Appendix

We review some basic facts and standard definitions and notations from the theory of differentiable manifolds and differential topology. Proofs will be omitted and can be found in [La] and [Hi].

Manifold will always mean a paracompact, smooth (meaning C^∞) manifold satisfying the second axiom of countability, and modeled on a hilbert space of finite or infinite dimension. Only in the final chapters do we deal explicitly with the infinite dimensional case, and before that the reader who feels more comfortable in the finite dimensional context can simply think of all the manifolds that arise as being finite dimensional. In particular when we assume that the model hilbert space is V, with inner product $\langle\ ,\ \rangle$ then the reader can assume $V = \boldsymbol{R}^n$ and $\langle x, y\rangle = x \cdot y = \sum_{i=1}^n x_i y_i$.

The tangent space to a smooth manifold X at x is denoted by TX_x, and if $F : X \to Y$ is a smooth map and $y = F(x)$ then $DF_x : TX_x \to TY_y$ denotes the differential of F at x. If Y is a hilbert space then as usual we canonically identify TY_y with Y itself. With this identification we denote the differential of F at x by $dF : TF_x \to Y$. In particular if $f : X \to \boldsymbol{R}$ is a smooth real valued function on X then, for each x in X its differential $df_x : TX_x \to \boldsymbol{R}$ is an element of T^*X_x , the cotangent space to X at x. Also if X is modelled on V and $\Phi : O \to V$ is a chart for X at p, we have an isomorphism $d\Phi_p : TX_p \to V$. A Riemannian structure for X is an assignment to each x in X of a continuous, positive definite inner product $\langle\ ,\ \rangle_x$ on TX_x, such that the associated norm is complete. If $\Phi : O \to V$ is a chart as above then for each x in O there is a uniquely determined bounded, positive, self-adjoint operator $g(x)$ on V such that for $u, v \in TX_x$,

$$\langle u, v, \rangle_x = \langle g(x) d\Phi(u), d\Phi(v)\rangle,$$

where $\langle\ ,\ \rangle$ is the inner product in V. The Riemannian structure is smooth if for each chart Φ the map $x \mapsto g(x)$ from O into the Banach space of self-adjoint operators on V is smooth. (When $V = \boldsymbol{R}^n$ this just means that the matrix elements $g_{ij}(x)$ are smooth functions of x.)

For a Riemannian manifold X there is a norm preserving duality isomorphism $\ell \mapsto \hat{\ell}$ of T^*X_x with TX_x, characterized by $\ell(u) = \langle u, \hat{\ell}\rangle_x$. In particular if $f : X \to \boldsymbol{R}$ is a smooth function, then the dual $(df_x)\hat{}$ of df_x is called the gradient of f at x and is denoted by ∇f. The vector field ∇f plays a central rôle in Morse theory, and we note that its characteristic property is that for each Y in TX_x, $Yf \stackrel{\text{def}}{\equiv} df(Y)$, the directional derivative of f at x in the direction Y, is given by $\langle Y, \nabla f_x\rangle$. It follows from the Schwarz inequality that if $df_x \neq 0$ then, among all the unit vectors Y at x, the directional derivative of f in the direction Y assumes its maximum, $\|\nabla f\|$, uniquely for $Y = \frac{1}{\|\nabla f\|}\nabla f$.

We recall that given a smooth map $F : X \to Y$ a point x of X is called a *regular point* of F if $DF_x : TX_x \to TY_{F(x)}$ is surjective. Other points of X are called *critical points* of F. A point y of Y is called a *critical value* of F if $F^{-1}(y)$ contains at least one critical point of F. Other points of Y are called *regular values*. (Note that if y is a non-value of F, i.e., if $F^{-1}(y)$ is empty, then y is nevertheless considered to be a "regular value" of F.) By the Implicit Function Theorem if x is a regular point of F and $y = F(x)$, then there is a neighborhood O of x in X such that $O \cap F^{-1}(y)$ is a smooth submanifold of X (of dimension $\dim(X) - \dim(Y)$ when $\dim(X) < \infty$). Thus if y is a regular value of F then $F^{-1}(y)$ is a (possibly empty) closed, smooth submanifold of X.

If X is an n-dimensional smooth manifold, then a subset S of X is said to have measure zero in X if for each chart $\Phi : O \to \boldsymbol{R}^n$ for X, $\Phi(S \cap O)$ has Lebesque measure zero in $\boldsymbol{R}^n$. Note that it follows that S has no interior.

Morse-Sard Theorem. *[DR, p.10] If X and Y are finite dimensional smooth manifolds and $F : X \to Y$ is a smooth map, then the set of critical values of F has measure zero in Y and in particular it has no interior.*

Corollary. *If X is compact then the set of regular values of F is open and dense in Y.*

If $f : X \to \boldsymbol{R}$ is a smooth function and $df_x \neq 0$, then since $\boldsymbol{R}$ is one-dimensional, $df_x : TX_x \to \boldsymbol{R}$ must be surjective, i.e., x is a regular value of f. Thus for a real valued smooth function the critical points are exactly the points where df_x is zero. Of course when X is Riemannian we can equally well characterize the critical points of f as the zeros of the vector field ∇f.

Let X be a smooth Riemannian manifold, and M a smooth submanifold of X with the induced Riemannian structure. If $F : X \to \boldsymbol{R}$ is a smooth function on X and $f = F|M$ is its restriction to M then, at a point x of M, df_x is the restriction to TM_x of dF_x, and it follows from this and the characterization of the gradient above that ∇f_x is the orthogonal projection onto TM_x of ∇F_x. Thus x is a critical point of f if and only if ∇f_x is orthogonal to TM_x. Now suppose c is a regular value of some other smooth, real valued function $G : X \to \boldsymbol{R}$ and $M = G^{-1}(c)$. Then $TM_x = \ker(dG_x) = \nabla G_x^{\perp}$, hence in this case $TM_x^{\perp}$ is spanned by ∇G_x. This proves:

Lagrange Multiplier Theorem. *Let F and G be two smooth real valued functions on a Riemannian manifold X, c a regular value of G, and $M = G^{-1}(c)$. Then x in M is a critical point of $f = F|M$ if and only if $\nabla F_x = \lambda \nabla G_x$ for some real λ.*

Let Y be a smooth vector field on a manifold X. A *solution curve* for Y is a smooth map σ of an open interval (a, b) into X such that $\sigma'(t) = Y_{\sigma(t)}$ for all $t \in (a, b)$. It is said to have *initial condition* x if $a < 0 < b$ and $\sigma(0) = x$, and it is called *maximal* if it is not the restriction of a solution

curve with properly larger domain. An equivalent condition for maximality is the following: either $b = \infty$ or else $\sigma(t)$ has no limit points as $t \to \infty$, and similarly either $a = -\infty$ or else $\sigma(t)$ has no limit points as $t \to -\infty$

Global Existence and Uniqueness Theorem for ODE. *If Y is a smooth vector field on a smooth manifold X, then for each x in X there is a unique maximal solution curve of Y, $\sigma_x : (\alpha(x), \beta(x)) \to X$, having x as initial condition.*

For $t \in \boldsymbol{R}$ we define $D(\varphi_t) = \{x \in X \mid \alpha(x) < t < \beta(x)\}$ and $\varphi_t : D(\varphi_t) \to X$ by $\varphi_t(x) = \sigma_x(t)$. Then $D(\varphi_t)$ is open in X and φ_t is a difeomorphism of $D(\varphi_t)$ onto its image. The collection $\{\varphi_t\}$ is called the flow generated by Y, and we call the vector field Y *complete* if $\alpha \equiv -\infty$ and $\beta \equiv \infty$. In this case $t \mapsto \varphi_t$ is a one parameter group of diffeomorphisms of X (i.e., a homomorphism of $\boldsymbol{R}$ into the group of diffeomorphisms of X.

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