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## Glossary

*action angle variables:* In an *integrable* Hamiltonian system with  $d$  degrees of freedom the level sets of regular values of the  $d$  integrals are *Lagrangian submanifolds*. In case these level sets are compact they are (unions of)  $d$ -tori. In the neighbourhood of such an invariant torus one can find *Darboux co-ordinates*  $(\varphi, I)$  such that the integrals (and in particular the Hamiltonian) depend only on the actions  $I$ . The values of  $(\varphi, I)$  form a product set  $\mathbb{T}^d \times \mathbb{Y}$ , with  $\mathbb{Y} \subseteq \mathbb{R}^d$  open. Every invariant  $d$ -torus  $I = \text{const}$  in the domain of this chart is parametrised by the angles  $\varphi$ .

*adjacent:* A *singularity*  $\mathcal{K}$  is adjacent to  $\mathcal{H}$  if  $\mathcal{H}$  is contained in the *universal unfolding* of  $\mathcal{K}$ .

*Arnol'd diffusion:* The existence of orbits in nearly integrable systems that connect distant parts of the phase space. Because of Nekhoroshev's theorem such motion must be very slow, and because of KAM theory the majority of initial conditions does not lead to such motions (but to quasi-periodic motions instead). In the literature this expression is sometimes also used for a special mechanism proposed by Arnol'd in [7] where such orbits are constructed on the basis of normally hyperbolic tori that are generated when the perturbation destroys resonant tori.

*Birkhoff normal form:* In case there are no *normal resonances* between the *normal frequencies*  $\alpha_j$  of a *centre* there is a formal change of variables that turns the Hamiltonian into a power series in the invariants  $I_j = \frac{1}{2}(x_j^2 + y_j^2)$ . Correspondingly, there is a "real" co-ordinate change into Birkhoff normal form  $H = \sum \alpha^{(k)} I^k + R$  up to order  $n$  with a remainder term satisfying  $D^k R(0) = 0 \quad \forall_{k \leq n}$ , and this latter result only requires absence of normal resonances  $k_j \alpha_j = 0$  up to order  $|k| \leq n$ .

*Cantor set, Cantor dust, Cantor family, Cantor fibration, Cantor stratification:*

Cantor dust is a separable locally compact space that is perfect, i.e. every point is in the closure of its complement, and totally disconnected. It can be shown that this determines Cantor dust up to homeomorphy. We use the term "Cantor set" for subsets of  $\mathbb{R}^n$  that are locally homeomorphic to a product of Cantor dust and a closed set.

On the real line  $\mathbb{R}$  one can define Cantor dust of positive measure by excluding around each rational number  $p/q$  an interval of size  $2\gamma/q^\tau$ ,  $\gamma > 0, \tau > 2$ . Similar

*Diophantine conditions* define Cantor sets in  $\mathbb{R}^n$ . Since these Cantor sets have positive measure their (Hausdorff)-dimension is  $n$ . We also use expressions like “Cantor family of tori” or “(ramified) Cantor fibration” for a (Whitney)-smooth collection of tori, parametrised over a Cantor set. Where the unperturbed system is stratified according to the co-dimension of occurring (bifurcating) tori, this leads to a Cantor stratification. See also Definition B.3.

*Casimir element*: A function  $f : \mathcal{P} \rightarrow \mathbb{R}$  is a Casimir element of the *Poisson algebra*  $C^\infty(\mathcal{P})$  if  $\{f, g\} = 0$  for all  $g \in C^\infty(\mathcal{P})$ . This induces a Poisson structure on the level sets  $f^{-1}(a)$ ,  $a \in \mathbb{R}$ . In case every point has a small neighbourhood on which the only Casimir elements are the constant functions, the Poisson structure on  $\mathcal{P}$  is non-degenerate, i.e.  $\mathcal{P}$  is a *symplectic manifold*.

*centre*: An equilibrium of a vector field is called a centre if all eigenvalues of the linearization are nonzero and on the imaginary axis.

*centre-saddle bifurcation*: Under variation of a parameter  $\lambda$  a *centre* and a *saddle* meet and vanish.

*Chern class*: Let  $\rho : \mathcal{P} \rightarrow B$  be a torus bundle with fibre  $\mathbb{T}^n$  and denote by  $\mathcal{G}$  the locally constant sheaf of first homotopy groups  $\pi_1(\mathbb{T}^n)$  of the fibres. The Chern class of the torus bundle is an element of  $H^2(B, \mathcal{G})$  that measures the obstruction to the existence of a global section of  $\rho$  – such a section exists if and only if the Chern class vanishes. An example with a non-vanishing Chern class is the Hopf fibration  $S^3 \rightarrow S^2$ .

*co-isotropic*: For a subspace  $U < V$  of a symplectic vector space  $(V, \omega)$  are equivalent: (i)  $U$  contains its  $\omega$ -orthogonal complement, (ii)  $U^{\perp\omega}$  is *isotropic*. If  $U$  satisfies one and thus both of these conditions it is called a co-isotropic subspace. A submanifold  $\mathcal{B} \subseteq \mathcal{P}$  of a symplectic manifold  $(\mathcal{P}, \omega)$  is called co-isotropic if all tangent spaces  $T_y\mathcal{B} < T_y\mathcal{P}$  are co-isotropic subspaces.

*conditionally periodic*: A motion  $t \mapsto \alpha(t) \in \mathcal{P}$  is conditionally periodic if there are frequencies  $\omega_1, \dots, \omega_k \in \mathbb{R}$  and a smooth embedding  $F : \mathbb{T}^k \rightarrow \mathcal{P}$  such that  $\alpha(t) = F(e^{2\pi i\omega_1 t}, \dots, e^{2\pi i\omega_k t})$ . We can think of the motion as a superposition of the periodic motions  $t \mapsto F(1, \dots, 1, e^{2\pi i\omega_j t}, 1, \dots, 1)$ . If the frequencies are rationally independent, the motion  $t \mapsto \alpha(t) \in \mathcal{P}$  lies dense on  $\text{im}F$  and this embedded torus is an invariant torus. In case there are *resonances* among the frequencies the motion is restricted to a subtorus.

A flow on a torus is parallel or conditionally periodic if there exist co-ordinates in which the vector field becomes constant.

*conjugacy*: Two flows  $\varphi_t$  and  $\psi_t$  of vector fields  $X$  and  $Y$  are (topologically) conjugate if there is a homeomorphism  $\eta$  with  $\eta \circ \varphi_t = \psi_t \circ \eta$ . In case the conjugacy  $\eta$  is a diffeomorphism one may differentiate this equation and obtains  $Y = T\eta \circ X \circ \eta^{-1}$ .

*conjugate*: (a) For flows: see *conjugacy*. (b) Two co-ordinates  $Q$  and  $P$  of a *Darboux* chart are (canonically) conjugate if  $\omega(X_Q, X_P) = \pm 1$ , i.e.  $X_Q$  and  $X_P$  span for every point  $x$  in the domain  $U$  of the chart a hyperbolic plane in  $T_x U$ .

*connection bifurcation*: Under variation of a parameter  $\lambda$  the stable and the unstable manifolds of two saddles approach and pass, coinciding for an isolated parameter value. This takes place in one degree of freedom where the Hamiltonian  $\frac{1}{2}p^2 + \frac{1}{2}q^2 - q^4 - \lambda q$  defines a standard example. In  $n$  degrees of freedom the saddles are replaced by normally hyperbolic  $(n-1)$ -tori and the system has to be *integrable*. While the connection bifurcation is robust in one degree of

freedom, in  $n \geq 2$  degrees of freedom a small *generic* perturbation may lead to transverse *heteroclinic orbits* and to *tangency bifurcations*.

**Darboux co-ordinates, Darboux basis:** In a Darboux basis  $\{e_1, \dots, e_n, f_1, \dots, f_n\}$  of a symplectic vector space  $(V, \omega)$  the symplectic product takes the simple form  $\omega(e_i, e_j) = 0 = \omega(f_i, f_j)$  and  $\omega(e_i, f_j) = \delta_{ij}$ . In Darboux co-ordinates  $(q_1, \dots, q_n, p_1, \dots, p_n)$  of a symplectic manifold  $(\mathcal{P}, \omega)$  the symplectic form becomes  $\omega = \sum dq_i \wedge dp_i$ .

**degree of freedom:** In so-called simple mechanical systems the *phase space* is the cotangent bundle of the configuration space and the dimension of the latter encodes “in how many directions the system can move”. For *symplectic manifolds* this notion is immediately generalized to one half of the dimension of the phase space. *Poisson spaces* are foliated by their symplectic leaves and the number of degrees of freedom is defined to be one half of the rank of the Poisson structure.

**differential space:** A topological space  $\mathcal{P}$  for which one singles out a subalgebra  $\mathcal{A} \subseteq C(\mathcal{P})$  of “smooth functions” that is locally defined, sufficiently rich to reproduce the topology and that is consistent with respect to compositions with smooth functions on Euclidian spaces. One then writes  $\mathcal{A} = C^\infty(\mathcal{P})$ . See also Definition B.2.

**Diophantine condition, Diophantine frequency vector:** A frequency vector  $\omega \in \mathbb{R}^n$  is called Diophantine if there are constants  $\gamma > 0$  and  $\tau > n - 1$  with

$$\bigwedge_{k \in \mathbb{Z}^n \setminus \{0\}} |\langle k, \omega \rangle| \geq \frac{\gamma}{|k|^\tau}.$$

The Diophantine frequency vectors satisfying this condition for fixed  $\gamma$  and  $\tau$  form a Cantor set of half lines. As the “Diophantine parameter”  $\gamma$  tends to zero (while  $\tau$  remains fixed), these half lines extend to the origin. The complement in any compact set of frequency vectors satisfying a Diophantine condition with fixed  $\tau$  has a measure of order  $O(\gamma)$  as  $\gamma \rightarrow 0$ .

**distinguished parameter:** Let  $H$  depend on the parameters  $I$  and  $\alpha$ . If we allow only re-parametrisations  $(I, \alpha) \mapsto (J, \beta)$  of the form  $J = J(I, \alpha)$  and  $\beta = \beta(\alpha)$ , we make  $I$  an internal or distinguished parameter, cf. [288]. This is appropriate if, for example,  $H$  is a reduced Hamiltonian,  $I$  the (fixed) value of the *momentum mapping* of the original system and  $\alpha$  an external parameter the original system depends upon. See also Section 1.1.2.

**dual bifurcation:** Where the bifurcation scenario depends on a sign  $\pm$  the “second choice” is called the dual case. An alternative division is that into *supercritical* and *subcritical* case.

**elliptic:** A periodic orbit/invariant torus is elliptic if all *Floquet multipliers/exponents* are on the unit circle/imaginary axis. An elliptic equilibrium is called a *centre*.

**energy shell:** The set of points that can be attained given a certain energy. If the phase space is a symplectic manifold, this set is given by the pre-image of that energy value under the Hamiltonian. For more general Poisson spaces this pre-image has to be intersected with a symplectic leaf.

**energy-momentum mapping:** Let  $\Gamma : G \times \mathcal{P} \rightarrow \mathcal{P}$  be a symplectic action of the Lie group  $G$  on the *symplectic manifold*  $\mathcal{P}$  with *momentum mapping*  $J : \mathcal{P} \rightarrow \mathfrak{g}^*$ . For a Hamiltonian function  $H$  that is invariant under the action  $\Gamma$  we call  $(H, J) : \mathcal{P} \rightarrow \mathbb{R} \times \mathfrak{g}^*$  the energy-momentum mapping.

*equivalence*: The meaning of “ $f$  is equivalent to  $g$ ” depends on the context. For  $f, g \in C^\infty(\mathcal{P})$  it means that there is a diffeomorphism  $\eta$  of  $\mathcal{P}$  with  $g \circ \eta = f$ . Sometimes this is called “right equivalence” and the term “left-right equivalence” is used if transformations are allowed in the range  $\mathbb{R}$  as well. See also Definitions 2.3 and 2.12. For vector fields one usually works with *topological equivalence*.

*ergodic*: A Hamiltonian system is ergodic if the relative measure within the *energy shell* of any invariant set is either zero or one.

*essentially quasi-homogeneous*: A *semi-quasi-homogeneous* planar *singularity* for which the  $C^\infty$ -*moduli* of higher (weighted) order are topologically irrelevant.

*filtration*: A filtration of an algebra  $\mathcal{A}$  is a series  $(\mathcal{F}_n)_{n \in \mathbb{N}_0}$  of subalgebras with  $\mathcal{F}_n \supseteq \mathcal{F}_m$  for  $n \leq m$  and  $\mathcal{F}_0 = \mathcal{A}$ . For a *graded* algebra  $\mathcal{A} = \prod_{k=0} \mathcal{A}_k$  a filtration is given by  $\mathcal{F}_n = \prod_{k \geq n} \mathcal{A}_k$ .

*finitely determined*: A singularity that is *equivalent* to its  $(\mu + 1)$ -*jet*, where  $\mu < \infty$  is the *multiplicity* (or Milnor number).

*Floquet exponent*: The eigenvalues of the normal matrix of an invariant  $n$ -torus in *Floquet form*. For  $n = 1$  exponentiation of these yields *Floquet multipliers*.

*Floquet form*: An invariant torus is in Floquet form if the *normal linear* behaviour does not depend on the toral angles.

*Floquet multipliers*: Let  $\gamma$  be a periodic orbit of the vector field  $X$  with flow  $\varphi_t$  and denote by  $\tau$  the period of  $\gamma$ . For  $x \in \gamma$  the eigenvalues of the linearization  $D\varphi_\tau$  are called Floquet multipliers of  $\gamma$  (they do not depend on the particular choice  $x \in \gamma$ ). The tangent vector spanning  $T_x\gamma$  is an eigenvector of  $D\varphi_\tau$  to the eigenvalue 1. The other eigenvalues (counted with multiplicity) are also the eigenvalues of a *Poincaré mapping* defined on a *Poincaré section* through  $x$ . In case the vector field is Hamiltonian the Floquet multipliers occur in pairs  $(\lambda, \lambda^{-1})$  of the same multiplicity and the multipliers  $\pm 1$  have even multiplicity. The “second” generalized eigenvector to the eigenvalue 1 is transverse to the energy shell of  $\gamma$ .

*frequency-halving bifurcation*: In the *supercritical* case the invariant torus loses its stability, giving rise to a stable torus with one of its frequencies halved. In the *dual* or *subcritical* case stability is lost through collision with an unstable invariant torus with one of its frequencies halved.

*generic*: A property is generic if the set it defines contains a countable intersection of open dense sets. The genericity conditions that we impose on our normal forms are usually only finitely many inequalities and thus define open dense sets themselves. Where the perturbation has to be generic this often does involve infinitely many conditions (and it is therefore much more involved to check that all these are indeed satisfied).

*genuine first and second order resonance*: A *centre* in  $k_1:k_2:\dots:k_\ell$  resonance in  $\ell$  degrees of freedom for which there are  $\ell - 1$  independent *resonances* of order  $\leq 3$  (resp.  $\leq 4$ ).

*germ*: The germ of  $H$  in  $z$  consists of all  $K$  that coincide with  $H$  on some neighbourhood of  $z$ . To prove results on germs one usually keeps shrinking the neighbourhood.

*gradation*: A gradation of a finite dimensional algebra  $\mathcal{A}$  is a direct sum representation

$$\mathcal{A} = \bigoplus_{k=0}^n \mathcal{A}_k$$

with  $\mathcal{A}_k \cdot \mathcal{A}_m \subseteq \mathcal{A}_{k+m}$ . For infinite dimensional algebras (like the algebra  $\mathbb{R}[x]$  of polynomials) one might need infinite direct sums ( $\mathbb{R}[x]$  is graded as a direct sum of polynomials with degree  $k$ ), or even infinite (direct) products (as for the algebra  $\mathbb{R}[[x]]$  of formal power series in one real variable).

*group action:* A mapping  $\Gamma : G \times \mathcal{P} \longrightarrow \mathcal{P}$  is a group action of the Lie group  $G$  on the phase space  $\mathcal{P}$  if  $\Gamma_e = \text{id}$  and  $\Gamma_{g \cdot h} = \Gamma_g \circ \Gamma_h$  for all Lie group elements  $g$  and  $h$ . For  $G = \mathbb{R}$  one can recover the generating vector field by means of  $X = \frac{d}{dt} \Gamma_t$ .

*Hamiltonian flip bifurcation:* On the cone  $\mathcal{B} \subseteq \mathbb{R}^3$  defined by  $2uv = \frac{1}{2}w^2$ ,  $u \geq 0, v \geq 0$  the singular equilibrium  $(u, v, w) = 0$  loses its stability giving rise to a regular *centre*, cf. Fig. 2.4. In the *dual* case  $(u, v, w) = 0$  gains stability and a *saddle* splits off.

*Hamiltonian Hopf bifurcation:* In a two-degree-of-freedom system a *centre* loses its stability: the two pairs of purely imaginary eigenvalues meet in a *Krein collision*. In a three-degree-of-freedom system the same can happen to two pairs  $e^{\pm\lambda}, e^{\pm\mu}$  of *Floquet multipliers* in a one-parameter family of periodic orbits – parametrised by the energy. For a quasi-periodic Hamiltonian Hopf bifurcation one needs at least four degrees of freedom.

*Hamiltonian pitchfork bifurcation:* In the *supercritical* case the equilibrium/periodic orbit/invariant torus loses its stability, giving rise to an additional pair of stable equilibria/periodic orbits/invariant tori. In the *dual* or *subcritical* case stability is lost as a pair of unstable equilibria/periodic orbits/invariant tori shrinks down.

*Hamiltonian system:* Newton's second law states  $F = m\ddot{q}$ . Suppose that  $F$  is a conservative force, with potential  $V$ . We write the equations of motion as a system of first order differential equations

$$\begin{aligned} \dot{q} &= \frac{1}{m}p \\ \dot{p} &= -\frac{\partial V}{\partial q} \end{aligned}$$

that has the total energy  $H(q, p) = \langle p, p \rangle / (2m) + V(q)$  as a first integral. This can be generalized. Given a Hamiltonian function  $H(q, p)$  we have the Hamiltonian vector field

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} \end{aligned}$$

with first integral  $H$ . Moreover, we can replace  $\mathbb{R}^{2n}$  by a *symplectic manifold* or by a *Poisson space*.

*heteroclinic orbit:* A point  $z$  is heteroclinic to two equilibria  $a$  and  $b$  if  $z$  lies in the intersection of the *stable manifold* of  $a$  and the *unstable manifold* of  $b$ . This implies that the whole orbit of  $z$  consists of heteroclinic points. This notion immediately generalizes to orbits heteroclinic to two invariant sets, e.g. periodic orbits or invariant tori.

*Hilbert basis:* The (smooth) invariants of an action of a compact group are all given as functions of finitely many “basic” invariants.

*homoclinic orbit:* A point  $z$  is homoclinic to an equilibrium  $a$  if  $z$  lies in the intersection of the *stable* and *unstable manifolds* of  $a$ . This implies that the whole orbit of  $z$  consists of homoclinic points. This notion immediately generalizes to orbits homoclinic to invariant sets, e.g. periodic orbits or invariant tori.

*hypo-elliptic:* We call an equilibrium hypo-elliptic if its linearization has both elliptic and hyperbolic eigenvalues, all nonzero. Similarly for periodic orbits and invariant tori.

*hyperbolic:* A periodic orbit/invariant torus is hyperbolic if no eigenvalue of the symplectic *normal linearization* is on the imaginary axis. A hyperbolic equilibrium is called a *saddle*.

*integrable system:* A *Hamiltonian system* with  $d$  degrees of freedom is (Liouville)-integrable if it has  $d$  functionally independent commuting integrals of motion. Locally this implies the existence of a (local) torus action.

*iso-energetic Poincaré mapping:* For a Hamiltonian system a *Poincaré mapping* leaves the *energy shells* invariant. Restricting to the intersection  $\Sigma \cap \{H = h\}$  of the *Poincaré section* with an energy shell we obtain an iso-energetic Poincaré mapping.

*isotropic:* For a subspace  $U < V$  of a symplectic vector space  $(V, \omega)$  are equivalent:

- (i)  $U$  is contained in its  $\omega$ -orthogonal complement, (ii) every basis of  $U$  can be extended to a *Darboux basis* of  $V$ . If  $U$  satisfies one and thus both of these conditions it is called an isotropic subspace. A submanifold  $\mathcal{B} \subseteq \mathcal{P}$  of a symplectic manifold  $(\mathcal{P}, \omega)$  is called isotropic if all tangent spaces  $T_x \mathcal{B} < T_x \mathcal{P}$  are isotropic subspaces.

$\ell$ -jet: The Taylor polynomial of order  $\ell$  – up to co-ordinate changes.

*Krein collision:* Two pairs of purely imaginary *Floquet exponents* meet in a double pair on the imaginary axis and split off to form a complex quartet  $\pm \Re \pm i \Im$ . This is a consequence of a transversality condition on the linear terms at 1:–1 *resonance*; an additional non-degeneracy condition on the non-linear part ensures that a ((quasi)-periodic) *Hamiltonian Hopf bifurcation* takes place.

*Lagrangian submanifold:* For a subspace  $U < V$  of a symplectic vector space  $(V, \omega)$  are equivalent: (i)  $U$  is *isotropic* and *co-isotropic*, (ii)  $U$  is a maximal isotropic subspace, (iii)  $U$  is a minimal co-isotropic subspace, (iv)  $V$  has a *Darboux basis*  $\{e_1, \dots, e_n, f_1, \dots, f_n\}$  with  $\text{span}\{e_1, \dots, e_n\} = U$ . If  $U$  satisfies one and thus all of these conditions it is called a Lagrangian subspace. A submanifold  $\mathcal{B} \subseteq \mathcal{P}$  of a symplectic manifold  $(\mathcal{P}, \omega)$  is called Lagrangian if all tangent spaces  $T_x \mathcal{B} < T_x \mathcal{P}$  are Lagrangian subspaces.

*Lie–Poisson structure:* Let  $\mathfrak{g}$  be a Lie algebra with structure constants  $\Gamma_{ij}^k$ . Then

$$\{\mu_i, \mu_j\} = \pm \sum \Gamma_{ij}^k \mu_k \text{ defines two Poisson structures on the dual space } \mathfrak{g}^*.$$

*linear centraliser unfolding:* The *universal unfolding*  $\Omega_\lambda$  of a matrix  $\Omega_0 \in M_{n \times n}(\mathbb{R})$  within  $M_{n \times n}(\mathbb{R})$  obtained as  $\Omega_\lambda = \Omega_0 + \sum \lambda_i \Gamma_i$  where the  $\Gamma_i$  span  $\ker(\text{Ad}_{\Omega_0^T}) = \{\Gamma \in M_{n \times n}(\mathbb{R}) \mid \Gamma \circ \Omega_0^T = \Omega_0^T \circ \Gamma\}$ .

*local algebra:* The quotient of all germs in  $z$  by the Jacobi ideal, generated by the partial derivatives of a given *singularity*. As explained in [15] this is the “algebra of functions on the infinitesimal pre-image of the point  $z$ ”.

*local bifurcation:* Bifurcations of equilibria can be studied within a small neighbourhood in the product of phase space and parameter space. The same is true

- for fixed points of a discrete dynamical system, but when suspended to a flow the corresponding bifurcating periodic orbits obtain a *semi-local* character.
- local group action:* The defining properties  $\Gamma_e = \text{id}$  and  $\Gamma_{g \cdot h} = \Gamma_g \circ \Gamma_h$  of a *group action* of the Lie group  $G$  on the phase space  $\mathcal{P}$  also characterise a local group action, but the mapping  $\Gamma$  may only be defined on a neighbourhood of  $\{e\} \times \mathcal{P} \subseteq G \times \mathcal{P}$ .
- modulus:* Two planar singularities are called right *equivalent* if there is a (smooth) co-ordinate change of the plane that transforms these singularities into each other. Invariants obstructing such an equivalence that vary continuously with the singularity are called moduli.
- momentum mapping:* Let  $\Gamma : G \times \mathcal{P} \longrightarrow \mathcal{P}$  be a symplectic action of the Lie group  $G$  on the *symplectic manifold*  $\mathcal{P}$ . A mapping  $J : \mathcal{P} \longrightarrow \mathfrak{g}^*$  into the dual space of the Lie algebra of  $G$  is a momentum mapping for the action if  $X_{\hat{J}(\xi)} = \xi_{\mathcal{P}}$  for all  $\xi \in \mathfrak{g}$ . Here  $\hat{J}(\xi) : \mathcal{P} \longrightarrow \mathbb{R}$  is defined by  $\hat{J}(\xi)(z) = J(z) \cdot \xi$  and  $\xi_{\mathcal{P}}$  is the infinitesimal generator of the action corresponding to  $\xi$ . The momentum mapping  $J$  is called  $\text{Ad}^*$ -equivariant provided that  $J \circ \Gamma_g = \text{Ad}_{g^{-1}}^* \circ J$ .
- monodromy:* Let  $\rho : \mathcal{P} \longrightarrow B$  be a torus bundle with fibre  $\mathbb{T}^k$  and choose a connection on this bundle. Then a closed curve in  $B$  with base point  $y$  gives rise to an automorphism of the torus  $\rho^{-1}(y)$ . The induced homomorphism  $\pi_1(B, y) \longrightarrow GL(H^1(\rho^{-1}(y), \mathbb{Z}))$  does not depend on the particular choice of the connection and is called the monodromy of (the connected component containing  $\rho^{-1}(y)$  of)  $\mathcal{P}$ . For a product of  $k$  connected circle bundles the monodromy becomes a mapping  $\pi_1(B) \longrightarrow GL_k(\mathbb{Z})$ .
- Morse function:* A function  $H \in C^2(\mathcal{P})$  for which all *singularities* are quadratic and have different values.
- multiplicity:* The (maximal) number of critical points “contained” in a *singularity*. See also Definition 2.10.
- Newton diagram:* For a planar *singularity* the integer points  $(j, k) \in \mathbb{N}_0^2$  represent monomials  $p^k q^j$  and in the *non-degenerate* case finitely many of these can be chosen as representants of a basis of the *local algebra*, cf. Fig. 2.2.
- non-degenerate integrable Hamiltonian system, function:* In *action angle variables*  $(\varphi, I)$  the Hamiltonian  $H$  only depends on the action variables  $I$  and the equations of motion become  $\dot{\varphi} = \omega(I)$ ,  $\dot{I} = 0$ , with frequencies  $\omega(I) = \partial H / \partial I$ . The system is non-degenerate at the invariant torus  $\{I = I_0\}$  if  $D^2 H(I_0)$  is invertible. In this case  $\omega$  defines near  $\{I = I_0\}$  an isomorphism between the actions and the angular velocities. Other conditions ensuring that most tori have *Diophantine* frequency vectors are iso-energetic non-degeneracy or Rüssmann-like conditions on higher derivatives.
- non-degenerate singularity:* A *singularity* for which every unfolding has only finitely many critical points. This means that the *multiplicity* is finite and is equivalent to *finite determinacy* and implies that there exists a *universal unfolding*.
- normal frequency:* Given an elliptic invariant torus of a Hamiltonian system, one can define the normal linearization on the symplectic normal bundle, see [159, 56]. The eigenvalues of the normal linearization being  $\pm i\alpha_1, \dots, \pm i\alpha_m$ , we call the  $\alpha_j$  the normal frequencies. Under the exponential mapping the eigenvalues of the normal linearization of a periodic orbit are mapped to *Floquet multipliers*.
- normal hyperbolicity, normally hyperbolic manifold:* An invariant submanifold of a dynamical system is called normally hyperbolic if the normal bundle admits

a splitting into attracting and repelling directions with uniform bounds. For Hamiltonian systems one also speaks of hyperbolic invariant  $n$ -tori if the symplectic normal bundle admits a splitting into attracting and repelling directions. The normal bundle itself is then split into the symplectic normal bundle and the  $n$  directions conjugate to the *isotropic* tangent space. In particular, the  $n$ -parameter family of tori parametrised by the actions conjugate to the toral angles does form a normally hyperbolic manifold.

*normal linearization:* The linearization within the normal bundle of an invariant submanifold. In the Hamiltonian context these are often *isotropic* and we further restrict to the symplectic normal linear behaviour, e.g. to identify *elliptic* and *hyperbolic* invariant tori.

*normal resonance:* A *centre* in  $\ell$  degrees of freedom is in  $k_1:k_2:\dots:k_\ell$  resonance if the linearization is *conjugate* to the superposition of  $\ell$  oscillators with frequencies  $k_1\omega, k_2\omega, \dots, k_\ell\omega$  for some  $\omega \in \mathbb{R}$ .

*orbit cylinder:* An embedding  $\Gamma : S^1 \times ]a, b[ \longrightarrow \mathcal{P}$  such that for all  $e \in ]a, b[$  the set  $\gamma_e = \Gamma(S^1 \times \{e\})$  is a periodic orbit of the Hamiltonian system  $X_H$  on  $\mathcal{P}$ . In case  $\Gamma$  is transverse to every *energy shell* the parameter  $e$  can be chosen to be the energy of  $\gamma_e$ .

*parabolic:* We call an equilibrium of a one-degree-of-freedom system parabolic if its linearization is nilpotent but nonzero. An invariant torus is parabolic if its symplectic *normal linearization* has a parabolic equilibrium. In particular the four *Floquet multipliers* of a parabolic periodic orbit in two degrees of freedom are all equal to 1.

*phase space:* By Newton's second law the equations of motion are second order differential equations. The trajectory is completely determined by the initial positions and the initial velocities, or, equivalently, the initial momenta. The phase space is the set of all possible combinations of initial positions and initial momenta.

*pinched torus:* The compact (un)stable manifold of a saddle in two degrees of freedom with a quartet  $\pm\Re \pm i\Im$  of hyperbolic eigenvalues resembles a torus  $\mathbb{T}^2 = S^1 \times S^1$  with one of the fibres  $\{x\} \times S^1$  reduced to a point.

*Poincaré mapping:* To a point  $z \in \Sigma$  on a *Poincaré section* with time of first return  $T(z)$  the Poincaré mapping assigns the point  $\varphi_{T(z)}(z) \in \Sigma$ . This yields a diffeomorphism between domain and range.

*Poincaré section:* A hypersurface  $\Sigma$  that is transverse to the orbits of the flow  $\varphi_t$ . For points  $z \in \Sigma$  one calls  $\min\{T > 0 \mid \varphi_T(z) \in \Sigma\}$  the time of first return.

*Poisson space, Poisson structure:* A Poisson algebra  $\mathcal{A}$  is a real Lie algebra that is also a commutative ring with unit. These two structures are related by Leibniz' rule  $\{f \cdot g, h\} = f \cdot \{g, h\} + g \cdot \{f, h\}$ .

A Poisson manifold  $\mathcal{P}$  has a Poisson bracket on  $C^\infty(\mathcal{P})$  that makes  $C^\infty(\mathcal{P})$  a Poisson algebra. If there are locally no *Casimir elements* other than constant functions this leads to a *symplectic* structure on  $\mathcal{P}$ , see [16, 81].

Poisson spaces naturally arise in *singular reduction*, this motivates us to allow varieties  $\mathcal{P}$  where the Poisson bracket is defined on a suitable subalgebra  $\mathcal{A}$  of  $C(\mathcal{P})$ .

Given a Hamiltonian function  $H \in \mathcal{A}$  one obtains for  $f \in \mathcal{A}$  the equations of motion  $\frac{d}{dt}f = \{f, H\}$ . For canonically *conjugate* co-ordinates  $(q, p)$  on  $\mathcal{P}$ , i.e. with  $\{q_i, q_j\} = 0 = \{p_i, p_j\}$  and  $\{q_i, p_j\} = \delta_{ij}$ , this amounts to



$$\begin{aligned}\dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} .\end{aligned}$$

*Poisson symmetry:* A symmetry that preserves the *Poisson structure*.

*proper degeneracy:* For the application of the KAM-theorem to a perturbation of an *integrable system* it is necessary that the integrable system is *non-degenerate*, so that the frequencies have maximal rank as function of the actions. If there are global conditions relating the frequencies, so that the *conditionally periodic* motion can be described by a smaller number of these, the system is properly degenerate. *Superintegrable systems* are a particular example.

*quasi-homogeneous:* A homogeneous polynomial  $H$  satisfies  $H(e^\tau z) = e^{d\tau} H(z)$  where  $d$  is the order of  $H$ . For quasi-homogeneous polynomials one allows for different weights  $\alpha_{z_j}$  for the co-ordinates  $z_j$ . See also Definition 2.11.

*quasi-periodic:* A *conditionally periodic* motion that is not periodic. The closure of the trajectory is an invariant  $k$ -torus  $\mathbb{T}$  with  $k \geq 2$ .

A parallel or conditionally periodic flow on a  $k$ -torus is called quasi-periodic if the frequencies  $\omega_1, \dots, \omega_k$  are rationally independent.

*ramified torus bundle:* Let a differentiable mapping  $f : \mathcal{P} \longrightarrow \mathbb{R}^m$  be given. According to Sard's Lemma almost all values  $a \in \mathbb{R}^m$  are regular. The connected components of the sets  $f^{-1}(a)$ ,  $a \in \text{im} f$  regular, define a foliation of an open subset of the  $(n+m)$ -dimensional manifold  $\mathcal{P}$ .

In our settings the components of  $f$  are the first integrals of an *integrable* Hamiltonian system with compact level sets. Then the regular fibres are  $n$ -tori, and their union is a torus bundle. The topology of this bundle is determined by the topology of the base space  $B$ , the *monodromy* and the *Chern class* of the bundle. In many examples that we encounter, the connected components of  $B$  are contractible, whence monodromy and Chern class are trivial.

For the geometry of the bundle one also wants to know how the singular fibres are distributed: where  $n$ -tori shrink to normally elliptic  $(n-1)$ -tori and where they are separated by stable and unstable manifolds of normally hyperbolic  $(n-1)$ -tori. The singular fibres are not necessarily manifolds, but may be stratified into  $X_H$ -invariant strata which are possibly non-compact.

One can continue and look for the singularities of these 'regular singular leaves'; the tori of dimension  $\leq n-2$  and the normally *parabolic*  $(n-1)$ -tori in which normally elliptic and normally hyperbolic  $(n-1)$ -tori meet in a quasi-periodic *centre-saddle bifurcation* or a *frequency-halving bifurcation*. The next "layer" is given by quasi-periodic *Hamiltonian Hopf bifurcations* and bifurcations of higher co-dimension.

*reduced phase space:* Let  $\Gamma : G \times \mathcal{P} \longrightarrow \mathcal{P}$  be a symplectic action of the Lie group  $G$  on the *symplectic manifold*  $\mathcal{P}$  with  $\text{Ad}^*$ -equivariant *momentum mapping*  $J : \mathcal{P} \longrightarrow \mathfrak{g}^*$ . For a regular value  $\mu \in \mathfrak{g}^*$  let the action of the isotropy group  $G_\mu = \{g \in G \mid \text{Ad}_{g^{-1}}^*(\mu) = \mu\}$  on  $J^{-1}(\mu)$  be free and proper. Then the quotient  $J^{-1}(\mu)/G_\mu$  is again a symplectic manifold, the reduced phase space. A  $\Gamma$ -invariant Hamiltonian function  $H$  on  $\mathcal{P}$  leads to a reduced Hamiltonian function  $H_\mu$  on the reduced phase space.

*reducible:* Periodic orbits are always reducible to *Floquet form*. For invariant tori reducibility is a consequence of *integrability*.

- relative equilibrium:** Let  $H$  be a Hamiltonian function that is invariant under the symplectic action  $G \times \mathcal{P} \longrightarrow \mathcal{P}$  and let  $\mu \in \mathfrak{g}^*$  be a regular value of the  $\text{Ad}^*$ -equivariant momentum mapping  $J : \mathcal{P} \longrightarrow \mathfrak{g}^*$ . Also assume that the isotropy group  $G_\mu$  under the  $\text{Ad}^*$  action on  $\mathfrak{g}^*$  acts freely and properly on  $J^{-1}(\mu)$ . Then  $X_H$  induces a Hamiltonian flow on the reduced phase space  $\mathcal{P}_\mu = J^{-1}(\mu)/G_\mu$ . The phase curves of the given Hamiltonian system on  $\mathcal{P}$  with momentum constant  $J = \mu$  that are taken by the projection  $J^{-1}(\mu) \longrightarrow \mathcal{P}_\mu$  into equilibrium positions of the reduced Hamiltonian system are called relative equilibria or stationary motions (of the original system).
- remove the degeneracy:** A perturbation of a *superintegrable system* removes the degeneracy if it is sufficiently mild to define an “intermediate system” that is still integrable and sufficiently wild to make that intermediate system *non-degenerate*. See also Definition 5.2.
- resonance:** If the frequencies of an invariant torus with *conditionally periodic* flow are rationally dependent this torus divides into invariant subtori. Such resonances  $\langle h, \omega \rangle = 0$ ,  $h \in \mathbb{Z}^k$ , define hyperplanes in  $\omega$ -space and, by means of the frequency mapping, also in phase space. The smallest number  $|h| = |h_1| + \dots + |h_k|$  is the order of the resonance. *Diophantine conditions* describe a measure-theoretically large complement of a neighbourhood of the (dense!) set of all resonances.
- reversible:** A vector field  $X$  is reversible with respect to an involution  $\gamma$  if  $D\gamma \circ X \circ \gamma = -X$ . For a Hamiltonian vector field  $X_H$  this is guaranteed if  $\gamma$  preserves  $H$  and multiplies the *Poisson structure* by  $-1$  or if  $\gamma$  preserves the Poisson bracket and satisfies  $H \circ \gamma = -H$ .
- saddle:** An equilibrium of a vector field is called a saddle if the linearization has no eigenvalues on the imaginary axis. On a small neighbourhood of a saddle the flow is topologically *conjugate* to its linearization.
- semi-local bifurcation:** Bifurcations of  $n$ -tori can be studied in a tubular neighbourhood. For  $n = 1$  a *Poincaré section* turns the periodic orbit into a fixed point of the *Poincaré mapping* and the bifurcation obtains a *local* character.
- semi-quasi-homogeneous:** A singularity for which the “lowest order terms” form a *quasi-homogeneous* polynomial that is *non-degenerate*. Because of the last condition not every quasi-homogeneous polynomial is semi-quasi-homogeneous. See also Definition 2.11.
- simple:** A *singularity* is simple if it has no *moduli*. See also Definition 2.7.
- singular reduction:** If  $\Gamma : G \times \mathcal{P} \longrightarrow \mathcal{P}$  is a *Poisson symmetry*, then the group action on  $\mathcal{P}$  makes  $\mathcal{B} = \mathcal{P}/G$  a *Poisson space* as well. Fixing the values of the resulting *Casimirs* yields the *reduced phase space*, which turns out to have singular points where the action  $\Gamma$  is not free.
- singularity:** A critical point of the (Hamiltonian) function, in one degree of freedom this yields an equilibrium. See also the first paragraph of Appendix A.
- solenoid:** Given a sequence  $f_j : S^1 \longrightarrow S^1$  of coverings  $f_j(\zeta) = \zeta^{\alpha_j}$  of the circle  $S^1$  the solenoid  $\Sigma_a \subseteq (S^1)^{\mathbb{N}_0}$ ,  $a = (\alpha_j)_{j \in \mathbb{N}_0}$  consists of all  $z = (\zeta_j)_{j \in \mathbb{N}_0}$  with  $\zeta_j = f_j(\zeta_{j+1}) \ \forall j \in \mathbb{N}_0$ .
- stable manifold:** For an equilibrium  $a \in \mathcal{P}$  of the vector field  $X$  with flow  $\varphi_t$  the stable manifold is the set  $W_s(a) = \{z \in \mathcal{P} \mid \lim_{t \rightarrow \infty} \varphi_t(z) = a\}$  of points that are asymptotic to this equilibrium. Given an invariant subset  $A \subseteq \mathcal{P}$  of the flow this notion generalizes to the stable manifold  $W_s(A)$  of points that are asymptotic to  $A$ . The unstable manifold  $W_u(A) = \{z \in \mathcal{P} \mid \lim_{t \rightarrow -\infty} d(\varphi_t(z), A) = 0\}$  is defined by time reversal.

*stratification*: The decomposition of a topological space into smaller pieces satisfying certain boundary conditions. With the exception of *Cantor stratifications* all stratifications in these notes are Whitney-stratifications, see Definition B.1.

*structurally stable*: A system is structurally stable if it is *topologically equivalent* to all nearby systems. A family is structurally stable if for every nearby family there is a re-parametrisation such that all corresponding systems are topologically equivalent. See also Definition 2.2.

*subcartesian*: Essentially the stratified subsets of  $\mathbb{R}^n$ , see Definition B.2.

*subcritical*: Where the bifurcation scenario depends on a sign  $\pm$  the subcritical case is the one where additional critical elements are created as the equilibrium/periodic orbit/invariant torus gains its stability. Alternatively one speaks of the *dual* case.

*supercritical*: Where the bifurcation scenario depends on a sign  $\pm$  the supercritical case is the one where additional critical elements are created as the equilibrium/periodic orbit/invariant torus loses its stability, making this a “soft” loss of stability.

*superintegrable system*: A Hamiltonian system with  $d$  degrees of freedom is superintegrable if it has  $d + 1$  functionally independent integrals of motion such that each of the first  $d - 1$  of them commutes with all  $d + 1$ . Such a *properly degenerate* system admits generalized action angle co-ordinates  $(\varphi_1, \dots, \varphi_{d-1}, I_1, \dots, I_{d-1}, q, p)$ , see [219]. In case the non-degeneracy condition  $\det D^2 H(I) \neq 0$  is satisfied almost everywhere the system is “minimally superintegrable”. In the other extreme of a “maximally superintegrable” system all motions are periodic.

*symplectic manifold*: A  $2n$ -dimensional manifold  $\mathcal{P}$  with a non-degenerate closed two-form  $\omega$ , i.e.  $d\omega = 0$  and  $\omega(u, v) = 0 \forall v \in T\mathcal{P} \Rightarrow u = 0$ . A diffeomorphism  $\psi$  of symplectic manifolds that respects the two-form(s) is called a symplectomorphism. Given a Hamiltonian function  $H \in C^\infty(\mathcal{P})$  one obtains through  $\omega(X_H, \cdot) = dH$  the Hamiltonian vector field  $X_H$ . For every  $x \in \mathcal{P}$  there are co-ordinates  $(q, p)$  around  $x$  with  $\omega = dq \wedge dp$ . In these *Darboux co-ordinates*  $X_H$  reads

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} . \end{aligned}$$

*syzygy*: A constraining equation that is identically fulfilled by the elements of a *Hilbert basis*.

*tangency bifurcations*: *Generically* stable and *unstable manifolds* of fixed points of a *symplectic* mapping intersect transversely. Within generic one-parameter families they may also touch – intersect with matching tangents – giving rise to a tangency bifurcation.

*(topologically) equivalent*: Two flows  $\varphi_t$  and  $\psi_t$  are equivalent if there is a homeomorphism  $\eta$  that maps trajectories of  $\varphi_t$  into trajectories of  $\psi_t$  – preserving the time direction. In other words, there is a monotonous re-parametrisation  $\tau(t, y)$  of the flow  $\psi_\tau$  such that the two flows become topologically *conjugate*:  $\eta(\varphi_t(z)) = \psi_{\tau(t, \eta(z))}(\eta(z)) \forall z$ . See also Definition 2.1. In case the phase space is not a smooth manifold we require  $\eta$  to map singular points to singular points.

*ultraviolet cut-off*: An upper bound of the order of the Fourier coefficients.

*unfolding*: Every family  $(f_\lambda)_{\lambda \in \Lambda}$  containing  $g$  for a particular parameter value  $\lambda_0$ , i.e. with  $f_{\lambda_0} = g$ , is an unfolding of  $g$ ; even the constant family  $f_\lambda = g \quad \forall \lambda \in \Lambda$  is an unfolding of  $g$ . However, we are mostly interested in *structurally stable* or versal unfoldings.

*unimodal*: A *singularity* with exactly one *modulus*.

*universal unfolding*: An *unfolding*  $g_\mu$  of  $h$  is universal if to every other unfolding  $f_\lambda$  of  $h$  there is a parameter change  $\mu(\lambda)$  such that  $g_{\mu(\lambda)}$  and  $f_\lambda$  are *equivalent*.

*(un)stable manifold*: In Hamiltonian systems with one degree of freedom the *stable manifold* and the unstable manifold of an equilibrium often coincide and thus consist of *homoclinic orbits*. In such a case we call it an (un)stable manifold. This carries over to the stable and the unstable manifold of a periodic orbit or an invariant torus in higher degrees of freedom if the system is *integrable*.

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