References

For Chapter I

Let us start with some representative classics:

BRIOT C., BOUQUET C. : Théorie des fonctions elliptiques, 2e éd. Paris, Gauthiers-Villars 1875, 2 vol.

FRICKE R. : Die elliptischen Funktionen und ihre Anwendungen, Leipzig-Berlin, Teubner 1922.

APPEL P., LACOUR E. : Principes de la théorie des fonctions elliptiques et applications, 2e éd. Paris, Gauthier -Villars 1922.

This last book should especially be consulted for applications such as spherical pendulum, plane elastic curve, heat theory,...

Among more recent books on function theory containing a chapter on elliptic functions, we quote only

WHITTAKER E.T., WATSON G.N. : (A course of) Modern analysis, Cambridge at the University Press, 4th ed. reprinted 1965.

But a reader interested in the most simple properties of elliptic functions only should start with

AHLFORS L.V. : Complex analysis, 2d ed. New-York, McGraw-Hill, 1966.

For the theory of theta functions, we have adopted notations close to those of Weil in

WEIL A. : Introduction à l'étude des variétés kählériennes, Act. Sc. et Industrielles 1267, Paris, Hermann 1958.

Let us also mention

WEIL A. : Théorèmes fondamentaux de la théorie des fonctions thêta, Séminaire Bourbaki Mai 1949.

SIEGEL C.L. : Vorlesungen über gewählte Kapitel der Funktionentheorie (Notes by Gottschling E., Klingen H.), Mathematische Institut der Universität, Göttingen, 1964.

For automorphic functions and forms,

FORD L.R. : Automorphic functions, 2^d ed. New-York, Chelsea Publ. 1951.

LEHNER J. : Discontinuous groups and automorphic functions, (Math. Survey 8) Amer. Math. Soc. Providence, 1964.

Shorter introductions are provided by

GUNNING R.C. : Lectures on modular forms (Notes by Brumer A.), Princeton University Press, Princeton N.J., 1962.

SERRE J.-P. : Cours d'arithmétique (Collection "Le Mathématicien"), Presses Universitaires de France, Paris 1970.

The formula $SL_2(\mathbb{Z})/(\pm 1) = \mathbb{Z}/(2) * \mathbb{Z}/(3)$ and interesting generalizations can be found in

SERRE J.-P. : Arbres, amalgames et SL_2, (notes rédigées avec la collaboration de Bass H.), to appear in the Springer-Verlag lecture notes in mathematics series, Berlin.

I have selected Siegel's method for the derivation of the infinite product of Δ (Jacobi's theorem) :

SIEGEL C.L. : A simple proof of $\eta(-1/\tau) = \eta(\tau)\sqrt{\tau/i}$ Mathematika 1, 1954, p.4 (Complete works, vol 2, p.188).

SIEGEL C.L. : Analytische Zahlentheorie (Notes by Kürten K.F., Köhler G.), Mathematische Institut der Universität, Göttingen, 1964.

For more formulas connecting elliptic functions to theta functions

ERDELYI-MAGNUS-OBERHETTINGER-TRICOMI : Higher Transcendental Functions, (Bateman Manuscript Project), New-York, McGraw-Hill, 1953, vol.2.

FOR CHAPTER II

Prerequisites on noetherian rings, integers, extension of valuations (and the "Nullstellensatz") are all contained in

LANG S. : Algebra, Addison-Wesley Publ.Co., Reading Mass. Third printing 1971 (World Student Series ed.).

For a good introduction to the theory of algebraic curves (Bezout and Riemann-Roch's theorem) on a relatively elementary level, a student could start with

FULTON W. : Algebraic curves, Benjamin Inc., New-York, 1969.

One can also look at

CHEVALLEY C. : Introduction to the theory of algebraic functions of one variable (Survey 6) Amer. Math. Soc. Providence, 1951.

EICHLER M. : Einführung in die Theorie der algebraischen Zahlen und Funktionen, Basel u. Stuttgart, Birkhäuser, 1963.

These books contain also an introduction to the algebraic theory of elliptic functions. For the classification of elliptic differential forms and integrals, one can consult

COURANT R. HUREWITZ A. : Funktionentheorie, (Die Grundlehren...Bd.3) Berlin, Springer-Verlag, 1922.

For analytic p-adic function theory, the basic material is contained in

DWORK B. : On the zeta function of a hypersurface, I.H.E.S. Publ. Math. No 12 , Bures-sur-Yvette, 1962.

GUNTZER A. : Zur Funktionentheorie einer veränderlichen über einem vollständigen nichtarchimedischen Grundkörper, Archiv d. Math., 17, 1966, pp.415-431.

Note however that in this last article, the groundfield is always supposed to be algebraically closed and complete, so that a little

bit more work has to be done to get Schnirelmann's theorem in its strongest form (II.4.16), or its corollary (II.4.18). Schnirelmann's original reference is

SCHNIRELMANN L. : Sur les fonctions dans les corps normés et algébriquement fermés, Bull. Acad. Sci. USSR. Math, vol. 5, pp.487-497.

Tate's p-adic elliptic curves are treated in

ROQUETTE P. : Analytic theory of elliptic functions over local fields, Göttingen, Vandenhoeck and Ruprecht, 1970.

A sketch of this theory is also given in

SERRE J.-P. : Abelian ℓ-adic representations and elliptic curves, New-York, Benjamin Inc., 1968.

For Chapter III

Division points on abelian varieties are treated in

LANG S. : Abelian varieties, Interscience Publ. Inc., New-York, 1959.

MUMFORD D. : Abelian varieties, Oxford University Press (for the Tata Institute of fundamental Research, Bombay), Oxford, 1970.

A more elementary point of view is adopted by

SHIMURA G. : Arithmetic theory of automorphic functions (Introduction to the...), Iwanami Shoten, Publ., and Princeton University Press, 1971,

who gives the classical proof for the integrality of singular invariants. We have also followed this book for the presentation of the second section. More on ℓ-adic representations will be found in

SERRE J.-P. : Abelian ℓ-adic representations and elliptic curves, New-York, Benjamin Inc., 1968,

already referred to for chapter II .

An introduction to the basic ideas of etale cohomology (for pedestrians...) is given in the talk

MUMFORD D. : Arithmetical algebraic geometry, Proceedings of a Conference held at Purdue University, ed. by O.F.G. Schilling, Harper & Row, Publ., New-York, 1965, pp. 33-81.

For the finiteness theorem for the rational torsion, we have given Weil's proof as indicated by J.W.S. Cassels in the excellent survey article

CASSELS J.W.S. : Diophantine equations with special reference to elliptic curves, Journal London Math. Soc.,41, 1966, pp.193-291.

For the original proof, see

LUTZ E. : Les solutions de l'équation $y^2 = x^3 - Ax - B$ dans les corps

p-<u>adiques</u>, C.R.Acad.Sc. Paris, 203,1936,pp.20-22.

WEIL A. : <u>Sur les fonctions elliptiques</u> p-<u>adiques</u>, loc.cit. pp.22-24.

Let us also quote

SERRE J.-P. : p-<u>torsion des courbes elliptiques</u>, Sém. Bourbaki, vol. 1969-70, exposé 380, pp.281-294.

Plenty of references on elliptic curves are given in the survey article by Cassels, and in the book on ℓ-adic representations by Serre. For more recent references, one can look at the bibliography of the book by Shimura and that of

SERRE J.-P. : <u>Propriétés galoisiennes des points d'ordre fini des courbes elliptiques</u>, Inv. Math., 15, 1972, pp. 259-331.

FOR CHAPTER IV

The Cartier operator is defined and studied in Eichler's book (reference given for Chapter II). A standard reference for the Hasse invariant of elliptic curves is

DEURING M. : Die Typen der Multiplikatorenringe elliptischer Funktionenkörper, Abh. Math. Sem. Univ. Hamburg, 14, 1941, pp.197-272.

We have chosen Igusa's method to prove that the roots of $H_p(\lambda) = 0$ are simple and to determine the number of supersingular invariants.

IGUSA J.I. : Class number of a definite quaternion with prime discriminant, Proc. Nat. Acad. Sc. USA, 44, 1958, pp.312-314 .

Another way of defining the zeta function of a curve over a finite field, using an adelic integral, is given in

WEIL A. : Basic Number Theory, (Die Grundlehren ... Bd.144) , Berlin, Springer-Verlag, 1967 .

Observe that he gives the functional equation with the explicit rational form of this function (Th.4, p.130), but omits to prove that the coefficients of the polynomial in the numerator are integers. For a proof that there exists only finitely many integral solutions on the curve $y^2 = x^3 + k$, look at

MORDELL L.J. : Diophantine equations, London - New-York, Academic Press, 1969.

Since I have omitted to define the absolute invariant j of an elliptic curve in characteristic 2, the interested reader will have to go back to the appendix of Roquette's book on p-adic elliptic functions (reference given for Chapter II), where he will find reduced forms for all characteristics. The original article is

DEURING M. : Invarianten und Normalformen elliptischer Funktionenkörper, Math. Zeitschrift, 47, 1941, pp. 47-56.

For finer reduction properties of elliptic curves, we refer to the third chapter of

NERON A. : Modèles minimaux des variétés abéliennes sur les corps locaux et globaux, I.H.E.S. Publ. Math. 21, Bures-sur-Yvette, 1964, .

and for applications of minimal models to

SERRE J.-P., TATE J. : Good reduction of abelian varieties, Ann. of Math., 88, 1968, pp. 492-517.

The Riemann hypothesis for the zeta function of elliptic curves over finite fields is proved in the book by S. Lang on abelian varieties. One can also come back to

WEIL A. : Sur les courbes algébriques et les variétés qui s'en déduisent, Act. Sc. et Industrielles 1041 , Paris, Hermann 1948.

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