

Comments and Remarks

The main body of this book is built on the works of the authors, with various collaboration with other researchers, on this subject since 1993. Some significant results of other researchers are also included to enhance the book. However, due to the limitation of our information, we inevitably might have overlooked some new development in this field while writing this book, for which we deeply regret.

In Chapter 1, the results on the pure BSDEs, especially the fundamental well-posedness result, are based on the method introduced in the seminal paper of Pardoux-Peng [1]. The results on nonsolvability of FBSDEs are inspired by the example of Antonelli [1]. The well-posedness results of FBSDEs over small duration is also based in the spirit of the work of Antonelli [1]. The whole Chapter 2 is based on the paper of Yong [4].

In Chapter 3 we begin to consider a general form of the FBSDE (1) with an arbitrarily given $T > 0$. The main references for this chapter are based on the works of Ma-Yong [1], virtually the first result regarding solvability of FBSDE in this generality; and Ma-Yong [4], in which the notion of *approximate solvability* is introduced. A direct consequence of the method of optimal control is the Four Step Scheme presented in Chapter 4. The finite horizon case is initiated by Ma-Protter-Yong [1]; and the infinite horizon case is the theoretical part of the work on “Black’s Consol Rate Conjecture” presented later in Chapter 8, by Duffie-Ma-Yong [1].

Chapter 5 can be viewed either as a tool needed to extend the Four Step Scheme to the situation when the coefficients are allowed to be random, or as an independent subject in stochastic partial differential equations. The main results come from the papers of Ma-Yong [2] and [3]; and the applications in finance (e.g, the stochastic Black-Scholes formula) are collected in Chapter 8.

The method of continuation of Chapter 6 is based on the paper of Hu-Peng [2], and its generalization by Yong [1]. The method adopted a widely used idea in the theory of partial differential equations. Compared to the Four Step Scheme, this method allows the randomness of the coefficients and the degeneracy of the forward diffusion, but requires some analysis which readers might find difficult in a different way.

Chapter 7 is based on the work of Cvitanic-Ma [2]. The idea for the forward SDER using the solution mapping of Skorohod problem is due to Anderson-Orey [1], while the Lipschitz property of such solution mapping is adopted from Dupuis-Ishii [1]. The proof of the backward SDER is a modification of the arguments of Pardoux-Rascanu [1], [2], as well as some arguments from Buckdahn-Hu [1]. The proof of the existence and uniqueness of FBSDER adopted the idea of Pardoux-Tang [1], a generalized method of contraction mapping theorem, which can be viewed as an independent method for solving FBSDE as well.

Chapter 8 collects some successful applications of the FBSDEs developed so far. The integral representation theorem is due to Ma-Protter-Yong [1]; the Nonlinear Feynman-Kac formula is in the spirit of Peng [4], but the argument of the proof follows more closely those of Cvitanic-Ma [2]. The Black's consol rate conjecture is due to Duffie-Ma-Yong [1]; while hedging contingent claims for large investors comes from Cvitanic-Ma [1] for unconstrained case, and from Buckdahn-Hu [1] for constraint case. The section on stochastic Black-Scholes formula is based on the results of Ma-Yong [2] and [3], and the American game option is from Cvitanic-Ma [2].

Finally, the numerical method presented in Chapter 9 is essentially the paper of Douglas-Ma-Protter [1], with slight modifications. We should point out that, to our best knowledge, the scheme presented here is the only numerical method for (strongly coupled) FBSDEs discovered so far, and even when reduced to the pure BSDE case, it is still one of the very few existing numerical methods that can be found in the literature.

In summary, FBSDE is a new type of stochastic differential equations that has its own mathematical flavor and many applications. Like a usual two-point boundary value problem, there is no generic theory for its solvability, and many interesting insights of the equations has yet to be discovered. In the meantime, although the theory exists only for such a short period of time (recall that the first paper on FBSDE was published in 1993!), many topics in theoretical and applied mathematics have already been found closely related to it, and its applicability is quite impressive. It is our hope that by presenting a lecture notes in the series of LNM, more attention would be drawn from the mathematics community, and the beauty of the problem would be further exposed.

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