C.I.M.E. Session on "Advanced numerical approximation of nonlinear hyperbolic equations"

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LIST OF C.I.M.E. SEMINARS | Publisher

1954 -	1. Analisi funzionale	C.I.M.E.
	2. Quadratura delle superficie e questioni connesse	"
	3. Equazioni differenziali non lineari	"
1955 -	4. Teorema di Riemann-Roch e questioni connesse	"
	5. Teoria dei numeri	"
	6. Topologia	"
	7. Teorie non linearizzate in elasticità, idrodinamica,aerodinamica	"
	8. Geometria proiettivo-differenziale	"
1956 -	9. Equazioni alle derivate parziali a caratteristiche reali	"
	10. Propagazione delle onde elettromagnetiche	"
	11. Teoria della funzioni di più variabili complesse e delle funzioni automorfe	"
1957 -	12. Geometria aritmetica e algebrica (2 vol.)	"
	13. Integrali singolari e questioni connesse	"
	14. Teoria della turbolenza (2 vol.)	"
1958 -	15. Vedute e problemi attuali in relatività generale	"
	16. Problemi di geometria differenziale in grande	"
	17. Il principio di minimo e le sue applicazioni alle equazioni funzionali	"
1959 -	18. Induzione e statistica	"
	19. Teoria algebrica dei meccanismi automatici (2 vol.)	"
	20. Gruppi, anelli di Lie e teoria della coomologia	"
1960 -	21. Sistemi dinamici e teoremi ergodici	"
	22. Forme differenziali e loro integrali	"
1961 -	23. Geometria del calcolo delle variazioni (2 vol.)	"
	24. Teoria delle distribuzioni	"
	25. Onde superficiali	"
1962 -	26. Topologia differenziale	"
	27. Autovalori e autosoluzioni	"
	28. Magnetofluidodinamica	"

1963 - 29. Equazioni differenziali astratte "
30. Funzioni e varietà complesse "
31. Proprietà di media e teoremi di confronto in Fisica Matematica "

1964 - 32. Relatività generale "
33. Dinamica dei gas rarefatti "
34. Alcune questioni di analisi numerica "
35. Equazioni differenziali non lineari "

1965 - 36. Non-linear continuum theories "
37. Some aspects of ring theory "
38. Mathematical optimization in economics "

1966 - 39. Calculus of variations Ed. Cremonese, Firenze
40. Economia matematica "
41. Classi caratteristiche e questioni connesse "
42. Some aspects of diffusion theory "

1967 - 43. Modern questions of celestial mechanics "
44. Numerical analysis of partial differential equations "
45. Geometry of homogeneous bounded domains "

1968 - 46. Controllability and observability "
47. Pseudo-differential operators "
48. Aspects of mathematical logic "

1969 - 49. Potential theory "
50. Non-linear continuum theories in mechanics and physics and their applications "
51. Questions of algebraic varieties "

1970 - 52. Relativistic fluid dynamics "
53. Theory of group representations and Fourier analysis "
54. Functional equations and inequalities "
55. Problems in non-linear analysis "

1971 - 56. Stereodynamics "
57. Constructive aspects of functional analysis (2 vol.) "
58. Categories and commutative algebra "

1972 -	59. Non-linear mechanics		"
	60. Finite geometric structures and their applications		"
	61. Geometric measure theory and minimal surfaces		"
1973 -	62. Complex analysis		"
	63. New variational techniques in mathematical physics		"
	64. Spectral analysis		"
1974 -	65. Stability problems		"
	66. Singularities of analytic spaces		"
	67. Eigenvalues of non linear problems		"
1975 -	68. Theoretical computer sciences		"
	69. Model theory and applications		"
	70. Differential operators and manifolds		"
1976 -	71. Statistical Mechanics		Ed Liguori, Napoli
	72. Hyperbolicity		"
	73. Differential topology		"
1977 -	74. Materials with memory		"
	75. Pseudodifferential operators with applications		"
	76. Algebraic surfaces		"
1978 -	77. Stochastic differential equations		"
	78. Dynamical systems		Ed Liguori, Napoli and Birhâuser Verlag
1979 -	79. Recursion theory and computational complexity		"
	80. Mathematics of biology		"
1980 -	81. Wave propagation		"
	82. Harmonic analysis and group representations		"
	83. Matroid theory and its applications		"
1981 -	84. Kinetic Theories and the Boltzmann Equation	(LNM 1048)	Springer-Verlag
	85. Algebraic Threefolds	(LNM 947)	"
	86. Nonlinear Filtering and Stochastic Control	(LNM 972)	"
1982 -	87. Invariant Theory	(LNM 996)	"
	88. Thermodynamics and Constitutive Equations	(LN Physics 228)	"
	89. Fluid Dynamics	(LNM 1047)	"

1983 -	90. Complete Intersections	(LNM 1092)	Springer-Verlag
	91. Bifurcation Theory and Applications	(LNM 1057)	"
	92. Numerical Methods in Fluid Dynamics	(LNM 1127)	"
1984 -	93. Harmonic Mappings and Minimal Immersions	(LNM 1161)	"
	94. Schrödinger Operators	(LNM 1159)	"
	95. Buildings and the Geometry of Diagrams	(LNM 1181)	"
1985 -	96. Probability and Analysis	(LNM 1206)	"
	97. Some Problems in Nonlinear Diffusion	(LNM 1224)	"
	98. Theory of Moduli	(LNM 1337)	"
1986 -	99. Inverse Problems	(LNM 1225)	"
	100. Mathematical Economics	(LNM 1330)	"
	101. Combinatorial Optimization	(LNM 1403)	"
1987 -	102. Relativistic Fluid Dynamics	(LNM 1385)	"
	103. Topics in Calculus of Variations	(LNM 1365)	"
1988 -	104. Logic and Computer Science	(LNM 1429)	"
	105. Global Geometry and Mathematical Physics	(LNM 1451)	"
1989 -	106. Methods of nonconvex analysis	(LNM 1446)	"
	107. Microlocal Analysis and Applications	(LNM 1495)	"
1990 -	108. Geometric Topology: Recent Developments	(LNM 1504)	"
	109. H_{∞} Control Theory	(LNM 1496)	"
	110. Mathematical Modelling of Industrial Processes	(LNM 1521)	"
1991 -	111. Topological Methods for Ordinary Differential Equations	(LNM 1537)	"
	112. Arithmetic Algebraic Geometry	(LNM 1553)	"
	113. Transition to Chaos in Classical and Quantum Mechanics	(LNM 1589)	"
1992 -	114. Dirichlet Forms	(LNM 1563)	"
	115. D-Modules, Representation Theory, and Quantum Groups	(LNM 1565)	"
	116. Nonequilibrium Problems in Many-Particle Systems	(LNM 1551)	"

1993 -	117. Integrable Systems and Quantum Groups	(LNM 1620)	Springer-Verlag
	118. Algebraic Cycles and Hodge Theory	(LNM 1594)	
	119. Phase Transitions and Hysteresis	(LNM 1584)	"
1994 -	120. Recent Mathematical Methods in Nonlinear Wave Propagation	(LNM 1640)	"
	121. Dynamical Systems	(LNM 1609)	"
	122. Transcendental Methods in Algebraic Geometry	(LNM 1646)	"
1995 -	123. Probabilistic Models for Nonlinear PDE's	(LNM 1627)	"
	124. Viscosity Solutions and Applications	(LNM 1660)	"
	125. Vector Bundles on Curves. New Directions	(LNM 1649)	"
1996 -	126. Integral Geometry, Radon Transforms and Complex Analysis	(LNM 1684)	"
	127. Calculus of Variations and Geometric Evolution Problems	to appear	"
	128. Financial Mathematics	(LNM 1656)	"
1997 -	129. Mathematics Inspired by Biology	to appear	"
	130. Advanced Numerical Approximation of Nonlinear Hyperbolic Equations	LNM 1697)	
	131. Arithmetic Theory of Elliptic Curves	to appear	"
	132. Quantum Cohomology	to appear	"
1998 -	133. Optimal Shape Design	to appear	
	134. Dynamix Sstems and Small Divisors	to appear	
	135. Mathematical Problems in Semiconductor Physics	to appear	
	136. Stochastic PDE's and Kolmogorov Equations in Infinite Dimension	to appear	
	137. Filtration in Porous Media and Industrial Applications	to appear	
	138. Quantum Cohomology	to appear	

FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

"Dynamical Systems and Small Divisors"

is the subject of the first 1998 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R.), the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.) and the European Community, will take place, under the scientific direction of Prof. STEFANO MARMI (Università di Firenze) and Prof. JEAN-CHRISTOPHE YOCCOZ (Université de Paris Sud, Orsay) at Grand Hotel San Michele, Cetraro (Cosenza), **from 13 to 20 June, 1998.**

Courses

a) **KAM-Theory for Linear Quasi-Periodic Systems** (8 lectures in English)
Prof. L. Hakan ELIASSON (KTH, Stockholm)

Abstract

The systems we shall consider are linear differential/difference-equations which have a quasi-periodic dependence in the unknown variable - often thought of as "time". These systems have a multiplicative part consisting of multiplication by a quasi-periodic scalar or matrix and a differential/difference part.

Two kinds of perturbation theory can be considered. If the multiplicative part is very small we perturb from a constant coefficient system which can be solved explicitly. In this case the perturbation theory aims to constructing quasi-periodic solutions. If the multiplicative part is very large this can often be thought of as having a very small differential/difference part, in which case we perturb from a pure multiplicative system. In this case one looks for "localized" solutions.

There is a common feature of these problems: the unperturbed part is an operator with a dense point spectrum and the problem is to construct point eigenvalues or even a dense point spectrum for the perturbed operator. The basic difficulty in such a perturbation theory turns out to be to control the "almost multiplicities" of the eigenvalues.

The standard example that will be considered is the discrete one-dimensional quasi-periodic Schrödinger equations but also other examples will be discussed.

Important contributions for these problems have been made by Dinaburg-Sinai, Moser-Pöschel, Krikorian, Frölich-Spencer-Wittver, Chuleavsky-Sinai and others.

For references one can consult my paper "One-dimensional quasi-periodic Schrödinger operators - dynamical systems and spectral theory" in the proceedings of ECM2 in Budapest 1996.

b) **Invariant Tori** (8 lectures in English)
Prof. Michael HERMAN (Université de Paris 7, Ecole Polytechnique)

Contents

Invariant tori are a crucial feature not only of hamiltonian dynamics, but also of volume - preserving dynamics and even general dynamics. Hamilton's implicit function theorem in Fréchet spaces provides an efficient and flexible setting in which to study the existence of such tori. It allows to give simple proofs of classical results, such as the existence of diophantine invariant lagrangian tori in Hamiltonian dynamics, as well as less classical but very basic ones, such as the existence of codimension one translated diophantine tori in general dynamics.

Many important applications, in the hamiltonian as well as the volume preserving context, will be considered.

References

[1] Jean-Benoit Bost, *Tores invariants des systèmes dynamiques hamiltoniens* (d'après Kolmogorov, Arnold, Moser, Rüssmann, Zehnder, Herman, Pöschel, ...), Séminaire Bourbaki, exposé n. 639 (1985), Astérisque 133/134, 113-157.

[2] Jean-Christophe Yoccoz, *Travaux de Herman sur les tores invariants*, Séminaire Bourbaki, exposé n. 754 (1992), Astérique 206 (1992), 311-340.

c) **Geometrical Methods in Small Divisors Problems** (8 lectures in English)
Prof. Jean-Christopher YOCCOZ (Université de Paris-Sud, Orsay)

Contents

- Diophantine approximation, continued fraction, Brjuno's condition, the arithmetical condition for analytic linearization of analytic circle diffeomorphisms.
- Circle diffeomorphisms: a short review on the topological and smooth theory.
- Analytic circle diffeomorphisms: topological stability and analytic linearizability, periodic orbits in the neighbourhood, statement of the main conjugacy result.
- Geometric renormalization: what is the "continued fraction" of an analytic circle diffeomorphism.
- The inverse construction. Non linearizable diffeomorphisms.
- Renormalization with parameters: how Hörmander's L^2 estimates on $\bar{\partial}$ is a substitute to one complex variable theory.

References

[1] R. Perez-Marco, *Solution complète au problème de Siegel de linéarisation d'une application holomorphe aux voisinage d'un point fixe* (d'après J.-C. Yoccoz), Séminaire Bourbaki, exposé n. 753 (1992), Astérisque 206 (1992), 273-310.

[2] J.-C. Yoccoz, *An introduction to small divisors problems* in "From number theory to physics", edited by M. Waldschmidt, P. Moussa, J.-M. Luck and C. Itzykson, Berlin, Springer Verlag, 1992, pp. 659-679.

[3] J.-C. Yoccoz, *Théorème de Siegel, nombres de Brjuno et polynômes quadratiques*, Astérisque 231 (1995), 3-88.

FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

"Mathematical Problems in Semiconductor Physics"

is the subject of the second 1998 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R.), the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.) and the European Community, will take place, under the scientific direction of Professors MARCELLO ANILE (Università di Catania), PIERRE DEGOND (Université Paul Sabatier, Toulouse) and PETER A. MARKOWICH (TU, Berlin) at Grand Hotel San Michele, Cetraro (Cosenza), **from 15 to 22 July, 1998.**

Courses

a) **Drift Diffusion Equations and Applications** (6 lectures in English)
Prof. Walter ALLEGRETTO (Univ. of Alberta, Canada)

- Drift Diffusion Equations (DDES): Introduction and mathematical results
- Numerical schemes for DDES: Introduction and analysis
- Recent mathematical results for DDES
- Practical microsensor applications, related and open mathematical problems

Background references:

[1] Mock, *Analysis of mathematical models for semiconductor devices*, Boole Press, 1983.
[2] Selberherr, *Analysis and simulation of semiconductor devices*, Springer, 1984.
[3] Markowich, Ringhofer and Schmeiser, *Semiconductor equations*, Springer, 1990.
[4] Ristic, *Sensor technology and devices*, Artech House, 1994.
[5] Jerome and Kerkhoven, *A finite element approximation theory for the drift diffusion semiconductor model*, SIAM J. Num. Anal. (1991), 403-422.
[6] Fang and Ito, *Global solutions of the time dependent drift-diffusion semiconductor equations*, J. Diff. Eqs (1995), 523-566.
[7] Gajewski and Gartner, *On the discretization of van Roosbroeck's equations with magnetic field*, ZAMM (1996), 247-264.
Other relevant references will be presented at the talks.

b) **An Introduction to Kinetic Theory** (6 lectures in English)
Prof. David LEVERMORE (University of Arizona, Tucson)

1. Basic Structure
   1.1. Kinetic Regimes: Mean Free Paths; Boltzmann-Grad Limit; Boltzmann Equation
   1.2. Boltzmann Collision Operator: Galilean Symmetry; Boltzmann Identity; Conservation; Entropy, Dissipation and Equilibria
   1.3. Abstract Collision Operators: Galilean Symmetry; Conservation; Entropy, Dissipation and Equilibria
   1.4. Examples and Generalizations: Germions and Bosons; Generalized BKG Operators; Fokker-Planck Operators; Polyatomic Molecules; Multispecies Mixtures

2. Fluid Dynamical Limits
   2.1. Fluid Dynamical Regimes: Knudsen Number; Compressible Euler Limit; Compressible Euler Equations

2.2. Corrections to the Euler Equations: Chapman-Enskog Expansions; Linearized Collision Operators; Compressible Navier-Stokes Equations; Burnett Equations
2.3. Incompressible Navier-Stokes Limits: Mach and Reynolds Numbers; Boussinesq Balance; Dominant Balance; Maxwell Balance
2.4 Beyond Navier-Stokes

3. Moment Closure Hierarchies
3.1. Transition Regimes: Skewness and Flatness; Moment Closures; Moment Realizability
3.2 Grad Closure: Hermite Polynomials; Diagonal Trick; Problems
3.3. Entropy-Based Closures: Entropy Minimizing Distributions; Hyperbolicity; Entropy, Dissipation, and Equilibria; Recovering Fluid Dynamics; Generalized BGK Operators
3.4. Numerical Aspects: Riemann Solvers; Boundary Conditions; Stiffness; Loss of Realizability

4. The Boltzmann-Poisson Equation
4.1. Carrier Transport in Semiconductors: Conduction Bands; Collision operators;. Parabolic Band Approximation
4.2. Drift-Diffusion Limit: Scaling; Moment Formulation; Drift-Diffusion Equation
4.3. High-Field Regimes: Homogeneous Steady-State; Scaling; Chapman-Enskog Expansion; High-Field Drift-Diffusion Equation
4.4. Entropy-Based Closures: Hydrodynamic Models; Gaussian Closures

Some backround reading

[1] C. Cercignani, R. Illner, and M. Pulvirenti, *The mathematical theory of dilute gases*, Appl. Math. Sciences 106, Springer Verlag, New York 1994.
[2] C. Bardos, F. Golse, D. Levermore, *Fluid dynamic limits of kinetic equations I: Formal derivations*, J. Stat. Phys. 63 (1991), 323-344.
[3] C. D. Levermore, *Moment closure hierarchies for kinetic theories*, J. Stat. Phys. 83 (1996), 1021-1065.
[4] P. Markowich, C. Ringhofer, and C. Schmeiser, *Semiconductor equations*, Springer Verlag, New York 1990.
[5] F. Poupaud, *Runaway phenomena and fluid approximation under high fields in semiconductor kinetic theory*, Z. Angew. Math. Mech. 72 (1992), 359-372.

c) **Transport Modelling in Semiconductors** (6 lectures in English)
Prof. Frederick POUPAUD (Université de Nice)

The goal of this course is to give a precise mathematical derivation of the various models used to describe transport phenomena in semiconductors.

The master equations of relativistic quantum dynamics are the Dirac equations. The main two asymptotics which can be performed starting from these equations are the (semi-) non relativistic limit (c, the velocity of light, tends to infinity) and the (semi-) classical limit (h, the Planck constant, tends to zero).

We first focus on the non relativistic limit which can be handled with classical tools of mathematical analysis. We show that in the limit the Dirac equations become the Schrödinger equation. We also prove that a better approximation is given by the Pauli equations.

Then we introduce new mathematical formalisms which allow to perform easily semiclassical limits of any hyperbolic system of equations. These formalisms are based on Wigner measures introduced independently by P. Gerard and P. L. Lions, T. Pauli in 91. We give the main properties of these objects and we state the general result obtained for hyperbolic systems.

Finally we apply this theory in the context of semiconductors. We show that the classical limit of Dirac or Schrödinger equations is the Vlasov Maxwell or Vlasov Poisson equation. The semiconductor medium is modelized at the quantum level by a periodic potential. In this case we show how the band structure appears and derive the semiclassical transport equation of semiconductors.

References

[1] Gerard, *Mesures semiclassiques et ondes de Blach*, Seminaire Ecole Polytechnique 90-91, exposé XVI.
[2] Lions, T. Paul, *Sur les mesures de Wigner*, Revista Mat. Iberoamericana, 9 (1993), 553-618.
[3] Markowich, N, J, Mauser, *The classical limit of a self-consistent quantum Vlasov equation in 3D*, Math. Meth. Mod. in Appl. Sci. 9 (1993), 109-124.
[4] Markowich, N. J. Mauser and F. Poupaud, *A Wigner function approach to (semi) classical limits: electrons in a periodic potential*, J. Math. Phys. 35 (1994), 1066-1094.
[5] Poupaud, C. Ringhofer, *Quantum hydrodynamic models in a crystal*, Appl. Math. Lett. 8 (6) (1995), 55-59.

[6] Poupaud and C. Ringhofer, *Semiclassical limits in a crystal with external potentials and effective mass theorems*, Comm. in P.D.E. 21 (1996), 1897-1918.
[7] Gérard, P. A. Markowich, N. Mauser and F. Poupaud, *Homogenization limits and Wigner transforms*, Comm. Pure Appl. Math. 50 (1997), 323-380.

d) **Foundations of Mathematical Models for Semiconductor** (5 lectures in English)
Prof. Christian RINGHOFER (Arizona State University, Tempe)

- Multi-particle systems, the Liouville and Vlasov equation, Schrödinger equation, Maxwell's equation (Refs: [2,9,11,16]).
- From multi-particle systems to single particle systems: The BBGKY hierarchy, the classical and semiclassical Boltzmann equation, semiconductors vs. gases, Bloch decomposition and energy bands, density matrices and Wigner functions (Refs: [1,11,14,17]).
- Properties of the Boltzmann equation, collision operators, macroscopic limits, the Hilbert expansion and the Chapman-Enskog expansion, the drift-diffusion system, the hydrodynamic equations, energy transport models (Refs: [2,3,7,8,9,12,13,14]).
- Properties of the Heisenberg and Wigner equations, classical limits, the effective mass approximation, quantum hydrodynamic models, relaxation time approximations and the Fokker-Planck collision term (Refs: [5,6,10,12,14,17,19]).
- Boundary conditions for the simulation of devices, rough surfaces, charge and current conservation considerations, wave absorption at contacts and insulators (Refs: [4,5,9,11,15,18]).

References

[1] A. Arnold, P. Degond, P. Markowich, H. Steinrueck, *The Wigner Poisson equation in a crystal*, Appl. Lect. Lett. 2 (1989), 187-191.
[2] N. Ashcroft, M. Mermin, *Solid state physics*, Holt-Saunders, New York, 1976.
[3] G. Baccarani, M. Wordeman, *An investigation of steady state velocity overshoot effects in Si and GaAs devices*, Solid State Electr. 28 (1985), 407-416.
[4] B. Engquist, A. Majda, *Absorbing boundary conditions for the numerical simulation of waves*, Math. Comp. 31 (1977), 629-651.
[5] D. Ferry, H. Grubin, *Modelling of quantum transport in semiconductor devices*, Solid State Phys. 49 (1995), 283-448.
[6] C. Gardner, *The quantum hydrodynamic model for semiconductor devices*, SIAM J. Appl. Math. 54 (1994), 409-427.
[7] H. Grad, *On the kinetic theory of rarefied gases*, Comm. Pure Appl. Math. 2 (1949), 331-407.
[8] H. Grad, *Principles of the kinetic theory of gases*, Handbooks Phys. 12 (1958), 205-294.
[9] A. Kersch, W. Morokoff, *Transport simulation in microelectronics*, Birkhäuser, Basel, 1995.
[10] P. Markowich, C. Ringhofer, *An analysis of the quantum Liouville equation*, ZAMM 69 (1989), 121-127.
[11] P. Markowich, C. Ringhofer, C. Schmeiser, *Semiconductor equations*, Springer, Berlin, 1990.
[12] P. Markowich, N. Mauser, F. Poupaud, *A Wigner function approach to semiclassical limits*, J. Math. Phys. 35 (1994), 1066-1094.
[13] F. Poupaud, *Diffusion approximation of the linear Boltzmann equation: Analysis of boundary layers*, Asympt. Anal. 4 (1991), 293-317.
[14] F. Poupaud, C. Ringhofer, *Quantum hydrodynamic models in semiconductor crystals*, Appl. Math. Lett. 8 (1995), 55-59.
[15] C. Ringhofer, D. Ferry, N. Kluksdahl, *Absorbing boundary condition for the simulation of quantum transport phenomena*, Transp. Theory and Stat. Phys. 18 (1989), 331-346.
[16] S. Selberherr, *Analysis of semiconductor devices*, 2nd ed., Wiley, NewYork, 1981.
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FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

"Stochastic PDE's and Kolmogorov Equations in Infinite Dimensions"

is the subject of the third 1998 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R.), the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.) and the European Community, will take place, under the scientific direction of Professor GIUSEPPE DA PRATO (S. N. S., Pisa) at Grand Hotel San Michele, Cetraro (Cosenza), **from August 24 to September 1, 1998.**

Courses

a) **Kolmogorov equations** (8 lectures in English)
Prof. N. V. KRYLOV (Univ. of Minnesota, Minneapolis)

1. Solvability of Itô's stochastic equations.
2 The Markov property of diffusion processes
3. A conditional version of Kolmogorov's equation
4. Differentiability of solutions of stochastic equations with respect to initial data
5. Kolmogorov's equations in the whole space
6-7. Some integral approximations of differential operators
8. Kolmogorov's equations in domains in the sense of distributions

References

[1] N. K. Krylov, *Introduction to the theory of diffusion processes*, American Math Soc. Providence RI, 1995

b) **L^p-analysis of finite and infinite dimensional diffusion operators** (8 lectures in English)
Prof. M. RÖCKNER (Universität Bielefeld)

1. Kolmogorov equations in $L^p(R^d;\mu)$:-the symmetric case
2. Some recent results on invariant measures
3. Kolmogorov equations in $L^p(R^d;\mu)$:- the non-symmetric case
4. Kolmogorov equations in $L^p(E\ ;\mu)$ for Banach spaces E
5. Uniqueness results
6. Relation to martingale problems and applications to stochastic quantization
7. Kolmogorov equations in $L^p(E\ ;\mu)$ for infinite dimensional manifolds E : a case study from continuum statistical mechanics.
8. Ergodicity

References

(Note: SFB--Preprints are available via internet SFB-Webpage:
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[2] S.Albeverio, Y.G. Kondratiev, M. Röckner, *Dirichlet operators via stochastic analysis*, J. Funct. Anal. 128, 102--138 (1995).

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[12] V. Liskevich, M. Röckner, *Strong uniqueness for a class of infinite dimensional Dirichlet operators and applications to stochastic quantization*, SFB-343--Preprint (1997).

[13] M. Röckner, T.S. Zhang, *Uniqueness of generalized Schrödinger operators and applications*. J. Funct. Anal. 105, 187--231 (1992).

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[15] W. Stannat, *(Non--symmetric) Dirichlet operators on L^1. existence, uniqueness and associated Markov processes*, SFB-343--Preprint (1997).

[16] W. Stannat, *The theory of Generalized Dirichlet forms and its Applications in Analysis and Stochastics*, Doktorate--Thesis. SFB-343--Preprint (1997).

c) **Kolmogorov equations with infinite numbers of variables** (8 lectures in English)
Prof. J. ZABCZYK (Polskiej Akademii Nauk, Warszawa)

1. Infinite dimensional stochastic diffusions
2. Heat equations on Hilbert spaces
3. Kolmogorov equations corresponding to Ornstein-Uhlenbeck processes
4. Bismut-Elworthy formula and its consequences
5. Equations in open subsets of Hilbert spaces
6. Kolmogorov semigroups and invariant measures
7. Bellman equations of stochastic control
8. Variational parabolic inequalities

References

[1] L.Gross, *Potential Theory in Hilbert spaces*, J. Funct. Anal. 1(1965), 123-189.

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FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

"Filtration in Porous Media and Industrial Applications"

is the subject of the fourth 1998 C.I.M.E. Session.

The session, sponsored by the Consiglio Nazionale delle Ricerche (C.N.R.), the Ministero dell'Università e della Ricerca Scientifica e Tecnologica (M.U.R.S.T.) and the European Community, will take place, under the scientific direction of Prof. ANTONIO FASANO (Università di Firenze) and Prof. HANS VAN DUIJN (University of Amsterdam) at Grand Hotel San Michele, Cetraro (Cosenza), **from August 24 to September 1, 1998.**

Courses

a) **Mathematical Models for Oil Reservoirs Engineering** (6 lectures in English)
Prof. Magne S. ESPEDAL (University of Bergen)

b) **Filtration Processes in Various Industrial Problems** (4 lectures in English)
Prof. Antonio FASANO (Università di Firenze)

1. Flows in porous media with migrating solid particles
2. Non-isothermal flows with a wetting front
3. Some free boundary problems in the manufacturing of composite materials
4. Filtration in media with hydrophile granules

Prerequisites: Basic theory of p.d.e.'s

References

[1] I. Barenblatt, V. M. Entov, V. M. Ryzhik, *Theory of fluid flows through natural rocks*, Kluwer 1990.
[2] J. Bear, *Duynamics of fluids in porous media*, American Elsevier, New York 1972.
[3] J. Bear, A. Verruijt, *Modelling ground water flow and pollution*, Reidel, Dordrecht 1987.
[4] A. Fasano, *Some non-standard one-dimensional filtration problems*, Bull Fac. Ed. Chiba Univ. 44 (1996), 5-29.
[5] L. Preziosi, *The theory of deformable porous media and its applications to composite material manufacturing*, Surveys Math. Ind. 6 (1996), 167-214.

c) **Reactive Transport Processes in Porous Media** (3 lectures in English)
Prof. Peter KNABER (Universität Erlangen-Nürnberg)

1. Modelling of Reactive Transport Processes in Porous Media.
Abstract: Transport by diffusion, convection and dispersion, homogeneous and inhomogeneous reactions; equilibrium and non-equilibrium; complexation; adsorption; ion exchange; precipitation/dissolution.

2. Qualitative Properties of the Diffusion-Convection-Adsorption Equation.
Abstract: Classification of isotherms, general theory: existence, uniqueness, regularity, comparison principle; special solutions: similarity solutions, travelling waves; finite speed of propagation; asymptotic states.

3. Identification of Constitutive Laws by Breakthrough Experiments.
Abstract: Ill-posed problems; experiment design and identifiability; output least-squares approach; regularization; multilevel algorithms.

Prerequisites: Basic non-linear functional analysis and PDE's.

d) **Homogenization Theory and Applications to Filtration Processes** (6 lectures in English)
Prof. A. MIKELIC (Université Lyon I)

1. An introduction to the multi-scale expansions for the equations of fluid mechanics in a porous medium.

Abstract: We present the derivation of Darcy's law and the double porosity models for the single phase incompressible flow through periodic porous media by homogenization.

2. Non-Newtonian single phase flow through a porous media.

Abstract: We consider the flow of the incompressible quasi-Newtonian fluid through a porous medium. The viscosity of the fluid is a monotone function of the norm of the deformation tensor. We derive the effective flow equation and rigorously establish the "duality" constitutive relations used for the directional flows in engineering literature.

3. Inertia effects for the single phase flow through a porous medium.

Abstract: We consider the fast incompressible single phase flow through a porous medium. It is shown that the effective filtration law is a non-local two-pressures generalization of the Navier-Stokes system. After establishing the analiticity, we show how this complicated law can be approximated by polynomial relations between the filtration velocity and the pressure gradient. As a consequence we show that Forchheimer's filtration law is not acceptable.

4. The interface law of Beavers, Joseph and Saffman.

Abstract: We consider finding of the boundary conditions for the interface between free fluid flow and flow in a porous medium. The corresponding boundary layers are presented in details. The modification of Beavers-Joseph law, due to Saffman, is justified and the parameters in the law are determined using the boundary layers.

5. Filtration through a porous medium with the permeable grains.

Abstract: We consider filtration through a strongly heterogeneous porous material being made of components ("fluid" and "grains") with different permeabilities k_f and k_F, respectively. Fluid penetrates partly the grains and we have a large number of propagating wetting fronts. We find the effective filtration laws.

6. On the modelling of compressible flows in a porous medium.

Abstract: We consider the compressible Navier-Stokes system in a porous medium and obtain variants of the porous medium equation as the effective filtration model.

Prerequisites: Basic applied non-linear functional analysis and PDE's. Basic knowledge of PDE's of hydrodynamics.

References

[1] H. Hornung, ed.: *Homogenization and porous media*, Interdisciplinary Applied Mathematics Series, Springer, New York, 1996. The chapter *"One-phase newtonian flow"* by G. Allaire is recommended as an introduction to the course.

[2] A. E. Scheidegger, *Hydrodynamics in porous media*, in ."Encyclopedia of Physics", vol. VIII/2, ed. S. Flügge, pp. 625-662, Springer, Berlin, 1963.

e) **Some Nonlinear Models Arising in Subsurface Transport** (6 lectures in English)
Prof. Hans VAN DUIJN (Delft University of Technology)

Abstract

In three lectures, each of two hours, the following topics will be discussed:

1. Elliptic free boundary problems in salt water intrusion models
2. Uniqueness conditions in a hyperbolic model for oil recovery by steam drive
3. Stability methods for density induced porous media flow

In the first two lectures, we introduce the interface between fresh and salt groundwater as the free boundary in the mathematical description. We discuss the problem of an upconed interface when wells are present in a two or three dimensional reservoir. We also present and discuss a salt water intrusion problem in a two dimensional vertical strip. We show existence, uniqueness various types of monotonicity and other qualitative properties.

In the second pair of lectures we deal with a 2x2 hyperbolic system describing steam injection into a one dimensional reservoir (a porous column) which originally contains oil. Since steam condenses into water, we are led to

consider a three-phase flow problem. We present a solution in terms of shock and rarefaction waves. Non-uniqueness occurs, which is resolved by introducing a particular regularization. It will be demonstrated that different regularisation lead to different solutions in the hyperbolic limit.

The third pair of lectures deals with an evaporation problem that describes the groundwater movement below salt lakes. We give two stability criteria. One is based on a linearization technique, the other on an energy method. The analytical results will be compared with both numerical and experimental resuls. Disregarding diffusion, we present a mathematical formulation which describes the occurrence of fingering as a continuous transition between the fluids.

References

[1] H. W. Alt & C. J. van Duijn, *A stationary flow of fresh and salt groundwater in a coastal aquifer*, Nonlinear Analysis TMA 14 (1990), 625-656.
[2] H. W. Alt & C. J. van Duijn, *A free boundary problem involving a cusp. Part 1: Global analysis*, European J. Appl. Math. 4 (1993), 39-63.
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[4] B. Straughan, *The energy method, stability and nonlinear convection*, New York, Springer, 1992.

FONDAZIONE C.I.M.E.
CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

"Optimal Shape Design"

is the subject of the first 1998 C.I.M.E.- C.I.M. (Centro International de Matematica, Coimbra, Portugal) Session.

The Session, sponsored by E. U. under TMR Programme, by C.I.M. and by C.I.M.E. will take place under the scientific direction of Professors Arrigo Cellina (University of Milan, Italy) and Antonio Ornelas (Universidade de Evora, Portugal) in Tróia, Portugal, **from June 1 to June 6, 1998**

Courses

a) **Some nonconvex optimal shape problems** (4 lectures in English)
Prof. Bernd KAWOHL, (Univ. Koeln, Germany)

Contents:
Newton's Problem of minimal resistance: how to find the shape of a body of minimal resistance moving in a rarefied fluid (1 hour);
The opaque square and the opaque circle: find a curve C of minimal length inside a square or a circle, having the property that every straight line transversing the square or the circle intersects C (1 hour);
Variational approaches to proving symmetry (1 hour).

References

[1] G.Buttazzo, B.Kawohl: On Newton's problem of minimal resistance, Mathematical Intelligencer, **15** No.4 (1993) pp. 7-12.
[2] G.Buttazzo V.Ferone, B.Kawohl: Minimum problems over sets of concave functions and related questions, Mathematische Nachrichten **173** (1995), pp. 71--89.
[3] F.Brock, V.Ferone, B.Kawohl: A symmetry problem in the calculus of variations, Calculus of Variations **4** (1996) pp. 593-599.
[4] B.Kawohl, C.Schwab: Convergent finite elements for a class of nonconvex variational problems, IMA J. Numer. Anal., in print

b) **Shape Control and Optimal Shape Design** (4 Lectures in English)
Olivier PIRONNEAU, Analyse Numerique,(Paris 6, France)

Contents:
We consider the connections between shape design and equivalent boundary control design.
A new branch of optimal shape design is emerging along with what is known as "active control": the shapes are now time dependent and the control is preferably closed loop. In particular we shall deal with: sound control; drag control and lift control

References

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[2] Y. Achdou, O.Pironneau, F. Valentin, *Etude de lois de paroi d'ordre 1 et 2 pour des parois rugueuses par dècomposition de domaine*, INRIA report submitted also to JCP, 1997.
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[8] F. Murat, J. Simon: Etude de problèmes d'optimum design. Proc. 7th IFIP conf. Lecture notes in Computer sciences, **41**, pp.54-62, 1976
[9] O. Pironneau , Optimal shape design for elliptic systems', Springer-Verlag, 1984.

c) **Homogenization methods in Optimal Design.** (4 Lectures in English)
Prof. Luc TARTAR,(Carnegie Mellon University, USA)

References

[1] L. Tartar: Control problems in the coefficients of PDE. In Lecture notes in Economics and Math systems. A. Bensoussan ed. Springer, 1974.

d) **Explicit solutions in Elastic Optimization** (4 lectures in English)
Prof. Piero VILLAGGIO,(Università di Pisa, Italy)

Contents:

The explicit solution of the boundary of an elastic body satisfying a given criterion of optimality is not available in general. However, some exact solutions (still useful for practical purposes) are obtained in plane elasticity by narrowing the class of boundaries in which the optimum is sought.

References

[1] N. V. Banichuk, Introduction to Optimization of Structures, Springer 1990
[2] H Neuber, Kerbspannungslehre, third Ed, Springer 1985
[3] P. Villaggio, Mathematical Models for Elastic Structures, Cambridge University Press 1997

e) **Optimal Shape Design: Theory, Modelling, Numerical Algorithms** (4 lectures in English)
Prof. Jean Paul ZOLESIO, CNRS, Sophia Antipolis, France

Contents

Applications will be given to find the optimal shape of a ship; to the optimal shaping of the mirror of the European Very Large telescope; to the optimal design of plates.

References

[1] J. Sokolowski, J.P. Zolesio: Introduction to Shape
[2] Optimization. Springer Series in Computational Mathematics. 1991.

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