

## References

- R. Adler and R. Tribe (1998). Uniqueness for a historical SDE with a singular interaction, *J. Theor. Prob.* 11, 515-533.
- D. Aldous (1993). Tree-based models for random distribution of mass. *J. Stat. Phys.* 73, 625-641.
- S. Athreya, M. T. Barlow, R. Bass and E. Perkins (2000). Degenerate stochastic differential equations and super-Markov chains. Preprint.
- P. Baras and M. Pierre (1984). Singularités éliminables pur des equations semilinéaires. *Ann. Inst. Fourier* 34, 185-206.
- M.T. Barlow, S.N. Evans and E. Perkins (1991). Collision local times and measure-valued diffusions. *Can. J. Math.* 43, 897-938.
- M.T. Barlow and E. Perkins (1994). On the filtration of historical Brownian motion. *Ann. Prob.* 22, 1273-1294.
- M. Bramson, J.T. Cox, and J.F. Le Gall (2001). Super-Browian limits of voter model clusters. To appear, *Ann. Prob.*
- M. Bramson, R. Durrett and G. Swindle (1989). Statistical mechanics of crabgrass. *Ann. Prob.* 17, 444-481.
- D.L. Burkholder (1973). Distribution function inequalities for martingales. *Ann. Prob.* 1, 19-42.
- Z. Ciesielski and S.J. Taylor (1962). First passage times and sojourn times for Brownian motion and exact Hausdorff measure of the sample path. *Trans. Amer. Math. Soc.* 103, 434-450.
- J.T. Cox, R. Durrett, E. Perkins (1999). Rescaled particle systems converging to super-Brownian motion. In: *Perplexing Problems in Probability—Festschrift in Honor of Harry Kesten*, pp. 269-284, Birkhäuser, Boston.
- J.T. Cox, R. Durrett, E. Perkins (2000). Rescaled voter models converge to super-Brownian motion. *Ann. Prob.* 28, 185-234.
- D.A. Dawson (1978). Geostochastic calculus. *Can. J. Statistics* 6, 143-168.
- D.A. Dawson (1992). Infinitely divisible random measures and superprocesses. In: *Stochastic Analysis and Related Topics*, pp. 1-130, Birkhäuser, Boston.
- D. A. Dawson (1993). Measure-valued Markov Processes, *Ecole d'Eté de Probabilités de Saint Flour 1991*, Lect. Notes in Math. 1541, Springer, Berlin.
- D. A. Dawson, A. Etheridge, K. Fleischmann, L. Mytnik. E. Perkins, and J. Xiong (2000a). Mutually catalytic branching in the plane. Preprint.
- D. A. Dawson, K. Fleischmann, L. Mytnik. E. Perkins, and J. Xiong (2000b). Mutually catalytic branching in the plane: uniqueness. Preprint.
- D.A. Dawson, I. Iscoe, E. Perkins (1989). Super-Brownian motion: path properties and hitting probabilities. *Probab. Th. Rel. Fields* 83, 135-205.

- D.A. Dawson and K.J. Hochberg (1979). The carrying dimension of a stochastic measure diffusion. *Ann. Prob.* 7, 693-703.
- D.A. Dawson and P. March (1995). Resolvent estimates for Fleming-Viot operators and uniqueness of solutions to related martingale problems. *J. Funct. Anal.* 132, 417-472.
- D.A. Dawson, E. Perkins (1991). *Historical Processes*. Mem. Amer. Math. Soc. 93 n. 454.
- D.A. Dawson and V. Vinogradov (1994). Almost sure path properties of  $(2, d, \beta)$ -superprocesses. *Stoch. Proc. Appl.* 51, 221-258.
- C. Dellacherie and P.A. Meyer (1978). *Probabilities and Potential*. North-Holland Math. Studies 29, North Holland, Amsterdam.
- E. Derbez and G. Slade (1998). The scaling limit of lattice trees in high dimensions. *Commun. Math. Phys.* 193, 69-104.
- J.S. Dhersin and J.F. Le Gall (1997). Wiener's test for super-Brownian motion and for the Brownian snake. *Probab. Th. Rel. Fields* 108, 103-129.
- P. Donnelly and T.G. Kurtz (1999). Particle representations for measure-valued population models. *Ann. Prob.* 27, 166-205.
- J. Dugundji (1966). *Topology*. Allyn and Bacon, Boston.
- R. Durrett and E. Perkins (1999). Rescaled contact processes converge to super-Brownian motion for  $d \geq 2$ . *Probab. Th. Rel. Fields* 114, 309-399.
- E.B. Dynkin (1991). A probabilistic approach to one class of nonlinear differential equations. *Probab. Th. Rel. Fields* 89, 89-115.
- E.B. Dynkin (1993). Superprocesses and partial differential equations. *Ann. Prob.* 20, 1185-1262.
- E.B. Dynkin (1994). *An Introduction to Branching Measure-Valued Processes*, CRM Monographs 6, Amer. Math. Soc., Providence.
- E.B. Dynkin and S.E. Kuznetsov (1996). Superdiffusions and removable singularities for quasilinear partial differential equations. *Comm. Pure. Appl. Math.* 48, 125-176.
- E.B. Dynkin and S.E. Kuznetsov (1998). Trace on the boundary for solutions of nonlinear differential equations. *Trans. Amer. Math. Soc.* 350, 4499-4519.
- P.E. Echevaria (1982). A criterion for invariant measures of Markov processes. *Z. f. Wahrsch. verw. Gebiete* 61, 1-16.
- N. El Karoui and S. Roelly (1991). Propriétés de martingales, explosion et représentation de Lévy-Khintchine d'une classe de processus de branchement à valeurs mesures. *Stoch. Process. Appl.* 38, 239-266.
- A. Etheridge (2000). *An Introduction to Superprocesses*, University Lecture Series vol.20, Amer. Math. Soc.

- A. Etheridge and P. March (1991). A note on superprocesses. *Probab. Th. Rel. Fields* 89, 141-147.
- S.N. Ethier and T.G. Kurtz (1986). *Markov Processes: Characterization and Convergence*. Wiley, N.Y.
- S.N. Ethier, and T.G. Kurtz (1993). Fleming-Viot processes in population genetics. *Siam. J. Cont. Opt.* 31, 345-386.
- S.N. Evans and E. Perkins (1991). Absolute continuity results for superprocesses. *Trans. Amer. Math. Soc.* 325, 661-681.
- S.N. Evans and E. Perkins (1994). Measure-valued branching diffusions with singular interactions. *Can. J. Math.* 46, 120-168.
- S.N. Evans and E. Perkins (1998). Collision local times, historical stochastic calculus and competing species. *Elect. J. Prob.* 3, paper 5.
- W. Feller (1939). Die Grundlagen der Volterraschen Theorie des Kampfes ums Dasein in Wahrscheinlichkeitstheoretischer Behandlung. *Acta Biotheoretica* 5, 11-40.
- W. Feller (1951). Diffusion processes in genetics. *Proc. Second Berkeley Symp. Math. Statist. Prob.*, Univ. of California Press Berkeley, pp. 227-246.
- P.J. Fitzsimmons (1988). Construction and regularity of measure-valued Markov processes. *Israel J. Math.* 64, 337-36.
- P.J. Fitzsimmons (1992). On the martingale problem for measure-valued branching processes. In: *Seminar on Stochastic Processes 1991*, pp. 39-51, Birkhäuser, Boston.
- I.I. Gihman and A.V. Skorohod (1975). *The Theory of Stochastic Processes II*. Springer, New York.
- T.E. Harris (1963). *The Theory of Branching Processes*. Springer, Berlin.
- R. van der Hofstad, G. Slade (2000). Convergence of critical oriented percolation to super-Brownian motion above 4+1 dimensions. Preprint.
- D.N. Hoover and H.J. Keisler (1984). Adapted probability distributions. *Trans. Amer. Math. Soc.* 286, 159-201.
- D.N. Hoover and E. Perkins (1983). Nonstandard construction of the stochastic integral and applications to stochastic differential equations I, II, *Trans. Amer. Math. Soc.* 275, 1-36, 37-58.
- N. Ikeda and S. Watanabe (1981). *Stochastic Differential Equations and Diffusion Processes*. North Holland, Amsterdam.
- I. Iscoe (1986). A weighted occupation time for a class of measure-valued branching processes. *Probab. Th. Rel. Fields* 71, 85-116.
- I. Iscoe (1988). On the supports of measure-valued critical branching Brownian motion. *Ann. Prob.* 16, 200-221.
- J. Jacod (1977). A general theorem of representation for martingales. In: *Proceedings of Symposia in Pure Math.* 31, pp. 37-54, Amer. Math. Soc, Providence.

- J. Jacod and A.N. Shiryaev (1987). *Limit theorems for Stochastic Processes*. Springer-Verlag, Berlin.
- A. Jakubowski (1986). On the Skorohod topology. *Ann. Inst. H. Poincaré B22*, 263-285.
- S. Kakutani (1944). On Brownian motion in  $n$ -space. *Proc. Imp. Acad. Tokyo* 20, 648-652.
- O. Kallenberg (1986). *Random Measures*. Academic Press, New York.
- F. Knight (1981). *Essentials of Brownian Motion and Diffusion*. Amer. Math. Soc., Providence.
- N. Konno and T. Shiga (1988). Stochastic differential equations for some measure-valued diffusions. *Probab. Th. Rel. Fields* 79, 201-225.
- N. Krylov (1997). On a result of Mueller and Perkins. *Probab. Th. Rel. Fields* 108, 543-557.
- N. Krylov (1997b). On SPDE's and superdiffusions. *Ann. Prob.* 25, 1789-1809.
- T.G. Kurtz (1975). Semigroups of conditioned shifts and approximation of Markov processes. *Ann. Prob.* 3, 618-642.
- T.G. Kurtz (1998). Martingale problems for conditional distributions of Markov processes. *Elect. J. Prob.* 3, paper 9.
- J.F. Le Gall (1993). A class of path-valued Markov processes and its applications to superprocesses. *Probab. Th. Rel. Fields* 95, 25-46.
- J.F. Le Gall (1993b). The uniform random tree in a Brownian excursion. *Probab. Th. Rel. Fields* 96, 369-383.
- J. F. Le Gall (1994). A lemma on super-Brownian motion with some applications. In: *The Dynkin Festschrift* pp. 237-251. Birkhäuser, Boston.
- J. F. Le Gall (1998). The Hausdorff measure of the range of super-Brownian motion. In: *Perplexing Problems in Probability—Festschrift in Honor of Harry Kesten*, pp. 285-314, Birkhäuser, Boston.
- J. F. Le Gall (1999). *Spatial Branching Processes, Random Snakes and Partial Differential Equations*. Lectures in Mathematics ETH Zurich, Birkhäuser Verlag, Basel.
- J. F. Le Gall and E. Perkins (1995). The Hausdorff measure of the support of two-dimensional super-Brownian motion. *Ann. Prob.* 232, 1719-1747.
- J. F. Le Gall, E. Perkins and S.J. Taylor (1995). The packing measure of the support of super-Brownian motion. *Stoch. Process. Appl.* 59, 1-20.
- Z. Li and T. Shiga (1995). Measure-valued branching diffusions: immigrations, excursions and limit theorems, *J. Math. Kyoto Univ.* 35, 233-274.
- M. Lopez (1996). Path properties and convergence of interacting superprocesses. Ph.D. thesis, UBC.

- M. Metivier (1987). Weak convergence of measure-valued processes using Sobolev-embedding techniques. In: Proc. Trento 1985 SPDE and Applications, Lect. Notes. Math. 1236, 172-183, Springer, Berlin.
- C. Mueller and E. Perkins (1992). The compact support property for solutions of the heat equation with noise. Probab. Th. Rel. Fields 93, 325-358.
- C. Mueller and R. Tribe (1994). Stochastic pde's arising from the long range contact and long range voter processes. Probab. Th. Rel. Fields 102, 519-546.
- L. Mytnik (1998). Weak uniqueness for the heat equation with noise. Ann. Prob. 26, 968-984.
- L. Mytnik (1999). Uniqueness for a competing species model. Can. J. Math. 51, 372-448.
- A. Pazy (1983). Semigroups of Linear Operators and Applications to Partial Differential Applications. Springer-Verlag, Berlin.
- E. Perkins (1988). A space-time property of a class of measure-valued branching diffusions. Trans. Amer. Math. Soc. 305, 743-795.
- E. Perkins (1989). The Hausdorff measure of the closed support of super-Brownian motion. Ann. Inst. H. Poincaré Stat. 25, 205-224.
- E. Perkins (1990). Polar sets and multiple points for super-Brownian motion. Ann. Prob. 18, 453-491.
- E. Perkins (1991). On the continuity of measure-valued processes. In: Seminar on Stochastic Processes 1990, pp. 261-268, Birkhäuser, Boston.
- E. Perkins (1992). Measure-valued branching diffusions with spatial interactions. Probab. Th. Rel. Fields 94, 189-245.
- E. Perkins (1992b). Conditional Dawson-Watanabe processes and Fleming-Viot processes. In: Seminar on Stochastic Processes 1991, pp.142-155, Birkhäuser, Boston.
- E. Perkins (1994). The Strong Markov Property of the Support of Super-Brownian Motion. In: The Dynkin Festschrift Markov Processes and their Applications, pp.307-326, Birkhäuser, Boston.
- E. Perkins (1995). On the martingale problem for interactive measure-valued diffusions. Mem. Amer. Math. Soc. 115 n. 549.
- E. Perkins (1995b). Measure-valued branching diffusions and interactions. Proceedings of the International Congress of Mathematicians, Zurich, 1994, pp.1036-1046, Birkhäuser Verlag, Basel.
- E. Perkins and S.J. Taylor (1998). The multifractal spectrum of super-Brownian motion. Ann. Inst. H. Poincaré Stat. 34, 97-138.
- S.C. Port and C. J. Stone (1978). Brownian Motion and Classical Potential Theory. Academic, New York.
- M. Reimers (1989). One-dimensional stochastic partial differential equations and the branching measure-diffusion. Probab. Th. Rel. Fields 81, 319-340.

- M. Reimers (1989b). Hyperfinite methods applied to the critical branching diffusion. *Probab. Th. Rel. Fields* 81, 11-27.
- D. Revuz and M. Yor (1991). *Continuous Martingales and Brownian Motion*. Springer, Berlin.
- S. Roelly-Coppoletta (1986). A criterion of convergence of measure-valued processes: application to measure branching processes. *Stochastics* 17,43-65.
- L.C.G. Rogers and D. Williams (1987). *Diffusions, Markov Processes and Martingales Vol. 2*. Wiley, Chichester.
- C. A. Rogers and S. J. Taylor (1961). Functions continuous and singular with respect to a Hausdorff measure. *Mathematika* 8, 1-31.
- H. Rost (1971). The stopping distributions of a Markov process. *Invent. Math.* 14, 1-16.
- W. Rudin (1974). *Real and Complex Analysis*. McGraw-Hill, New York.
- A. Schied (1996). Sample path large deviations for super-Brownian motion. *Probab. Th. Rel. Fields* 104, 319-347.
- M. Sharpe (1988). *General Theory of Markov Processes*. Academic, Boston.
- T. Shiga (1990). A stochastic equation based on a Poisson system for a class of measure-valued diffusion processes. *J. Math. Kyoto. Univ.* 30, 245-279.
- D.W. Stroock and S.R.S. Varadhan (1979). *Multidimensional Diffusion Processes*, Springer-Verlag, Berlin.
- S. Sugitani (1987). Some properties for the measure-valued diffusion process. *J. Math. Soc. Japan* 41, 437-462.
- A. Szulga (1982). On minimal metrics in the space of random variables. *Teor. Veroyatn. Primen* 27, 401-405.
- S.J. Taylor (1961). On the connections between generalized capacities and Hausdorff measures. *Proc. Cambridge Philos. Soc.* 57, 524-531.
- S.J. Taylor (1964). The exact Hausdorff measure of the sample path for planar Brownian motion. *Proc. Cam. Phil. Soc.* 60, 253-258.
- S.J. Taylor and N.A. Watson (1985). A Hausdorff measure classification of polar sets for the heat equation. *Math. Proc. Cam. Phil. Soc.* 47, 325-344.
- R. Tribe (1991). The connected components of the closed support of super-Brownian motion. *Probab. Th. Rel. Fields* 84, 75-87.
- R. Tribe (1992). The behavior of superprocesses near extinction. *Ann. Prob.* 20, 286-311.
- R. Tribe (1994). A representation for super Brownian motion, *Stoch. Proc. Appl.* 51, 207-219.
- J. Walsh (1986). *An Introduction to Stochastic Partial Differential Equations*, Ecole d'Été de Probabilités de Saint Flour 1984, Lect. Notes. in Math. 1180, Springer, Berlin.

S. Watanabe (1968). A limit theorem of branching processes and continuous state branching processes, J. Math. Kyoto U. 8, 141-167.

T. Yamada and S. Watanabe (1971). On the uniqueness of solutions of stochastic differential equations. J. Math. Kyoto U. 11, 155-167.

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