A

Appendix: A Short History of Universality

Thus it appears that universality is a generic phenomenon in analysis.
K.-G. Grosse-Erdmann

We conclude with a brief historical overview on the phenomenon of universality. We refer to Grosse-Erdmann [109] for a detailed and interesting survey on more or less all known types of universalities and an approach to unify them all.

The first *universal* object appearing in the mathematical literature was discovered by Fekete in 1914/15 (see [287]); he proved that there exists a real power series $\sum_{n=1}^{\infty} a_n x^n$ with the property that for any continuous function $f(x)$ on the interval $[-1, 1]$ satisfying $f(0) = 0$ there is a sequence of positive integers m_k such that

$$\sum_{n=1}^{m_k} a_n x^n \underset{k\to\infty}{\longrightarrow} f(x) \quad \text{uniformly on} \quad [-1, 1].$$

The proof relies essentially on Weierstrass' approximation theorem which states that every continuous function on a compact interval is the limit of a uniformly convergent sequence of polynomials (see [367]).

We illustrate Fekete's theorem and its proof by a p-adic version. The p-adic numbers were introduced by Hensel [131] in 1897. Given a prime number p, we define the p-adic absolute value $|\,.\,|_p$ on $\mathbb{Z}$ by

$$|\alpha|_p = \begin{cases} p^{-\nu(\alpha;p)} & \text{if } \alpha \neq 0, \\ 0 & \text{otherwise;} \end{cases}$$

here $\nu(\alpha; p)$ is the exponent of p in the prime factorization of $\alpha \in \mathbb{Z}$. It is an easy task to extend $|\cdot|_p$ from $\mathbb{Z}$ to $\mathbb{Q}$. The p-adic absolute values satisfies the *strong* triangle inequality: for any $\alpha, \beta \in \mathbb{Q}$,

$$|\alpha + \beta|_p \le \max\{|\alpha|_p, |\beta|_p\}$$

in contrast to the standard absolute value. The p-adic absolute value is non-archimedean (i.e., $\sup\{|n|_p : n \in \mathbb{N}\}$ is bounded). The integers that are p-adically close to zero are precisely the ones that are *highly* divisible by p. The set of p-adic numbers is the completion of $\mathbb{Q}$ with respect to the p-adic absolute value, and is denoted by $\mathbb{Q}_p$. The p-adic absolute value extends to a non-archimedean absolute value on $\mathbb{Q}_p$, and $\mathbb{Q}_p$ becomes a complete, locally compact, totally disconnected Hausdorff space. Similarly to the completion $\mathbb{R}$ of $\mathbb{Q}$ with respect to the standard (archimedean) absolute value, $\mathbb{Q}_p$ is a field, and its elements have a p-adically convergent series representation: for any $\alpha \in \mathbb{Q}_p$, there exist integers ν and a_k such that

$$\alpha = \sum_{k \geq \nu(\alpha;p)} a_k p^k \qquad \text{with} \quad 0 \leq a_k < p.$$

The set of p-adic numbers α with $|\alpha|_p \leq 1$ forms a ring, the ring $\mathbb{Z}_p$ of p-adic integers. Another construction of p-adic numbers uses the representation of $\mathbb{Z}_p$ as a projective limit of the ring of residue classes mod p^k. It is customary to write $|\cdot|_\infty$ for the standard absolute value on $\mathbb{Q}$, $\mathbb{Q}_\infty$ for $\mathbb{R}$, and then call $\mathbb{R}$ the completion of $\mathbb{Q}$ at *the infinite prime* $p = \infty$. Two absolute values are called equivalent if they induce the same topology. Ostrowski (see [279, Sect. II.3]) showed that every non-trivial absolute value on $\mathbb{Q}$ is equivalent to one of the absolute values $|\cdot|_p$, where p is a prime number or $p = \infty$. Thus, completion of $\mathbb{Q}$ with respect to its non-equivalent absolute values yields a family of complete, locally compact topological fields $\mathbb{Q}_p$ which contain $\mathbb{Q}$, one for each place $p \leq \infty$:

$$\mathbb{Q} \quad \hookrightarrow \quad \mathbb{Q}_2,\ \mathbb{Q}_3,\ \mathbb{Q}_5,\ \ldots, \quad \text{and} \quad \mathbb{Q}_\infty = \mathbb{R}.$$

p-adic analysis is quite different from real analysis. A non-archimedean absolute value induces a curious (ultrametric) topology. In p-adic analysis, the role of the intervals in $\mathbb{R}$ are played by the balls

$$a + p^\nu \mathbb{Z}_p := \{\alpha \in \mathbb{Q}_p : |\alpha - a|_p \leq p^{-\nu}\},$$

where $a \in \mathbb{Q}_p$ and $\nu \in \mathbb{Z}$. These balls are *clopen* sets, i.e., they are *clo*sed and o*pen*. It follows that each point inside a ball is center of the ball, and that any two balls are either disjoint or contained one in another.

The algebraic closure $\overline{\mathbb{Q}}_p$ of $\mathbb{Q}_p$ has infinite degree over $\mathbb{Q}_p$, but is not complete, and so it is not the right field for doing analysis. However, the completion $\mathbb{C}_p$ of $\overline{\mathbb{Q}}_p$ is algebraically closed. If

$$n = \alpha_0 + \alpha_1 p + \alpha_2 p^2 + \cdots \quad \text{and} \quad k = \beta_0 + \beta_1 p + \beta_2 p^2 + \cdots$$

are the p-adic expansions of the integers n and k, then one can show that

$$\binom{n}{k} \equiv \binom{\alpha_0}{\beta_0}\binom{\alpha_1}{\beta_1} \cdots \mod p.$$

In view of the binomic inversion formula this leads for any continuous functions $f : \mathbb{Z}_p \to \mathbb{C}_p$ to the representation

$$f(n) = \sum_{k=0}^{n} a_k(f) \binom{n}{k},$$

where $n \in \mathbb{N}$ and

$$a_k(f) := \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} f(j).$$

It is remarkable that this yields a series representation, the so-called Mahler series of f. For $k \in \mathbb{N}_0$, we define the finite difference operator $\bigtriangledown$ by

$$\bigtriangledown^k f(x) = \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} f(x+j).$$

Then $a_k = \bigtriangledown^k f(0)$ for $k \in \mathbb{N}$. Mahler [233] proved

Theorem A.1. *Let $f : \mathbb{Z}_p \to \mathbb{C}_p$ be a continuous function. Then $a_k \to 0$ as $k \to \infty$, and*

$$\sum_{k=0}^{n} a_k \binom{x}{k} \underset{n\to\infty}{\longrightarrow} f(x) \quad \textit{uniformly on } \mathbb{Z}_p.$$

A proof can be found in Robert [311, Sect. 4.2.4]. Notice that this theorem does not hold over $\mathbb{R}$.

In particular, Mahler's Theorem A.1 implies a p-adic version of Weierstrass' approximation theorem: for any continuous function $f : \mathbb{Z}_p \to \mathbb{C}_p$, there exists a sequence of polynomials f_n with coefficients in $\mathbb{C}_p$ that converges uniformly to f. Further applications of Mahler's theorem are the construction of p-adic analogues of classical functions like the exponential function, the logarithm or the Gamma-function; for details we refer again to Robert's monograph [311].

Now, it is not too surprising that an analogue of Fekete's universality theorem holds in the p-adic case too. Steuding [341] obtained

Theorem A.2. *There exists a p-adic power series $\sum_{n=1}^{\infty} a_n x^n$ with the property that for any ball $a+p^\nu\mathbb{Z}_p$, where $a, \nu \in \mathbb{Z}$, $0 < a < p$, and any continuous function f, defined on $a+p^\nu\mathbb{Z}_p$ and satisfying $f(a) = 0$, there exists a sequence of positive integers m_k such that*

$$\sum_{n=1}^{m_k} a_n x^n \underset{k\to\infty}{\longrightarrow} f(x) \quad \textit{uniformly on} \quad a + p^\nu \mathbb{Z}_p.$$

We follow Luh [227] in his proof of a version of Fekete's theorem.

Proof. Let $\{Q_n\}$ be a sequence of all polynomials with integral coefficients. We construct a sequence of polynomials $\{P_n\}$ as follows: let $P_0 = Q_0$ and assume that $P_0, \ldots, P_{n-1}$ are known. Denote by d_n the degree of P_{n-1}. Now let ϕ_n be a continuous function on $\mathbb{Z}_p$ such that

$$\phi_n(x) = \left(Q_n(x) - \sum_{k=0}^{n-1} P_k(x)\right) x^{-d_n-1}$$

for $x \in a+p^{-n}\mathbb{Z}_p$. In view of Mahler's Theorem A.1 there exists a polynomial f_n so that

$$\max_{x\in a+p^{-n}\mathbb{Z}_p} |f_n(x) - \phi_n(x)|_p \le p^{-d_n}.$$

Setting $P_n(x) = f_n(x)x^{d_n+1}$, the sequence $\{P_n\}$ is constructed. We note

$$\begin{aligned}\max_{x\in a+p^{-n}\mathbb{Z}_p} \left|\sum_{k=0}^{n} P_k(x) - Q_n(x)\right|_p &= \max_{x\in a+p^{-n}\mathbb{Z}_p} |(f_n(x) - \phi_n(x))x^{d_n+1}|_p \\ &\le p^{-d_n}.\end{aligned}$$

By construction, distinct P_n have no powers in common. Thus we can rearrange formally the polynomial series into a power series:

$$\sum_{n=1}^{\infty} a_n x^n := \sum_{n=0}^{\infty} P_n(x).$$

Again by Mahler's Theorem A.1, for any continuous function f on $a+p^{\nu}\mathbb{Z}_p$ there exists a sequence of positive integers n_k, tending to infinity with k, such that

$$\max_{x\in a+p^{-n_k}\mathbb{Z}_p} |f(x) - Q_{n_k}(x)|_p \le p^{-n_k}.$$

For sufficiently large k the ball $a+p^{\nu}\mathbb{Z}_p$ is contained in $a+p^{-n_k}\mathbb{Z}_p$. In view of the above estimates we obtain

$$\begin{aligned}&\max_{x\in a+p^{-n_k}\mathbb{Z}_p} \left|f(x) - \sum_{n=1}^{d_{n_k}+1} a_n x^n\right|_p = \max_{x\in a+p^{-n_k}\mathbb{Z}_p} \left|f(x) - \sum_{n=0}^{n_k} P_n(x)\right|_p \\ &\le \max_{x\in a+p^{-n_k}\mathbb{Z}_p} \left\{|f(x) - Q_{n_k}(x)|_p, \left|Q_{n_k}(x) - \sum_{n=0}^{n_k} P_n(x)\right|_p\right\} \\ &\le \max\{p^{-n_k}, p^{-d_{n_k}}\},\end{aligned}$$

which tends to zero as $k \to \infty$. Thus, putting $m_k = d_{n_k} + 1$, the assertion of the theorem follows. □

In the years after Fekete's discovery many universal objects were found. For instance, Birkhoff [23] proved in 1929 the existence of an entire function $f(z)$ with the property that to any given entire function $g(z)$ there exists a sequence of complex numbers a_n such that

$$f(z+a_n) \underset{n\to\infty}{\longrightarrow} g(z) \qquad \text{uniformly on compacta in } \mathbb{C}.$$

The proof relies in the main part on Runge's approximation theorem. This type of universality is very similar to the one of the Riemann zeta-function and other Dirichlet series. Birkhoff's theorem states the existence of an entire function with wild behaviour near infinity. Luh [228] constructed *holomorphic monsters*, that are holomorphic functions with an extraordinary wild boundary behaviour in arbitrary simply connected open sets. More precisely: let $\mathcal{G}$ be a proper open subset of $\mathbb{C}$ with simply connected components. Then there exists a function f holomorphic on $\mathcal{G}$ such that for every boundary point z of $\mathcal{G}$, every compact subset $\mathcal{K}$ with connected complement and every continuous function g on $\mathcal{K}$ which is holomorphic in the interior of $\mathcal{K}$, there exist linear transformations $\tau_n(z) = a_n z + b_n$ with $\tau_n(\mathcal{K}) \subset \mathcal{G}$ and $\operatorname{dist}(\tau_n(\mathcal{K}), z) \to 0$ as $n \to \infty$ for which

$$f(\tau_n(z)) \underset{n\to\infty}{\longrightarrow} g(z) \qquad \text{uniformly on } \mathcal{K};$$

in addition, each derivative of f and each antiderivative of f of arbitrary order has the boundary behaviour described above. We shall explain the notion antiderivative of a holomorphic function f defined on a simply connected open subset of $\mathbb{C}$; in fact, this notion is not unique. Here, for a *negative* integer j, the jth antiderivative $f^{(j)}$ of f with order $|j|$ is defined by

$$\frac{\mathrm{d}^{|j|}}{\mathrm{d}^{|j|}z} f^{(j)}(z) = f(z).$$

Any other antiderivative of f with the same order $|j|$ differs from $f^{(j)}$ on each component of $\mathcal{G}$ by some polynomial of degree less than $|j|$. Moreover, in [230], Luh proved the existence of multiply universal functions, that are holomorphic functions that satisfy, along with their derivatives and antiderivatives, six universal properties at the *same time*.

Marcinkiewicz [237] was in 1935 the first to use the notion *universality* when he proved the existence of a continuous function whose difference quotients approximate any measurable function in the sense of convergence almost everywhere. This should be compared with the result of Blair and Rubel [25] who proved that there exists an entire function f such that the set $\{f^{(n)} : n \in \mathbb{N}_0\}$ of all derivatives of f is dense in the space of all entire functions in the topology of uniform convergence on compact subsets of the complex plane. Other universal objects are, for example, conformal mappings composed with universal functions, discovered by Luh [229]. However, for a long time no *explicit* example of a universal object was found until Voronin discovered in 1975 that the Riemann zeta-function is universal!

In all these examples of universality there are two characteristic aspects of universality, namely the existence of a single object which

- is *maximal* divergent
- (via a countable process) allows to approximate a *maximal* class of objects.

This observation led to understand universality as a phenomenon which occurs quite naturally in certain limiting processes. Meanwhile it turned out that the phenomenon of universality is anything but a rare event in analysis! Many analytical processes which diverge or behave irregularly in some cases produce universal objects.

Grosse-Erdmann gave a rather general description of universality as follows. There is a topological space $\mathcal{X}$ of objects, a topological space $\mathcal{Y}$ of elements to be approximated, and a family of continuous mappings $T_j : \mathcal{X} \to \mathcal{Y}$ for $j \in J$. Then an object $x \in \mathcal{X}$ is called universal if every element $y \in \mathcal{Y}$ can be approximated by certain $T_j(x)$, i.e., the set $\{T_j(x) : j \in J\}$ lies dense in $\mathcal{Y}$. In the special case when the mappings T_j form a group of homeomorphisms, the concept of universality is well-known in topological dynamics under the name of topological transitivity (this reminds us of Bagchi's reformulation of Riemann's hypothesis in Sect. 8.2). In operator theory, where iterates T^j of an operator T are studied, universal elements are said to be hypercyclic. The general setting of Grosse-Erdmann covers quite many universality results. In [108, 109] he proved the following universality criterion.

Theorem A.3. *Suppose that $\mathcal{X}$ is a Baire space and $\mathcal{Y}$ is of the second category. Then the following assertions are equivalent:*

- *The set $\mathcal{U}$ of universal elements is residual in $\mathcal{X}$.*
- *The set $\mathcal{U}$ of universal elements is dense in $\mathcal{X}$.*
- *To every pair of non-empty sets $V \subset \mathcal{X}$ and $W \subset \mathcal{Y}$ there exists some $j \in J$ with*
$$T_j(V) \cap W \neq \emptyset.$$

If one of these conditions holds, then $\mathcal{U}$ is a dense G_δ-subset of $\mathcal{X}$.

We briefly explain the topological notions before we discuss a special case of this theorem; for details we refer to Grosse-Erdmann [108], Kelley's monograph [169] and Rudin's monograph [312]. In 1899, Baire introduced the notion of category to measure the size of subsets of topological spaces. A subset $\mathcal{E}$ of a topological space $\mathcal{X}$ is called nowhere dense if its closure $\overline{\mathcal{E}}$ contains no non-empty open subset of $\mathcal{X}$. Any countable union of nowhere dense sets is called a set of the first category (meager); all other subsets of $\mathcal{X}$ are said to be of the second category (non-meager). The complement of a set of the first category is called residual (co-meager). A topological space $\mathcal{X}$ is said to be a Baire space if the intersection of any countable family of open and dense subsets of $\mathcal{X}$ is dense in $\mathcal{X}$. Countable intersections of open sets are called G_δ-sets. The theorem of Baire states that any complete metric space is a Baire space.

In many applications of Theorem A.3, both $\mathcal{X}$ and $\mathcal{Y}$ are metric spaces. Then the first two assumptions of the theorem are fulfilled if $\mathcal{X}$ is complete and $\mathcal{Y}$ is separable. Furthermore, the third assertion can be rewritten as follows:

- For every $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ there exist sequences $\{x_n\}$ in $\mathcal{X}$ and $\{j_n\}$ in J such that $T_{j_n}(x_n)$ tends to y as $x_n \to x$.

For a verification of this statement one needs a suitable approximation theorem; for example, Weierstrass' approximation theorem for Fekete's universality theorem, respectively, Runge's approximation theorem for Birkhoff's universality result. With regard to universality for Dirichlet series, this fits to the denseness Theorem 5.10 for the space of analytic functions (which is a complete separable metric space by Theorem 3.15) and the use of Mergelyan's approximation Theorem 5.15, in the proof of the general universality Theorem 5.14, for $\tilde{\mathcal{S}}$. There is another remarkable aspect of Theorem A.3: if we observe the phenomenon of universality in some space $\mathcal{X}$, then the set of universal elements is dense. This observation supports the Linnik–Ibragimov conjecture (see Sect. 1.6)! This can be stated in a more explicit way: Nestoridis and Papadimitropoulos [278] proved the existence of a Dirichlet series $\sum_{n=1}^{\infty} \frac{a_n}{n^s}$, absolutely convergent for any $\sigma > 0$, with the property that for every admissible set $K \subset \{s \in \mathbb{C} : \sigma \leq 0\}$ and every entire function g, there exists a sequence of positive integers $\{\ell_j\}_j$ such that for any $\ell = 0, 1, 2, \ldots$

$$\left(\sum_{n=1}^{\ell_j} \frac{a_n}{n^s}\right)^{(\ell)} \to g(s)^{(\ell)} \qquad \text{uniformly on } K,$$

as $j \to \infty$. Furthermore, the set of such Dirichlet series is dense and G_δ (in the Baire sense) in the space of absolutely convergent Dirichlet series in the right half-plane. Here a set K is called admissible if K is compact with connected complement, and K is the finite union of sets K_r each of which contained in a vertical strip of width less than $\frac{1}{2}$. Their approach does not produce any explicit example of a universal Dirichlet series.

Universality is far away from being completely understood. In particular, the discovery of explicit examples of universal objects (zeta- and L-functions) has led to many new and interesting questions. It seems that universality of general Dirichlet series is not an arithmetic phenomenon at all, but it is much easier to find universal Dirichlet series explicitly among those associated with arithmetic objects.

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Notations

We indicate here some of the notations and conventions used in these notes; most of them are standard. However, this list is not complete; we omit notions which only appear in one chapter (where they are defined in situ) or which are covered by the index or which are standard.

As usual, we denote by $\mathbb{N} = \{1, 2, 3, \ldots\}$ the set of positive integers. The sets of integers, rational numbers, real numbers, and complex numbers are denoted by $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$, and $\mathbb{C}$, respectively. Following an old tradition in the theory of the zeta-function, the complex variable is given by a mixture of greek and latin letters: we write $s = \sigma + \mathrm{i}t$, where $\sigma, t \in \mathbb{R}$, and 'i' is the imaginary unit $\sqrt{-1}$ in the upper half-plane.

The letter p, with and without a subscript, denotes a prime number. We write $d \mid n$ (respectively, $d \nmid n$) if the integer d divides (respectively, does not divide) the integer n. The symbol $\equiv$ stands either for some congruence or it denotes that a function is constant. The number of elements of a finite set $\mathcal{A}$ is denoted by $\sharp\mathcal{A}$. The function $\pi(x)$ counts the number of primes $p \leq x$. Besides, we use many other arithmetical functions; in our notation we follow the classic [120].

The logarithm is, as usual in number theory, always taken to the basis $\mathrm{e} = \exp(1)$. The integer part and fractional part of a real number x are indicated by $[x]$ and $\{x\}$, respectively. Very convenient is the use of the Landau- and Vinogradov-symbols. Given two functions $f(x)$ and $g(x)$, both defined for $x \in X$, where $g(x)$ is positive for all $x \in X$, we write:

- $f(x) = \mathrm{O}(g(x))$ and $f(x) \ll g(x)$, respectively, if there exists a constant $C \geq 0$ such that
$$|f(x)| \leq Cg(x) \quad \text{for all} \quad x \in X;$$
- $f(x) \asymp g(x)$ if $f(x) \ll g(x) \ll |f(x)|$;

here X is specified either explicitly or implicitly. Usually, the set X is an interval $[\xi, \infty)$ for some real number ξ; in this case we also write:

- $f(x) \sim g(x)$ if the limit
$$\lim_{x\to\infty} \frac{|f(x)|}{g(x)}$$
exists and is equal to 1.
- $f(x) = \mathrm{o}(g(x))$ if the latter limit exists and is equal to zero.
- $f(x) = \Omega(g(x))$ if
$$\liminf_{x\to\infty} \frac{|f(x)|}{g(x)} > 0$$
(this is the negation of $f(x) = \mathrm{O}(g(x))$).

Sometimes the limit $x \to \infty$ is replaced by another limit $x \to x_0$, where x_0 is some complex number; in this case the limit x_0 is explicitly stated. In estimates, ϵ always denotes a small positive number, not necessarily the same at each appearance.

The letters $\mathbf{P}$ and $\mathbf{Q}$ always denote probability measures, often with a subscript. meas$(\mathcal{A})$ is the Lebesgue measure and $\mathrm{m}(\mathcal{A})$ stands for the Haar measure of a measurable set $\mathcal{A}$. By $N_{\mathcal{L}}(\,\cdot\,)$ and $N_{\mathcal{L}}^c(\,.\,)$ we denote the numbers of zeros and of c-values of the function $\mathcal{L}(s)$ in some region, often specified by some data in the brackets or by subscripts.

Finally, we list all axioms which were used in these notes:

(i), (1) *Ramanujan hypothesis.* $a(n) \ll n^{\epsilon}$ for any $\epsilon > 0$, where the implicit constant may depend on ϵ.

(ii) *Analytic continuation.* there exists a real number $\sigma_{\mathcal{L}}$ such that $\mathcal{L}(s)$ has an analytic continuation to the half-plane $\sigma > \sigma_{\mathcal{L}}$ with $\sigma_{\mathcal{L}} < 1$ except for at most a pole at $s = 1$.

(2) *Analytic continuation.* There exists a non-negative integer k such that $(s-1)^k\mathcal{L}(s)$ is an entire function of finite order.

(iii) *Finite order.* There exists a constant $\mu_{\mathcal{L}} \geq 0$ such that, for any fixed $\sigma > \sigma_{\mathcal{L}}$ and any $\epsilon > 0$,
$$\mathcal{L}(\sigma + \mathrm{i}t) \ll |t|^{\mu_{\mathcal{L}}+\epsilon} \quad \text{as} \quad |t| \to \infty;$$
the implicit constant may depend on ϵ.

(3) *Functional equation.* $\mathcal{L}(s)$ satisfies a functional equation of type
$$\Lambda_{\mathcal{L}}(s) = \omega\overline{\Lambda_{\mathcal{L}}(1-\overline{s})},$$
where
$$\Lambda_{\mathcal{L}}(s) := \mathcal{L}(s)Q^s \prod_{j=1}^{f} \Gamma(\lambda_j s + \mu_j)$$
with positive real numbers Q, λ_j, and complex numbers μ_j, ω with $\mathrm{Re}\,\mu_j \geq 0$ and $|\omega| = 1$.

(iv) *Polynomial Euler product.* There exists a positive integer m and for every prime p, there are complex numbers $\alpha_j(p), 1 \leq j \leq m$, such that

$$\mathcal{L}(s) = \prod_p \prod_{j=1}^{m} \left(1 - \frac{\alpha_j(p)}{p^s}\right)^{-1}.$$

(4) *Euler product.* $\mathcal{L}(s)$ satisfies

$$\mathcal{L}(s) = \prod_p \mathcal{L}_p(s), \quad \text{where} \quad \mathcal{L}_p(s) = \exp\left(\sum_{k=1}^{\infty} \frac{b(p^k)}{p^{ks}}\right)$$

with suitable coefficients $b(p^k)$ satisfying $b(p^k) \ll p^{k\theta}$ for some $\theta < \frac{1}{2}$.

(v) *Prime mean-square.* There exists a positive constant κ such that

$$\lim_{x\to\infty} \frac{1}{\pi(x)} \sum_{p \leq x} |a(p)|^2 = \kappa.$$

We denote by $\tilde{\mathcal{S}}$ the class of Dirichlet series satisfying the axioms (i)–(v) and by $\mathcal{S}^\sharp$ the so-called extended Selberg class of Dirichlet series satisfying (2) and (3); the subclass of the latter class of all elements satisfying additionally the axioms (1) and (4) is the Selberg class $\mathcal{S}$.

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