε-meaningful matching pairs

Meaningful Matching Pairs

STEP 4: GROUPING

4.1. Build a background model on the set of similarities associated to the matching pairs (Sect. 8.3.2)
4.2. The similarity transforms (Sect. 8.2.1) associated with the matching pairs form a point data set
4.3. From this set a clustering tree is built (Sect. A.1.3) using the dissimilarity measure of Def. 17 (similarity case)
4.4. Maximal groups are computed using the grouping algorithm in Sect. 7.4.1 and Def. 16

Maximal Meaningful Groups of Matches

References

1. Abbasi, S., Mokhtarian, F.: Retrieval of similar shapes under affine transformation. In: Proceedings of International Conference on Visual Information Systems, pp. 566–574. Amsterdam, The Netherlands (1999)
2. Almansa, A., Desolneux, A., Vamech, S.: Vanishing point detection without any a priori information. IEEE Transactions on Pattern Analysis and Machine Intelligence **25**(4), 502–507 (2003)
3. Alt, H., Guibas, L.: Discrete geometric shapes: Matching, interpolation, and approximation. In: J.R. Sack, J. Urrutia (eds.) Handbook of Computational Geometry, pp. 121–153. Elsevier Science Publishers (1999)
4. Alt, H., Knauer, C., Wenk, C.: Matching polygonal curves with respect to the Fréchet distance. In: Proceedings of the 18th International Symposium on Theoretical Aspects of Computer Science, pp. 63–74. Dresden, Germany (2001)
5. Alvarez, L., Guichard, F., Lions, P.L., Morel, J.M.: Axioms and fundamental equations of image processing: Multiscale analysis and P.D.E. Archive for Rational Mechanics and Analysis **16**(9), 200–257 (1993)
6. Alvarez, L., Mazorra, L., Santana, F.: Geometric invariant shape representations using morphological multiscale analysis and applications to shape representation. Journal of Mathematical Imaging and Vision **18**(2), 145–168 (2002)
7. Angenent, S., Sapiro, G., Tannenbaum, A.: On the affine heat flow for nonconvex curves. Journal of the American Mathematical Society (1998)
8. Ansari, N., Delp, E.J.: Partial shape recognition: A landmark-based approach. IEEE Transactions on Pattern Analysis and Machine Intelligence **12**(5), 470–483 (1990)
9. Arnaud, N., Cavalier, F., Davier, M., Hello, P.: Detection of gravitational wave bursts by interferometric detectors. Physical review D **59**(8), 082,002–1 – 082,002–9 (1999)
10. Asada, H., Brady, M.: The curvature primal sketch. IEEE Transactions on Pattern Analysis and Machine Intelligence **8**(1), 2–14 (1986)
11. Åström, K.: Affine and projective normalization of planar curves and regions. In: Proceedings of European Conference on Computer Vision, vol. 2, pp. 439–448. Stockholm, Sweden (1994)
12. Åström, K.: Fundamental limitations on projective invariants of planar curves. IEEE Transactions on Pattern Analysis and Machine Intelligence **17**(1), 77–81 (1995)
13. Attneave, F.: Some informational aspects of visual perception. Psychological review **61**(3), 183–193 (1954)
14. Ballard, D.: Generalizing the Hough transform to detect arbitrary shapes. Pattern Recognition **13**(2), 111–122 (1981)
15. Ballester, C., Caselles, V., Monasse, P.: The tree of shapes of an image. ESAIM: Control, Optimisation and Calculus of Variations **9**, 1–18 (2003)

16. Barles, G., Souganidis, P.: Convergence of approximation schemes for fully nonlinear second order equations. Asymptotic Analysis **4**, 271–283 (1991)
17. Barlow, H.: What is the computational goal of the neocortex? Large-scale neuronal theories of the brain. MIT Press, Cambridge, MA (1994)
18. Basri, R., Costa, L., Geiger, D., Jacobs, D.: Determining the similarity of deformable shapes. Vision Research **38**(15-16), 2365–2385 (1998)
19. Baumberg, A.: Reliable feature matching across widely separated views. In: Proceedings of the International Conference on Computer Vision and Pattern Recognition, pp. 774–781. Washington DC, USA (2004)
20. Beg, M.F., Miller, M., Trouvé, A., Younes, L.: Computing large deformation metric mappings via geodesic flows of diffeomorphisms. Internation Journal of Computer Vision **61**(2), 139–157 (2005)
21. Belongie, S., Carson, C., Greenspan, H., Malik, J.: Color- and texture-based image segmentation using the Expectation-Maximization algorithm and its application to content-based image retrieval. In: Proceedings of the International Conference on Computer Vision, pp. 675–682. Mumbai, India (1998)
22. Belongie, S., Malik, J., Puzicha, J.: Shape matching and object recognition using shape contexts. IEEE Transactions on Pattern Analysis and Machine Intelligence **24**(5), 509–522 (2002)
23. Bienenstock, E., Geman, S., Potter, D.: Compositionality, MDL priors, and object recognition. In: M. Mozera, M. Jordan, T. Petsche (eds.) Advances in Neural Information Processing Systems 9. MIT Press (1998)
24. Bock, H.: On some significance tests in cluster analysis. Journal of Classification **2**, 77–108 (1985)
25. Bottou, L., Bengio, Y.: Convergence properties of the k-means algorithms. In: G. Tesario, D. Touretzky (eds.) Advances in Neural Information Processing Systems, vol. 7, pp. 585–592. Denver, Colorado, USA (1995)
26. Calinski, T., Harabasz, J.: A dendrite method for cluster analysis. Communications in statistics **3**(1), 1–27 (1974)
27. Canny, J.: A variational approach to edge detection. In: National Conference on Artificial Intelligence, pp. 54–58. Washington DC, USA (1983)
28. Canny, J.: A computational approach to edge detection. IEEE Transactions on Pattern Analysis and Machine Intelligence **8**(6), 679–698 (1986)
29. Cao, F.: Geometric Curve Evolution and Image Processing, *Lecture Notes in Mathematics*, vol. 1805. Springer Verlag (2003)
30. Cao, F.: Good continuations in digital images. In: Proceeding of International Conference on Computer Vision, vol. 1, pp. 440–447. Nice, France (2003)
31. Cao, F.: Application of the Gestalt principles to the detection of good continuations and corners in image level lines. Computing and Visualisation in Science **7**(1), 3–13 (2004)
32. Cao, F., Bouthemy, P.: A general criterion for image similarity detection. Technical Report IRISA 1732 (2005)
33. Cao, F., Delon, J., Desolneux, A., Musé, P., Sur, F.: A unified framework for detecting groups and application to shape recognition. Journal of Mathematical Imaging and Vision **27**(2), 91–119 (2007)
34. Cao, F., Moisan, L.: A geometrical scheme for curve evolution driven by curvature. SIAM Journal of Numerical Analysis **39**(2), 624–646 (2000)
35. Cao, F., Musé, P., Sur, F.: Extracting meaningful curves from images. Journal of Mathematical Imaging and Vision **22**(2-3), 159–181 (2005)
36. Caselles, V., Coll, B., Morel, J.M.: A Kanizsa program. Progress in Nonlinear Differential Equations and their Applications **25**, 35–55 (1996)
37. Caselles, V., Coll, B., Morel, J.M.: Topographic maps and local contrast changes in natural images. International Journal of Computer Vision **33**(1), 5–27 (1999)
38. Chapple, P., Bertilone, D., Caprari, R., Newsam, G.: Stochastic model-based processing for detection of small targets in non-gaussian natural imagery. IEEE Transactions on Image Processing **10**(4), 554–564 (2001)

39. Cohen, F., Huang, Z., Yang, Z.: Invariant matching and identification of curves using B-Splines curve representation. IEEE Transactions on Image Processing **4**(1), 1–10 (1995)
40. Cohignac, T.: Reconnaissance de formes planes. Ph.D. thesis, Ceremade, Université Paris IX Dauphine (1994)
41. Cohignac, T., Lopez, C., Morel, J.M.: Integral and local affine invariant parameter and application to shape recognition. In: International Conference on Pattern Recognition, pp. A:164–168. Jerusalem, Israel (1994)
42. Cortadellas, J., Amat, J., de la Torre, F.: Robust normalization of silhouettes for recognition applications. Pattern Recognition Letters **25**(5), 591–601 (2004)
43. Cox, J., Karron, D.: Digital Morse theory (1998). Manuscript available at http://www.casi.net/D.DMT/D.Overview/AcademicPressPaper14-03
44. Day, W., Edelsbrunner, H.: Efficient algorithms for agglomerative hierarchical clustering methods. Journal of Classification **1**(1), 7–24 (1984)
45. Debled-Rennesson, I., Rémy, J.L., Rouyer-Degli, J.: Segmentation of discrete curves into fuzzy segments. Tech. Rep. 4989, INRIA (2003)
46. Dempster, A., Laird, N., Rubin, D.: Maximum likelihood from incomplete data via EM algorithm. Journal of the Royal Statistical Society, Series B **39**, 1–38 (1977)
47. Deriche, R.: Using Canny's criteria to derive a recursively implemented optimal edge detector. International Journal of Computer Vision **1**(2), 167–187 (1987)
48. Deriche, R., Faugeras, O.: Tracking line segments. Image and Vision Computing **8**(4), 261–270 (1990)
49. Desolneux, A., Ladjal, S., Moisan, L., Morel, J.M.: Dequantizing image orientation. IEEE Transactions on Image Processing **11**(10), 1129–1140 (2002)
50. Desolneux, A., Moisan, L., Morel, J.M.: Meaningful alignments. International Journal of Computer Vision **40**(1), 7–23 (2000)
51. Desolneux, A., Moisan, L., Morel, J.M.: Edge detection by Helmholtz principle. Journal of Mathematical Imaging and Vision **14**(3), 271–284 (2001)
52. Desolneux, A., Moisan, L., Morel, J.M.: Computational gestalts and perception thresholds. Journal of Physiology - Paris **97**(2-3), 311–322 (2003)
53. Desolneux, A., Moisan, L., Morel, J.M.: A grouping principle and four applications. IEEE Transactions on Pattern Analysis and Machine Intelligence **25**(4), 508–513 (2003)
54. Desolneux, A., Moisan, L., Morel, J.M.: Gestalt Theory and Image Analysis, a Probabilistic Approach. Interdisciplinary Applied Mathematics series, Springer Verlag (2007). Preprint available at http://www.cmla.ens-cachan.fr/Utilisateurs/morel/lecturenote.pdf
55. Devijver, P., Kittler, J.: Pattern recognition - A statistical approach. Prentice Hall (1982)
56. Donoser, M., Bischof, H.: Efficient Maximally Stable Extremal Region (MSER) Tracking. Proceedings of the 2006 IEEE Computer Society Conference on Computer Vision and Pattern Recognition-Volume 1 pp. 553–560 (2006)
57. Dryden, I.: General shape and registration analysis. Tech. rep., University of Leeds, Department of Statistics (1996)
58. Dubes, R..C.: How many clusters are best? – an experiment. Pattern Recognition **20**(6), 645–663 (1987)
59. Duda, R., Hart, P.: Pattern Classification and Scene Analysis. John Wiley and Sons (1973)
60. Duda, R., Hart, P., Stork, D.: Pattern Classification. John Wiley and Sons (2000). 2nd edition
61. Dudani, S., Breeding, K., McGhee, R.: Aircraft identification by moment invariants. IEEE Transactions on Computers **26**(1), 39–46 (1977)
62. Dudek, G., Tsotsos, J.: Shape representation and recognition from multiscale curvature. Computer Vision and Image Understanding **2**(68), 170–189 (1997)
63. Faugeras, O., Keriven, R.: Some recent results on the projective evolution of 2d curves. In: Proceedings of IEEE International Conference on Image Processing, vol. 3, pp. 13–16. Washington DC, USA (1995)
64. Fischler, M., Bolles, R.: Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography. Communications of the Association for Computing Machinery **24**(6), 381–395 (1981)

65. Fischler, M., Bolles, R.: Perceptual organization and curve partitioning. IEEE Transactions on pattern analysis and machine intelligence **8**(1), 100–105 (1986)
66. Frosini, P., Landi, C.: Size theory as a topological tool for computer vision. Pattern Recognition and Image Analysis **9**(4), 596–603 (1999)
67. Frosini, P., Landi, C.: Size functions and formal series. Applicable Algebra in Engineering, Communication and Computing **12**, 327–349 (2001)
68. Gdalyahu, Y., Weinshall, D.: Flexible syntactic matching of curves and its application to automatic hierarchical classification of silhouettes. IEEE Transactions on Pattern Analysis and Machine Intelligence **21**(12), 1312–1328 (1999)
69. Geman, S., McClure, D.: Statistical methods for tomographic image reconstruction. Bulletin of the International Statistical Institute **52**, 5–21 (1987)
70. Giraudon, G.: Chaînage efficace de contour. Tech. Rep. 0605, INRIA (1987)
71. Golub, G., Loan, C.V.: Matrix Computations. Johns Hopkins University Press (1989)
72. Gordon, A.: Null models in cluster validation. In: W. Gaul, D. Pfeifer (eds.) From Data to Knowledge: Theoretical and Practical Aspects of Classification, Data Analysis, and Knowledge Organization, pp. 32–44. Springer Verlag (1996)
73. Gordon, A.: Classification. Monographs on Statistics and Applied Probability 82, Chapman & Hall (1999)
74. Gousseau, Y.: Comparaison de la composition de deux images, et application à la recherche automatique. In: proceedings of GRETSI 2003. Paris, France (2003)
75. Grimson, W., Huttenlocher, D.: On the sensitivity of the Hough transform for object recognition. IEEE Transactions on Pattern Analysis and Machine Intelligence **12**(3), 255–274 (1990)
76. Grimson, W., Huttenlocher, D.: On the verification of hypothesized matches in model-based recognition. IEEE Transactions on Pattern Analysis and Machine Intelligence **13**(12), 1201–1213 (1991)
77. Guigues, L.: Modèles multi-échelles pour la segmentation d'images. Ph.D. thesis, Université de Cergy-Pontoise (2003)
78. Guy, G., Medioni, G.: Inferring global perceptual contours from local features. International Journal on Computer Vision **20**(1), 113–133 (1996)
79. Haralick, R.: Digital step edges from zero crossing of second directional derivatives. IEEE Transactions on Pattern Analysis and Machine Intelligence **6**(1), 58–68 (1984)
80. Harris, C., Stephens, M.: A combined corner and edge detector. In: 4th Alvey Vision Conference, Manchester, pp. 189–192 (1988)
81. Hartley, R., Zisserman, A.: Multiple View Geometry in Computer Vision. Cambridge University Press (2000)
82. Hastie, T., Tibshirani, R., Friedman, J.: The Elements of Statistical Learning Data Mining, Inference, and Prediction. Springer Series in Statistics (2001)
83. Helmholtz, H.: Treatise on Physiological Optics. Dover, New York (1962 (first published in 1867))
84. Hoeffding, W.: Probability Inequalities for Sums of Bounded Random Variables. Journal of the American Statistical Association **58**(301), 13–30 (1963)
85. Hough, P.: Methods and means for recognizing complex patterns (1962). U.S. Patent 3,069,654
86. Hu, M.: Visual pattern recognition by moments invariants. IRE Transactions on Information Theory **8**, 179–187 (1962)
87. Huang, Z., Cohen, F.: Affine-invariant B-spline moments for curve matching. IEEE Transactions on Image Processing **5**(10), 1473–1480 (1996)
88. Huttenlocher, D., Klanderman, G., Rucklidge, W.: Comparing images using the Hausdorff distance. IEEE Transactions on Pattern Analysis and Machine Intelligence **15**(9), 850–863 (1993)
89. Huttenlocher, D., Ullman, S.: Object recognition using alignment. In: International Conference of Computer Vision, pp. 267–291. London, UK (1987)
90. Hyvärinen, A.: Survey on independent component analysis. Neural Computing Surveys **2**, 94–128 (1999)

91. Illingworth, J., Kittler, J.: A survey of the Hough transform. Computer Vision, Graphics, and Image Processing **44**(1), 87–116 (1988)
92. Jacobs, D.: Robust and efficient detection of salient convex groups. IEEE Transactions on Pattern Analysis and Machine Intelligence **18**(1), 23–37 (1996)
93. Jain, A.K., Murty, M.N., Flynn, P.J.: Data clustering: a review. ACM Computing Surveys **31**(3), 264–323 (1999)
94. Joag-Dev, K., Proschan, F.: Negative association of random variables, with applications. Annals of Statistics **11**(1), 286–295 (1983)
95. Kanizsa, G.: Organization in Vision: Essays on Gestalt Perception. Praeger (1979)
96. Kaufman, L., Rousseeuw, P.J.: Finding groups in data: an introduction to cluster analysis. John Wiley and Sons (1990)
97. Khalil, M., Bayoumi, M.: A dyadic wavelet affine invariant function for 2d shape recognition. IEEE Transactions on Pattern Analysis and Machine Intelligence **23**(10) (2001)
98. Koenderink, J.: The structure of images. Biological Cybernetics **50**, 363–370 (1984)
99. Koepfler, G., Moisan, L.: Geometric multiscale representation of numerical images. In: Second International Conference on Scale Space Theories in Computer Vision, *Lecture Notes in Computer Science*, vol. 1682, pp. 339–350. Springer, Corfu, Greece (1999)
100. Krzyzak, A., Leung, S., Suen, C.: Reconstruction of two-dimensional patterns from Fourier descriptors. Machine Vision and Applications **2**, 123–140 (1989)
101. Lamdan, Y., Schwartz, J., Wolfson, H.: Object recognition by affine invariant matching. In: Proceedings of IEEE International Conference on Computer Vision and Pattern Recognition, pp. 335–344. Ann Arbor, Michigan, U.S.A. (1988)
102. Lamdan, Y., Wolfson, H.: Geometric hashing: a general and efficient model-based recognition scheme. In: Proceedings of IEEE International Conference on Computer Vision, pp. 238–249. Tampa, Florida, USA (1988)
103. Lance, G., Williams, W.: A general theory of classificatory sorting strategies. I. hierarchical systems. Computer Journal **9**, 373–370 (1967)
104. Lin, C., Chellappa, R.: Classification of partial 2-d shapes using Fourier descriptors. In: Proceedings of the International Conference on Computer Vision and Pattern Recognition, pp. 344–350. Miami Beach, Florida, USA (1986)
105. Lindeberg, T.: Feature detection with automatic scale selection. International Journal of Computer Vision **30**(2), 77–116 (1998)
106. Lindenbaum, M.: An integrated model for evaluating the amount of data required for reliable recognition. IEEE Trans. Pattern Analysis Machine Intelligence **19**(11), 1251–1264 (1997)
107. Lindenbaum, M., Bruckstein, A.: On recursive, $O(N)$ partitioning of a digitized curve into digital straight segments. Transactions on Pattern Analysis and Machine Intelligence **15**(9) (1993)
108. Lisani, J.: Shape based automatic images comparison. Ph.D. thesis, Université Paris 9 Dauphine, France (2001)
109. Lisani, J., Moisan, L., Monasse, P., Morel, J.M.: On the theory of planar shape. SIAM Multiscale Modeling and Simulation **1**(1), 1–24 (2003)
110. Lisani, J., Monasse, P., Rudin, L.: Fast shape extraction and applications. Tech. Rep. 2001-16, CMLA, ENS Cachan (2001)
111. Loncaric, S.: A survey of shape analysis techniques. Pattern Recognition **31**(8), 983–1001 (1998)
112. Lowe, D.: Perceptual Organization and Visual Recognition. Kluwer Academic Publisher (1985)
113. Lowe, D.: Object recognition from local scale-invariant features. In: Proceedings of IEEE International Conference on Computer Vision, pp. 1150–1157. Corfu, Greece (1999)
114. Lowe, D.: Distinctive image features from scale-invariant key points. International Journal of Computer Vision **60**(2), 91–110 (2004)
115. MacQueen, J.B.: Some methods for classification and analysis of multivariate observations. In: Proceedings of the 5th Berkeley Symposium on Mathematical Statistics and Probability, vol. 1, pp. 63–74 (1967)

116. Marr, D.: Vision. Freeman Publishers (1982)
117. Marr, D., Hildreth, E.: Theory of edge detection. Proceeding of the Royal Society of London **B-207**, 187–207 (1980)
118. Matas, J., Chum, O., Urban, M., Pajdla, T.: Robust wide baseline stereo from maximally stable extremal regions. Image and Vision Computing **22**(10), 761–767 (2004)
119. Matheron, G.: Random Sets and Integral Geometry. John Wiley and Sons (1975)
120. Mikolajczyk, K., Schmid: A performance evaluation of local descriptors. IEEE Transactions on Pattern Analysis and Machine Intelligence **27**(10), 1615–1630 (2005)
121. Mikolajczyk, K., Schmid, C.: Scale & affine invariant interest point detectors. International Journal of Computer Vision **60**(1), 63–86 (2004)
122. Mikolajczyk, K., Tuytelaars, T., Schmid, C., Zisserman, A., Matas, J., Schaffalitzky, F., Kadir, T., Van Gool, L.: A comparison of affine region detectors. International Journal of Computer Vision **65**(1-2), 43–72 (2005)
123. Miller, M., Trouvé, A., Younes, L.: On the metrics and Euler-Lagrange equations of computational anatomy. Annual Review of Biomedical Engineering **4**, 375–405 (2002)
124. Miller, M., Trouvé, A., Younes, L.: Geodesic shooting for computational anatomy. Journal of Mathematical Imaging and Vision **24**(2), 209–228 (2006)
125. Milligan, G., Cooper, M.: An examination of procedures for determining the number of clusters in a data set. Psychometrika **50**(2), 159–179 (1985)
126. Milnor, J.: Topology from the Differentiable Viewpoint. Princeton University Press (1997)
127. Moisan, L.: Affine plane curve evolution: A fully consistent scheme. IEEE Transactions on Image Processing **7**(3), 411–420 (1998)
128. Moisan, L., Stival, B.: A probabilistic criterion to detect rigid point matches between two images and estimate the fundamental matrix. International Journal of Computer Vision **57**(3), 201–218 (2004)
129. Mokhtarian, F.: Silhouette-based isolated object recognition through curvature scale space. IEEE Transactions on Pattern Analysis and Machine Intelligence **17**(5), 539–544 (1995)
130. Mokhtarian, F., Abbasi, S., Kittler, J.: Efficient and robust retrieval by shape content through curvature scale space. In: Proceedings of International Workshop on Image Databases and MultiMedia Search, pp. 35–42. Amsterdam, The Netherlands (1996)
131. Mokhtarian, F., Abbasi, S., Kittler, J.: Robust and efficient shape indexing through curvature scale space. In: Proceedings of British Machine Vision Conference, pp. 53–62. Edinburgh, UK (1996)
132. Mokhtarian, F., Mackworth, A.: A theory of multiscale, curvature-based shape representation for planar curves. IEEE Transactions on Pattern Analysis and Machine Intelligence **14**(8), 789–805 (1992)
133. Monasse, P.: Contrast invariant image registration. In: Proceedings of International Conference on Acoustics, Speech and Signal Processing, vol. 6, pp. 3221–3224. Phoenix, Arizona, USA (1999)
134. Monasse, P.: Représentation morphologique d'images numériques et application au recalage, morphological representation of digital images and application to registration. Ph.D. thesis, Université Paris 9 Dauphine, France (2000)
135. Monasse, P., Guichard, F.: Fast computation of a contrast invariant image representation. IEEE Transactions on Image Processing **9**(5), 860–872 (2000)
136. Monasse, P., Guichard, F.: Scale-space from a level lines tree. Journal of Visual Communication and Image Representation **11**, 224–236 (2000)
137. Morel, J.M., Solimini, S.: Variational Methods in Image Segmentation. Birkhauser (1995)
138. Murphy-Chutorian, E., Trivedi, M.: N-tree Disjoint-Set Forests for Maximally Stable Extremal Regions. In: Proceedings of the British Machine Vision Conference. Edinburgh, United Kingdom (2006)
139. Musé, P., Sur, F., Cao, F., Gousseau, Y.: Unsupervised thresholds for shape matching. In: Proceedings of IEEE International Conference on Image Processing. Barcelona, Spain (2003)
140. Musé, P., Sur, F., Cao, F., Gousseau, Y., Morel, J.M.: An A Contrario Decision Method for Shape Element Recognition. International Journal of Computer Vision **69**(3), 295–315 (2006)

141. Musé, P., Sur, F., Morel, J.M.: Sur les seuils de reconnaissance des formes. Traitement du Signal **20**(3), 279–294 (2003)
142. Niethammer, M., Betelu, S., Sapiro, G., Tannenbaum, A., Giblin, P.: Area-based medial axis of planar curves. International Journal of Computer Vision **60**(3), 203–224 (2004)
143. Obdržálek, S., Matas, J.: Local affine frames for image retrieval. In: CIVR'02: Proceedings of International Conference The Challenge of Image and Video Retrieval, pp. 318–327. Springer-Verlag (2002)
144. Olson, C., Huttenlocher, D.: Automatic target recognition by matching oriented edge pixels. IEEE Transactions on Image Processing **6**(12), 103–113 (1997)
145. Orrite, C., Herrero, J.: Shape matching of partially occluded curves invariant under projective transformation. Computer Vision and Image Understing **93**(1), 34–64 (2004)
146. Papoulis, A., Bertran, M.: Digital filtering and prolate functions. IEEE Transactions on Circuits and Systems **19**(6), 674–681 (1972)
147. Pennec, X.: Toward a generic framework for recognition based on uncertain geometric features. Videre: Journal of Computer Vision Research **1**(2), 58–87 (1998)
148. Persoon, E., Fu, K.: Shape discrimination using Fourier descriptors. SMC **7**(3), 170–179 (1977)
149. Poor, H.: An Introduction to Signal Detection and Estimation, 2nd edn. Springer Texts in Electrical Engineering. Springer Verlag (1994)
150. Rabin, J., Gousseau, Y., Delon, J.: A contrario matching of local descriptors. Tech. Rep. hal-00168285, Ecole Nationale Supérieure des Télécommunications, Paris, France (2007)
151. Rothwell, C.: Object Recognition Through Invariant Indexing. Oxford Science Publications (1995)
152. Rothwell, C., Zisserman, A., Forsyth, D., Mundy, J.: Planar object recognition using projective shape representation. International Journal of Computer Vision **16**, 57–99 (1995)
153. Rubin, E.: Visuell wahrgenommene Figuren. Copenhagen, Gyldendals (1921)
154. Salembier, P., Serra, J.: Flat zones filtering, connected operators, and filters by reconstruction. IEEE Transactions on Image Processing **4**(8), 1153–1160 (1995)
155. Sapiro, G., Tannenbaum, A.: Affine invariant scale-space. International Journal of Computer Vision **11**(1), 25–44 (1993)
156. Sato, J., Cipolla, R.: Quasi-invariant parameterisations and matching of curves in images. International Journal of Computer Vision **28**(2), 117–138 (1998)
157. Schmid, C., G., Dorko, Lazebnik, S., Mikolajczyk, K., Ponce., J.: Pattern recognition with local invariant features. In: C. Chen, e. P.S.P Wang (eds.) Handbook of Pattern Recognition and Computer Vision. World Scientific Publishing Co. (2005). 3rd edition
158. Schmid, C., Mohr, R.: Combining greyvalue invariants with local constraints for object recognition. In: Proc. CVPR96, pp. 872–877. San Francisco, California (1996)
159. Schmid, C., Mohr, R.: Local greyvalue invariants for image retrieval. IEEE Transaction on Pattern Analysis and Machine Intelligence **19**(5), 530–535 (1997)
160. Sclaroff, S., Pentland, A.: Modal matching for correspondence and recognition. IEEE Transactions on Pattern Analysis and Machine Intelligence **17**(6), 545–561 (1995)
161. Serra, J.: Image Analysis and Mathematical Morphology. Academic Press (1982)
162. Shen, D., Ip, H.: Discriminative wavelet shape descriptors for recognition of 2-d patterns. Pattern Recognition **32**(8), 151–165 (1999)
163. Sivic, J., Zisserman, A.: Video Google: a text retrieval approach to object matching in videos. In: Ninth IEEE International Conference on Computer Vision, pp. 1470–1477 (2003)
164. Sklansky, J., Gonzalez, V.: Fast polygonal approximation of digitized curves. Pattern Recognition **12**, 327–331 (1980)
165. Small, C.: The Statistical Theory of Shapes. Springer Verlag (1996)
166. Song, Y.: A Top-down Algorithm for Computation of Level Line Trees. IEEE Transactions on Image Processing **16**(8), 2107–2116 (2007)
167. Song, Y., Zhang, A.: Analyzing scenery images by monotonic tree. Multimedia Systems **8**(6), 495–511 (2003)
168. Stewart, C.: MINPRAN: a new robust estimator for computer vision. IEEE Transactions on Pattern Analysis and Machine Intelligence **17**(10), 925–938 (1995)

169. Stockman, G., Kopstein, S., Benett, S.: Matching images to models for registration and object detection via clustering. IEEE Transactions on Pattern Analysis and Machine Intelligence **4**(3), 229–241 (1982)
170. Tan, P., Steinbach, M., Kumar, V.: Introduction to Data Mining. Addison-Wesley (2005)
171. de la Torre, F., Black, M.: A framework for robust subspace learning. International Journal of Computer Vision **54**(1-2-3), 117–142 (2003)
172. Veltkamp, R.: Shape matching: similarity measures and algorithms. In: Proceedings of International Conference on Shape Modeling and Applications, pp. 188–197. Genova, Italy (2001)
173. Veltkamp, R., Hagedoorn, M.: State-of-the-art in shape matching. In: M. Lew (ed.) Principles of Visual Information Retrieval, vol. 19. Springer Verlag (2001)
174. Veltkamp, R., Tanase, M.: Content-based image retrieval systems: A survey. Tech. Rep. UU-CS-2000-34, Utrecht University (2000)
175. Venters, C.C., Cooper, M.D.: A review of content-based image retrieval systems. Tech. Rep. jtap-054, University of Manchester, UK (2000)
176. Ward, J.H.J.: Hierarchical grouping to optimize an objective function. Journal of the American Statistical Association **58**(2), 236–244 (1963)
177. Watson, G., Watson, S.: Detection of unusual events in intermittent non-gaussian images using multiresolution background models. Optical Engineering **35**(11), 3159–3171 (1996)
178. Weiss, I.: Noise-resistant invariants of curves. IEEE Transactions on Pattern Analysis and Machine Intelligence **15**(9), 943–948 (1993)
179. Wertheimer, M.: Untersuchungen zur Lehre der Gestalt, II. Psychologische Forschung **4**, 301–350 (1923). Translation published as Laws of Organization in Perceptual Forms, in Ellis, W. (1938). A source book of Gestalt psychology (pp. 71-88). Routledge & Kegan Paul
180. Winter, A., Nastar, C.: Differential feature distribution maps for image segmentation and region queries in image databases. In: CBAIVL Workshop at Conference on Computer Vision and Pattern Recognition. Fort Collins, Colorado, USA (1999)
181. Witkin, A.: Scale space filtering. In: Proceedings of International Joint Conference on Artificial Intelligence, pp. 1019–1021. Karlsruhe, Germany (1983)
182. Wolfson, H.: Model-based object recognition by Geometric Hashing. In: Proceedings of the European Conference on Computer Vision, pp. 526–536. Lecture Notes in Computer Vision 427, Springer Verlag, Antibes, France (1990)
183. Wolfson, H.: On curve matching. IEEE Transactions on Pattern Analysis and Machine Intelligence **12**(5), 483–489 (1990)
184. Wolfson, H., Rigoutsos, I.: Geometric hashing: an overview. IEEE Computational Science & Engineering **4**(4), 10–21 (1997)
185. Zahn, C., Roskies, R.: Fourier descriptors for plane closed curves. IEEE Transactions on Computers **C-21**(3), 269–281 (1972)

Index

T

W

Lecture Notes in Mathematics

For information about earlier volumes
please contact your bookseller or Springer
LNM Online archive: springerlink.com

Vol. 1766: H. Hennion, L. Hervé, Limit Theorems for Markov Chains and Stochastic Properties of Dynamical Systems by Quasi-Compactness (2001)
Vol. 1767: J. Xiao, Holomorphic Q Classes (2001)
Vol. 1768: M. J. Pflaum, Analytic and Geometric Study of Stratified Spaces (2001)
Vol. 1769: M. Alberich-Carramiñana, Geometry of the Plane Cremona Maps (2002)
Vol. 1770: H. Gluesing-Luerssen, Linear Delay-Differential Systems with Commensurate Delays: An Algebraic Approach (2002)
Vol. 1771: M. Émery, M. Yor (Eds.), Séminaire de Probabilités 1967-1980. A Selection in Martingale Theory (2002)
Vol. 1772: F. Burstall, D. Ferus, K. Leschke, F. Pedit, U. Pinkall, Conformal Geometry of Surfaces in S^4 (2002)
Vol. 1773: Z. Arad, M. Muzychuk, Standard Integral Table Algebras Generated by a Non-real Element of Small Degree (2002)
Vol. 1774: V. Runde, Lectures on Amenability (2002)
Vol. 1775: W. H. Meeks, A. Ros, H. Rosenberg, The Global Theory of Minimal Surfaces in Flat Spaces. Martina Franca 1999. Editor: G. P. Pirola (2002)
Vol. 1776: K. Behrend, C. Gomez, V. Tarasov, G. Tian, Quantum Comohology. Cetraro 1997. Editors: P. de Bartolomeis, B. Dubrovin, C. Reina (2002)
Vol. 1777: E. García-Río, D. N. Kupeli, R. Vázquez-Lorenzo, Osserman Manifolds in Semi-Riemannian Geometry (2002)
Vol. 1778: H. Kiechle, Theory of K-Loops (2002)
Vol. 1779: I. Chueshov, Monotone Random Systems (2002)
Vol. 1780: J. H. Bruinier, Borcherds Products on O(2,1) and Chern Classes of Heegner Divisors (2002)
Vol. 1781: E. Bolthausen, E. Perkins, A. van der Vaart, Lectures on Probability Theory and Statistics. Ecole d' Eté de Probabilités de Saint-Flour XXIX-1999. Editor: P. Bernard (2002)
Vol. 1782: C.-H. Chu, A. T.-M. Lau, Harmonic Functions on Groups and Fourier Algebras (2002)
Vol. 1783: L. Grüne, Asymptotic Behavior of Dynamical and Control Systems under Perturbation and Discretization (2002)
Vol. 1784: L. H. Eliasson, S. B. Kuksin, S. Marmi, J.-C. Yoccoz, Dynamical Systems and Small Divisors. Cetraro, Italy 1998. Editors: S. Marmi, J.-C. Yoccoz (2002)
Vol. 1785: J. Arias de Reyna, Pointwise Convergence of Fourier Series (2002)
Vol. 1786: S. D. Cutkosky, Monomialization of Morphisms from 3-Folds to Surfaces (2002)
Vol. 1787: S. Caenepeel, G. Militaru, S. Zhu, Frobenius and Separable Functors for Generalized Module Categories and Nonlinear Equations (2002)
Vol. 1788: A. Vasil'ev, Moduli of Families of Curves for Conformal and Quasiconformal Mappings (2002)
Vol. 1789: Y. Sommerhäuser, Yetter-Drinfel'd Hopf algebras over groups of prime order (2002)
Vol. 1790: X. Zhan, Matrix Inequalities (2002)
Vol. 1791: M. Knebusch, D. Zhang, Manis Valuations and Prüfer Extensions I: A new Chapter in Commutative Algebra (2002)
Vol. 1792: D. D. Ang, R. Gorenflo, V. K. Le, D. D. Trong, Moment Theory and Some Inverse Problems in Potential Theory and Heat Conduction (2002)
Vol. 1793: J. Cortés Monforte, Geometric, Control and Numerical Aspects of Nonholonomic Systems (2002)
Vol. 1794: N. Pytheas Fogg, Substitution in Dynamics, Arithmetics and Combinatorics. Editors: V. Berthé, S. Ferenczi, C. Mauduit, A. Siegel (2002)
Vol. 1795: H. Li, Filtered-Graded Transfer in Using Noncommutative Gröbner Bases (2002)
Vol. 1796: J.M. Melenk, hp-Finite Element Methods for Singular Perturbations (2002)
Vol. 1797: B. Schmidt, Characters and Cyclotomic Fields in Finite Geometry (2002)
Vol. 1798: W.M. Oliva, Geometric Mechanics (2002)
Vol. 1799: H. Pajot, Analytic Capacity, Rectifiability, Menger Curvature and the Cauchy Integral (2002)
Vol. 1800: O. Gabber, L. Ramero, Almost Ring Theory (2003)
Vol. 1801: J. Azéma, M. Émery, M. Ledoux, M. Yor (Eds.), Séminaire de Probabilités XXXVI (2003)
Vol. 1802: V. Capasso, E. Merzbach, B. G. Ivanoff, M. Dozzi, R. Dalang, T. Mountford, Topics in Spatial Stochastic Processes. Martina Franca, Italy 2001. Editor: E. Merzbach (2003)
Vol. 1803: G. Dolzmann, Variational Methods for Crystalline Microstructure – Analysis and Computation (2003)
Vol. 1804: I. Cherednik, Ya. Markov, R. Howe, G. Lusztig, Iwahori-Hecke Algebras and their Representation Theory. Martina Franca, Italy 1999. Editors: V. Baldoni, D. Barbasch (2003)
Vol. 1805: F. Cao, Geometric Curve Evolution and Image Processing (2003)
Vol. 1806: H. Broer, I. Hoveijn. G. Lunther, G. Vegter, Bifurcations in Hamiltonian Systems. Computing Singularities by Gröbner Bases (2003)
Vol. 1807: V. D. Milman, G. Schechtman (Eds.), Geometric Aspects of Functional Analysis. Israel Seminar 2000-2002 (2003)
Vol. 1808: W. Schindler, Measures with Symmetry Properties (2003)
Vol. 1809: O. Steinbach, Stability Estimates for Hybrid Coupled Domain Decomposition Methods (2003)
Vol. 1810: J. Wengenroth, Derived Functors in Functional Analysis (2003)
Vol. 1811: J. Stevens, Deformations of Singularities (2003)

Vol. 1812: L. Ambrosio, K. Deckelnick, G. Dziuk, M. Mimura, V. A. Solonnikov, H. M. Soner, Mathematical Aspects of Evolving Interfaces. Madeira, Funchal, Portugal 2000. Editors: P. Colli, J. F. Rodrigues (2003)
Vol. 1813: L. Ambrosio, L. A. Caffarelli, Y. Brenier, G. Buttazzo, C. Villani, Optimal Transportation and its Applications. Martina Franca, Italy 2001. Editors: L. A. Caffarelli, S. Salsa (2003)
Vol. 1814: P. Bank, F. Baudoin, H. Föllmer, L.C.G. Rogers, M. Soner, N. Touzi, Paris-Princeton Lectures on Mathematical Finance 2002 (2003)
Vol. 1815: A. M. Vershik (Ed.), Asymptotic Combinatorics with Applications to Mathematical Physics. St. Petersburg, Russia 2001 (2003)
Vol. 1816: S. Albeverio, W. Schachermayer, M. Talagrand, Lectures on Probability Theory and Statistics. Ecole d'Eté de Probabilités de Saint-Flour XXX-2000. Editor: P. Bernard (2003)
Vol. 1817: E. Koelink, W. Van Assche (Eds.), Orthogonal Polynomials and Special Functions. Leuven 2002 (2003)
Vol. 1818: M. Bildhauer, Convex Variational Problems with Linear, nearly Linear and/or Anisotropic Growth Conditions (2003)
Vol. 1819: D. Masser, Yu. V. Nesterenko, H. P. Schlickewei, W. M. Schmidt, M. Waldschmidt, Diophantine Approximation. Cetraro, Italy 2000. Editors: F. Amoroso, U. Zannier (2003)
Vol. 1820: F. Hiai, H. Kosaki, Means of Hilbert Space Operators (2003)
Vol. 1821: S. Teufel, Adiabatic Perturbation Theory in Quantum Dynamics (2003)
Vol. 1822: S.-N. Chow, R. Conti, R. Johnson, J. Mallet-Paret, R. Nussbaum, Dynamical Systems. Cetraro, Italy 2000. Editors: J. W. Macki, P. Zecca (2003)
Vol. 1823: A. M. Anile, W. Allegretto, C. Ringhofer, Mathematical Problems in Semiconductor Physics. Cetraro, Italy 1998. Editor: A. M. Anile (2003)
Vol. 1824: J. A. Navarro González, J. B. Sancho de Salas, $\mathscr{C}^\infty$ – Differentiable Spaces (2003)
Vol. 1825: J. H. Bramble, A. Cohen, W. Dahmen, Multiscale Problems and Methods in Numerical Simulations, Martina Franca, Italy 2001. Editor: C. Canuto (2003)
Vol. 1826: K. Dohmen, Improved Bonferroni Inequalities via Abstract Tubes. Inequalities and Identities of Inclusion-Exclusion Type. VIII, 113 p, 2003.
Vol. 1827: K. M. Pilgrim, Combinations of Complex Dynamical Systems. IX, 118 p, 2003.
Vol. 1828: D. J. Green, Gröbner Bases and the Computation of Group Cohomology. XII, 138 p, 2003.
Vol. 1829: E. Altman, B. Gaujal, A. Hordijk, Discrete-Event Control of Stochastic Networks: Multimodularity and Regularity. XIV, 313 p, 2003.
Vol. 1830: M. I. Gil', Operator Functions and Localization of Spectra. XIV, 256 p, 2003.
Vol. 1831: A. Connes, J. Cuntz, E. Guentner, N. Higson, J. E. Kaminker, Noncommutative Geometry, Martina Franca, Italy 2002. Editors: S. Doplicher, L. Longo (2004)
Vol. 1832: J. Azéma, M. Émery, M. Ledoux, M. Yor (Eds.), Séminaire de Probabilités XXXVII (2003)
Vol. 1833: D.-Q. Jiang, M. Qian, M.-P. Qian, Mathematical Theory of Nonequilibrium Steady States. On the Frontier of Probability and Dynamical Systems. IX, 280 p, 2004.
Vol. 1834: Yo. Yomdin, G. Comte, Tame Geometry with Application in Smooth Analysis. VIII, 186 p, 2004.
Vol. 1835: O.T. Izhboldin, B. Kahn, N.A. Karpenko, A. Vishik, Geometric Methods in the Algebraic Theory of Quadratic Forms. Summer School, Lens, 2000. Editor: J.-P. Tignol (2004)
Vol. 1836: C. Năstăsescu, F. Van Oystaeyen, Methods of Graded Rings. XIII, 304 p, 2004.
Vol. 1837: S. Tavaré, O. Zeitouni, Lectures on Probability Theory and Statistics. Ecole d'Eté de Probabilités de Saint-Flour XXXI-2001. Editor: J. Picard (2004)
Vol. 1838: A.J. Ganesh, N.W. O'Connell, D.J. Wischik, Big Queues. XII, 254 p, 2004.
Vol. 1839: R. Gohm, Noncommutative Stationary Processes. VIII, 170 p, 2004.
Vol. 1840: B. Tsirelson, W. Werner, Lectures on Probability Theory and Statistics. Ecole d'Eté de Probabilités de Saint-Flour XXXII-2002. Editor: J. Picard (2004)
Vol. 1841: W. Reichel, Uniqueness Theorems for Variational Problems by the Method of Transformation Groups (2004)
Vol. 1842: T. Johnsen, A. L. Knutsen, K_3 Projective Models in Scrolls (2004)
Vol. 1843: B. Jefferies, Spectral Properties of Noncommuting Operators (2004)
Vol. 1844: K.F. Siburg, The Principle of Least Action in Geometry and Dynamics (2004)
Vol. 1845: Min Ho Lee, Mixed Automorphic Forms, Torus Bundles, and Jacobi Forms (2004)
Vol. 1846: H. Ammari, H. Kang, Reconstruction of Small Inhomogeneities from Boundary Measurements (2004)
Vol. 1847: T.R. Bielecki, T. Björk, M. Jeanblanc, M. Rutkowski, J.A. Scheinkman, W. Xiong, Paris-Princeton Lectures on Mathematical Finance 2003 (2004)
Vol. 1848: M. Abate, J. E. Fornaess, X. Huang, J. P. Rosay, A. Tumanov, Real Methods in Complex and CR Geometry, Martina Franca, Italy 2002. Editors: D. Zaitsev, G. Zampieri (2004)
Vol. 1849: Martin L. Brown, Heegner Modules and Elliptic Curves (2004)
Vol. 1850: V. D. Milman, G. Schechtman (Eds.), Geometric Aspects of Functional Analysis. Israel Seminar 2002-2003 (2004)
Vol. 1851: O. Catoni, Statistical Learning Theory and Stochastic Optimization (2004)
Vol. 1852: A.S. Kechris, B.D. Miller, Topics in Orbit Equivalence (2004)
Vol. 1853: Ch. Favre, M. Jonsson, The Valuative Tree (2004)
Vol. 1854: O. Saeki, Topology of Singular Fibers of Differential Maps (2004)
Vol. 1855: G. Da Prato, P.C. Kunstmann, I. Lasiecka, A. Lunardi, R. Schnaubelt, L. Weis, Functional Analytic Methods for Evolution Equations. Editors: M. Iannelli, R. Nagel, S. Piazzera (2004)
Vol. 1856: K. Back, T.R. Bielecki, C. Hipp, S. Peng, W. Schachermayer, Stochastic Methods in Finance, Bressanone/Brixen, Italy, 2003. Editors: M. Fritelli, W. Runggaldier (2004)
Vol. 1857: M. Émery, M. Ledoux, M. Yor (Eds.), Séminaire de Probabilités XXXVIII (2005)
Vol. 1858: A.S. Cherny, H.-J. Engelbert, Singular Stochastic Differential Equations (2005)
Vol. 1859: E. Letellier, Fourier Transforms of Invariant Functions on Finite Reductive Lie Algebras (2005)
Vol. 1860: A. Borisyuk, G.B. Ermentrout, A. Friedman, D. Terman, Tutorials in Mathematical Biosciences I. Mathematical Neurosciences (2005)

Vol. 1861: G. Benettin, J. Henrard, S. Kuksin, Hamiltonian Dynamics – Theory and Applications, Cetraro, Italy, 1999. Editor: A. Giorgilli (2005)
Vol. 1862: B. Helffer, F. Nier, Hypoelliptic Estimates and Spectral Theory for Fokker-Planck Operators and Witten Laplacians (2005)
Vol. 1863: H. Führ, Abstract Harmonic Analysis of Continuous Wavelet Transforms (2005)
Vol. 1864: K. Efstathiou, Metamorphoses of Hamiltonian Systems with Symmetries (2005)
Vol. 1865: D. Applebaum, B.V. R. Bhat, J. Kustermans, J. M. Lindsay, Quantum Independent Increment Processes I. From Classical Probability to Quantum Stochastic Calculus. Editors: M. Schürmann, U. Franz (2005)
Vol. 1866: O.E. Barndorff-Nielsen, U. Franz, R. Gohm, B. Kümmerer, S. Thorbjønsen, Quantum Independent Increment Processes II. Structure of Quantum Lévy Processes, Classical Probability, and Physics. Editors: M. Schürmann, U. Franz, (2005)
Vol. 1867: J. Sneyd (Ed.), Tutorials in Mathematical Biosciences II. Mathematical Modeling of Calcium Dynamics and Signal Transduction. (2005)
Vol. 1868: J. Jorgenson, S. Lang, $\mathrm{Pos}_n(\mathrm{R})$ and Eisenstein Series. (2005)
Vol. 1869: A. Dembo, T. Funaki, Lectures on Probability Theory and Statistics. Ecole d'Eté de Probabilités de Saint-Flour XXXIII-2003. Editor: J. Picard (2005)
Vol. 1870: V.I. Gurariy, W. Lusky, Geometry of Müntz Spaces and Related Questions. (2005)
Vol. 1871: P. Constantin, G. Gallavotti, A.V. Kazhikhov, Y. Meyer, S. Ukai, Mathematical Foundation of Turbulent Viscous Flows, Martina Franca, Italy, 2003. Editors: M. Cannone, T. Miyakawa (2006)
Vol. 1872: A. Friedman (Ed.), Tutorials in Mathematical Biosciences III. Cell Cycle, Proliferation, and Cancer (2006)
Vol. 1873: R. Mansuy, M. Yor, Random Times and Enlargements of Filtrations in a Brownian Setting (2006)
Vol. 1874: M. Yor, M. Émery (Eds.), In Memoriam Paul-André Meyer - Séminaire de Probabilités XXXIX (2006)
Vol. 1875: J. Pitman, Combinatorial Stochastic Processes. Ecole d'Eté de Probabilités de Saint-Flour XXXII-2002. Editor: J. Picard (2006)
Vol. 1876: H. Herrlich, Axiom of Choice (2006)
Vol. 1877: J. Steuding, Value Distributions of *L*-Functions (2007)
Vol. 1878: R. Cerf, The Wulff Crystal in Ising and Percolation Models, Ecole d'Eté de Probabilités de Saint-Flour XXXIV-2004. Editor: Jean Picard (2006)
Vol. 1879: G. Slade, The Lace Expansion and its Applications, Ecole d'Eté de Probabilités de Saint-Flour XXXIV-2004. Editor: Jean Picard (2006)
Vol. 1880: S. Attal, A. Joye, C.-A. Pillet, Open Quantum Systems I, The Hamiltonian Approach (2006)
Vol. 1881: S. Attal, A. Joye, C.-A. Pillet, Open Quantum Systems II, The Markovian Approach (2006)
Vol. 1882: S. Attal, A. Joye, C.-A. Pillet, Open Quantum Systems III, Recent Developments (2006)
Vol. 1883: W. Van Assche, F. Marcellàn (Eds.), Orthogonal Polynomials and Special Functions, Computation and Application (2006)
Vol. 1884: N. Hayashi, E.I. Kaikina, P.I. Naumkin, I.A. Shishmarev, Asymptotics for Dissipative Nonlinear Equations (2006)
Vol. 1885: A. Telcs, The Art of Random Walks (2006)
Vol. 1886: S. Takamura, Splitting Deformations of Degenerations of Complex Curves (2006)
Vol. 1887: K. Habermann, L. Habermann, Introduction to Symplectic Dirac Operators (2006)
Vol. 1888: J. van der Hoeven, Transseries and Real Differential Algebra (2006)
Vol. 1889: G. Osipenko, Dynamical Systems, Graphs, and Algorithms (2006)
Vol. 1890: M. Bunge, J. Funk, Singular Coverings of Toposes (2006)
Vol. 1891: J.B. Friedlander, D.R. Heath-Brown, H. Iwaniec, J. Kaczorowski, Analytic Number Theory, Cetraro, Italy, 2002. Editors: A. Perelli, C. Viola (2006)
Vol. 1892: A. Baddeley, I. Bárány, R. Schneider, W. Weil, Stochastic Geometry, Martina Franca, Italy, 2004. Editor: W. Weil (2007)
Vol. 1893: H. Hanßmann, Local and Semi-Local Bifurcations in Hamiltonian Dynamical Systems, Results and Examples (2007)
Vol. 1894: C.W. Groetsch, Stable Approximate Evaluation of Unbounded Operators (2007)
Vol. 1895: L. Molnár, Selected Preserver Problems on Algebraic Structures of Linear Operators and on Function Spaces (2007)
Vol. 1896: P. Massart, Concentration Inequalities and Model Selection, Ecole d'Été de Probabilités de Saint-Flour XXXIII-2003. Editor: J. Picard (2007)
Vol. 1897: R. Doney, Fluctuation Theory for Lévy Processes, Ecole d'Été de Probabilités de Saint-Flour XXXV-2005. Editor: J. Picard (2007)
Vol. 1898: H.R. Beyer, Beyond Partial Differential Equations, On linear and Quasi-Linear Abstract Hyperbolic Evolution Equations (2007)
Vol. 1899: Séminaire de Probabilités XL. Editors: C. Donati-Martin, M. Émery, A. Rouault, C. Stricker (2007)
Vol. 1900: E. Bolthausen, A. Bovier (Eds.), Spin Glasses (2007)
Vol. 1901: O. Wittenberg, Intersections de deux quadriques et pinceaux de courbes de genre 1, Intersections of Two Quadrics and Pencils of Curves of Genus 1 (2007)
Vol. 1902: A. Isaev, Lectures on the Automorphism Groups of Kobayashi-Hyperbolic Manifolds (2007)
Vol. 1903: G. Kresin, V. Maz'ya, Sharp Real-Part Theorems (2007)
Vol. 1904: P. Giesl, Construction of Global Lyapunov Functions Using Radial Basis Functions (2007)
Vol. 1905: C. Prévôt, M. Röckner, A Concise Course on Stochastic Partial Differential Equations (2007)
Vol. 1906: T. Schuster, The Method of Approximate Inverse: Theory and Applications (2007)
Vol. 1907: M. Rasmussen, Attractivity and Bifurcation for Nonautonomous Dynamical Systems (2007)
Vol. 1908: T.J. Lyons, M. Caruana, T. Lévy, Differential Equations Driven by Rough Paths, Ecole d'Été de Probabilités de Saint-Flour XXXIV-2004 (2007)
Vol. 1909: H. Akiyoshi, M. Sakuma, M. Wada, Y. Yamashita, Punctured Torus Groups and 2-Bridge Knot Groups (I) (2007)
Vol. 1910: V.D. Milman, G. Schechtman (Eds.), Geometric Aspects of Functional Analysis. Israel Seminar 2004-2005 (2007)
Vol. 1911: A. Bressan, D. Serre, M. Williams, K. Zumbrun, Hyperbolic Systems of Balance Laws. Cetraro, Italy 2003. Editor: P. Marcati (2007)

Vol. 1912: V. Berinde, Iterative Approximation of Fixed Points (2007)
Vol. 1913: J.E. Marsden, G. Misiołek, J.-P. Ortega, M. Perlmutter, T.S. Ratiu, Hamiltonian Reduction by Stages (2007)
Vol. 1914: G. Kutyniok, Affine Density in Wavelet Analysis (2007)
Vol. 1915: T. Bıyıkoğlu, J. Leydold, P.F. Stadler, Laplacian Eigenvectors of Graphs. Perron-Frobenius and Faber-Krahn Type Theorems (2007)
Vol. 1916: C. Villani, F. Rezakhanlou, Entropy Methods for the Boltzmann Equation. Editors: F. Golse, S. Olla (2008)
Vol. 1917: I. Veselić, Existence and Regularity Properties of the Integrated Density of States of Random Schrödinger (2008)
Vol. 1918: B. Roberts, R. Schmidt, Local Newforms for GSp(4) (2007)
Vol. 1919: R.A. Carmona, I. Ekeland, A. Kohatsu-Higa, J.-M. Lasry, P.-L. Lions, H. Pham, E. Taflin, Paris-Princeton Lectures on Mathematical Finance 2004. Editors: R.A. Carmona, E. Çinlar, I. Ekeland, E. Jouini, J.A. Scheinkman, N. Touzi (2007)
Vol. 1920: S.N. Evans, Probability and Real Trees. Ecole d'Été de Probabilités de Saint-Flour XXXV-2005 (2008)
Vol. 1921: J.P. Tian, Evolution Algebras and their Applications (2008)
Vol. 1922: A. Friedman (Ed.), Tutorials in Mathematical BioSciences IV. Evolution and Ecology (2008)
Vol. 1923: J.P.N. Bishwal, Parameter Estimation in Stochastic Differential Equations (2008)
Vol. 1924: M. Wilson, Littlewood-Paley Theory and Exponential-Square Integrability (2008)
Vol. 1925: M. du Sautoy, L. Woodward, Zeta Functions of Groups and Rings (2008)
Vol. 1926: L. Barreira, V. Claudia, Stability of Nonautonomous Differential Equations (2008)
Vol. 1927: L. Ambrosio, L. Caffarelli, M.G. Crandall, L.C. Evans, N. Fusco, Calculus of Variations and Non-Linear Partial Differential Equations. Cetraro, Italy 2005. Editors: B. Dacorogna, P. Marcellini (2008)
Vol. 1928: J. Jonsson, Simplicial Complexes of Graphs (2008)
Vol. 1929: Y. Mishura, Stochastic Calculus for Fractional Brownian Motion and Related Processes (2008)
Vol. 1930: J.M. Urbano, The Method of Intrinsic Scaling. A Systematic Approach to Regularity for Degenerate and Singular PDEs (2008)
Vol. 1931: M. Cowling, E. Frenkel, M. Kashiwara, A. Valette, D.A. Vogan, Jr., N.R. Wallach, Representation Theory and Complex Analysis. Venice, Italy 2004. Editors: E.C. Tarabusi, A. D'Agnolo, M. Picardello (2008)
Vol. 1932: A.A. Agrachev, A.S. Morse, E.D. Sontag, H.J. Sussmann, V.I. Utkin, Nonlinear and Optimal Control Theory. Cetraro, Italy 2004. Editors: P. Nistri, G. Stefani (2008)
Vol. 1933: M. Petković, Point Estimation of Root Finding Methods (2008)
Vol. 1934: C. Donati-Martin, M. Émery, A. Rouault, C. Stricker (Eds.), Séminaire de Probabilités XLI (2008)
Vol. 1935: A. Unterberger, Alternative Pseudodifferential Analysis (2008)
Vol. 1936: P. Magal, S. Ruan (Eds.), Structured Population Models in Biology and Epidemiology (2008)
Vol. 1937: G. Capriz, P. Giovine, P.M. Mariano (Eds.), Mathematical Models of Granular Matter (2008)
Vol. 1938: D. Auroux, F. Catanese, M. Manetti, P. Seidel, B. Siebert, I. Smith, G. Tian, Symplectic 4-Manifolds and Algebraic Surfaces. Cetraro, Italy 2003. Editors: F. Catanese, G. Tian (2008)
Vol. 1939: D. Boffi, F. Brezzi, L. Demkowicz, R.G. Durán, R.S. Falk, M. Fortin, Mixed Finite Elements, Compatibility Conditions, and Applications. Cetraro, Italy 2006. Editors: D. Boffi, L. Gastaldi (2008)
Vol. 1940: J. Banasiak, V. Capasso, M.A.J. Chaplain, M. Lachowicz, J. Miękisz, Multiscale Problems in the Life Sciences. From Microscopic to Macroscopic. Będlewo, Poland 2006. Editors: V. Capasso, M. Lachowicz (2008)
Vol. 1941: S.M.J. Haran, Arithmetical Investigations. Representation Theory, Orthogonal Polynomials, and Quantum Interpolations (2008)
Vol. 1942: S. Albeverio, F. Flandoli, Y.G. Sinai, SPDE in Hydrodynamic. Recent Progress and Prospects. Cetraro, Italy 2005. Editors: G. Da Prato, M. Röckner (2008)
Vol. 1943: L.L. Bonilla (Ed.), Inverse Problems and Imaging. Martina Franca, Italy 2002 (2008)
Vol. 1944: A. Di Bartolo, G. Falcone, P. Plaumann, K. Strambach, Algebraic Groups and Lie Groups with Few Factors (2008)
Vol. 1945: F. Brauer, P. van den Driessche, J. Wu (Eds.), Mathematical Epidemiology (2008)
Vol. 1946: G. Allaire, A. Arnold, P. Degond, T.Y. Hou, Quantum Transport. Modelling, Analysis and Asymptotics. Cetraro, Italy 2006. Editors: N.B. Abdallah, G. Frosali (2008)
Vol. 1947: D. Abramovich, M. Mariño, M. Thaddeus, R. Vakil, Enumerative Invariants in Algebraic Geometry and String Theory. Cetraro, Italy 2005. Editors: K. Behrend, M. Manetti (2008)
Vol. 1948: F. Cao, J.-L. Lisani, J.-M. Morel, P. Musé, F. Sur, A Theory of Shape Identification (2008)
Vol. 1949: H.G. Feichtinger, B. Helffer, M.P. Lamoureux, N. Lerner, J. Toft, Pseudo-differential Operators. Cetraro, Italy 2006. Editors: L. Rodino, M.W. Wong (2008)
Vol. 1950: M. Bramson, Stability of Queueing Networks, Ecole d' Eté de Probabilités de Saint-Flour XXXVI-2006 (2008)

Recent Reprints and New Editions

Vol. 1702: J. Ma, J. Yong, Forward-Backward Stochastic Differential Equations and their Applications. 1999 – Corr. 3rd printing (2007)
Vol. 830: J.A. Green, Polynomial Representations of GL_n, with an Appendix on Schensted Correspondence and Littelmann Paths by K. Erdmann, J.A. Green and M. Schoker 1980 – 2nd corr. and augmented edition (2007)
Vol. 1693: S. Simons, From Hahn-Banach to Monotonicity (Minimax and Monotonicity 1998) – 2nd exp. edition (2008)
Vol. 470: R.E. Bowen, Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms. With a preface by D. Ruelle. Edited by J.-R. Chazottes. 1975 – 2nd rev. edition (2008)
Vol. 523: S.A. Albeverio, R.J. Høegh-Krohn, S. Mazzucchi, Mathematical Theory of Feynman Path Integral. 1976 – 2nd corr. and enlarged edition (2008)
Vol. 1764: A. Cannas da Silva, Lectures on Symplectic Geometry 2001 – Corr. 2nd printing (2008)

LECTURE NOTES IN MATHEMATICS

Springer

Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

Editorial Policy (for the publication of monographs)

1. Lecture Notes aim to report new developments in all areas of mathematics and their applications - quickly, informally and at a high level. Mathematical texts analysing new developments in modelling and numerical simulation are welcome.

   Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They may be based on specialised lecture courses. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character. For similar reasons it is unusual for doctoral theses to be accepted for the Lecture Notes series, though habilitation theses may be appropriate.

2. Manuscripts should be submitted either to Springer's mathematics editorial in Heidelberg, or to one of the series editors. In general, manuscripts will be sent out to 2 external referees for evaluation. If a decision cannot yet be reached on the basis of the first 2 reports, further referees may be contacted: The author will be informed of this. A final decision to publish can be made only on the basis of the complete manuscript, however a refereeing process leading to a preliminary decision can be based on a pre-final or incomplete manuscript. The strict minimum amount of material that will be considered should include a detailed outline describing the planned contents of each chapter, a bibliography and several sample chapters.

   Authors should be aware that incomplete or insufficiently close to final manuscripts almost always result in longer refereeing times and nevertheless unclear referees' recommendations, making further refereeing of a final draft necessary.

   Authors should also be aware that parallel submission of their manuscript to another publisher while under consideration for LNM will in general lead to immediate rejection.

3. Manuscripts should in general be submitted in English. Final manuscripts should contain at least 100 pages of mathematical text and should always include

   – a table of contents;
   – an informative introduction, with adequate motivation and perhaps some historical remarks: it should be accessible to a reader not intimately familiar with the topic treated;
   – a subject index: as a rule this is genuinely helpful for the reader.

   For evaluation purposes, manuscripts may be submitted in print or electronic form, in the latter case preferably as pdf- or zipped ps-files. Lecture Notes volumes are, as a rule, printed digitally from the authors' files. To ensure best results, authors are asked to use the LaTeX2e style files available from Springer's web-server at:

   ftp://ftp.springer.de/pub/tex/latex/svmonot1/ (for monographs).

Additional technical instructions, if necessary, are available on request from: lnm@springer.com.

4. Careful preparation of the manuscripts will help keep production time short besides ensuring satisfactory appearance of the finished book in print and online. After acceptance of the manuscript authors will be asked to prepare the final LaTeX source files (and also the corresponding dvi-, pdf- or zipped ps-file) together with the final printout made from these files. The LaTeX source files are essential for producing the full-text online version of the book (see www.springerlink.com/content/110312 for the existing online volumes of LNM).

   The actual production of a Lecture Notes volume takes approximately 12 weeks.

5. Authors receive a total of 50 free copies of their volume, but no royalties. They are entitled to a discount of 33.3% on the price of Springer books purchased for their personal use, if ordering directly from Springer.

6. Commitment to publish is made by letter of intent rather than by signing a formal contract. Springer-Verlag secures the copyright for each volume. Authors are free to reuse material contained in their LNM volumes in later publications: a brief written (or e-mail) request for formal permission is sufficient.

Addresses:

Professor J.-M. Morel, CMLA,
École Normale Supérieure de Cachan,
61 Avenue du Président Wilson, 94235 Cachan Cedex, France
E-mail: Jean-Michel.Morel@cmla.ens-cachan.fr

Professor F. Takens, Mathematisch Instituut,
Rijksuniversiteit Groningen, Postbus 800,
9700 AV Groningen, The Netherlands
E-mail: F.Takens@math.rug.nl

Professor B. Teissier, Institut Mathématique de Jussieu,
UMR 7586 du CNRS, Équipe "Géométrie et Dynamique",
175 rue du Chevaleret
75013 Paris, France
E-mail: teissier@math.jussieu.fr

For the "Mathematical Biosciences Subseries" of LNM:

Professor P.K. Maini, Center for Mathematical Biology,
Mathematical Institute, 24-29 St Giles,
Oxford OX1 3LP, UK
E-mail: maini@maths.ox.ac.uk

Springer, Mathematics Editorial I, Tiergartenstr. 17
69121 Heidelberg, Germany,
Tel.: +49 (6221) 487-8410
Fax: +49 (6221) 4876-8259
E-mail: lnm@springer.com