Appendix

HEREDITARILY MAJORIZABLE FUNCTIONALS OF FINITE TYPE

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The purpose of the following is * to show that the Dialectica interpretation (chapter III, § 5) of the simplest nontrivial case $\forall y^2 E_2(y)$ of the axiom of extensionality cannot be carried out by use of a primitive recursive functional (theorem 3.2). To accomplish this we introduce the notion of hereditary majorizability and show that if a functional is hereditarily majorizable, then it does not satisfy the functional interpretation of the axiom of extensionality (§ 2). Then we show that every primitive recursive functional is hereditarily majorizable (§ 3).

The functional interpretation of the next most simple case $\forall y^3 E_3(y)$ of the axiom of extensionality is discussed briefly in § 4. In contrast with the case of $\forall y^2 E_2(y)$, the existence of a functional satisfying the functional interpretation of $\forall y^3 E_3(y)$ depends strongly on the class of functionals over which the variables are taken to range. Indeed, if the variables are taken to range over the class of ordinary set-theoretic functionals, then the existence of a functional providing the functional interpretation of $\forall y E_3(y)$ implies the axiom of choice for sets of number-theoretic functionals (theorem 4.1). Hence by known results, there are models of Zermelo - Fraenkel set theory (without the axiom of choice) in which there is no functional satisfying the functional interpretation of $\forall y^3 E_3(y)$.

§ 1. Extensionality.

Supposing $X, W, Z_1, \ldots, Z_s$ to be variables ** such that $XZ_1 \ldots Z_s$ and $WZ_1 \ldots Z_s$ are terms of type 0, let $X =_e W$ denote extensional equality, namely, $(\forall Z_1 \ldots Z_s)(XZ_1 \ldots Z_s = WZ_1 \ldots Z_s)$ (as in 2.7.2). Now let $X_1, \ldots, X_k$

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* References with three numbers refer to this volume outside this appendix. References such as 4.1, § 2 etc. refer to this appendix.

** In this appendix also capitals are used as variables for objects of finite type.

be variables of types $\sigma_1,\ldots,\sigma_k$, respectively, and let σ denote $(\sigma_1)(\sigma_2)\ldots(\sigma_k)0$. A functional G of type σ is said to be <u>extensional</u> if

$$(1.1)\qquad (\forall X_1\ldots X_k)(\forall W_1\ldots W_k)(\forall i[X_i =_e W_i] \rightarrow GX_1\ldots X_k = GW_1\ldots W_k),$$

where $\forall i[X_i =_e W_i]$ denotes the conjunction of $X_1 =_e W_1,\ldots,X_k =_e W_k$. Let us abbreviate (1.1) by $E_\sigma(G)$. The <u>axiom of extensionality</u> for functionals of type σ (2.7.2) is here taken in the form $\forall y^\sigma E_\sigma(y)$. The simplest non-trivial case of the axiom of extensionality is $\forall y^2 E_2(y)$; namely,

$$(1.2)\qquad \forall Y\alpha\beta(\forall u[\alpha u = \beta u] \rightarrow Y\alpha = Y\beta),$$

where α and β have type 1 and Y has type 2. A functional F of type $(2)(1)(1)0$ satisfies the <u>Dialectica functional interpretation</u> of (1.2) if

$$(1.3)\qquad \forall Y\alpha\beta[Y\alpha \neq Y\beta \rightarrow \alpha(FY\alpha\beta) \neq \beta(FY\alpha\beta)].$$

We will work in $\underset{\sim}{\mathrm{HA}}^\omega$ (1.6.15). We use the formalism of typed combinators because it simplifies the exposition of § 3. The λ-operator is assumed to be defined by the rules of 1.6.8.

As a point of methodology we note here that the three theorems of § 2, and all instances of the two theorems of § 3, are derivable in $\underset{\sim}{\mathrm{HA}}^\omega$ (see remark 3.1). Thus the theorems of §§ 2 - 3 are valid for all models of $\underset{\sim}{\mathrm{HA}}^\omega$: in particular (cf. chapter II): the set-theoretical model, the models HRO and HEO based on partial recursive function application, the models ICF and ECF based on continuous function application, the term models CTM and CTNF, and Kleene's general recursive functionals (cf. 2.8.2). Not that the theorems which deal directly with the functional interpretation of the axiom of extensionality (namely, theorems 2.2, 2.3 and 3.2) are of much interest in the case of nonextensional functionals: after all, the negation of the axiom (1.2) of entensionality implies the negation of the functional interpretation of (1.2) trivially. But theorems 2.1 and 3.1 on boundedness and hereditary majorizability appear to be of independent interest. (Also, the intensional continuous functionals of types 1 and 2 are extensional.)

§ 2. Hereditarily majorizable functionals.

A relation F^* maj F (F^* hereditarily majorizes F) will now be defined between functionals F^* and F of the same type σ. The definition is by induction on $\underset{\approx}{T}$. If σ is 0, then F^* and F are numbers n and m: we define n maj m to mean $n \geq m$. If σ is $(\tau)\rho$ then F^* maj F means

$\forall G^*G(G^* \text{ maj } G \rightarrow F^*G^* \text{ maj } FG)$. We say that a functional F is hereditarily majorizable if there exists a functional F^* such that F^* maj F.

The following three remarks are easily verified.

Remark 2.1. If F^* maj F and G^* maj G, then F^*G^* maj FG.

Remark 2.2. If G_r^* maj G_r for $0 \leq r \leq p$, then $G_0^*G_1^*\ldots G_p^*$ maj $G_0G_1\ldots G_p$.

Remark 2.3. Suppose $HX_1\ldots X_p$ has type 0. If $H^*X_1^*\ldots X_p^* \geq HX_1\ldots X_p$ for all $X_1^*,\ldots,X_p^*, X_1,\ldots,X_p$ such that X_r^* maj X_r for $1 \leq r \leq p$, then H^* maj H.

In the following theorem, X_r and $Z_1,\ldots,Z_{s(r)}$ are variables such that $X_rZ_1\ldots Z_{s(r)}$ has type 0.

Theorem 2.1. Suppose a functional F of type $(\sigma_1)(\sigma_2)\ldots(\sigma_p)0$ is hereditarily majorizable. Let k be fixed and, for $1 \leq r \leq p$, let $\underline{M}_r$ denote the set of functionals X_r of type σ_r such that $(\forall Z_1\ldots Z_{s(r)})(X_rZ_1\ldots Z_{s(r)} \leq k)$. Then

$$(2.1) \qquad \exists m(\forall X_1 \in \underline{M}_1)\ldots(\forall X_p \in \underline{M}_p)(FX_1\ldots X_p \leq m).$$

Proof. By assumption there exists F^* such that F^* maj F. For $1 \leq r \leq p$, let G_r^* denote $(\lambda Z_1\ldots Z_{s(r)}).k$. Then $(\forall X_r \in \underline{M}_r)(G_r^* \text{ maj } X_r)$ by remark 2.3. Hence

$$(\forall X_1 \in \underline{M}_1)\ldots(\forall X_p \in \underline{M}_p)(F^*G_1^*\ldots G_p^* \geq FX_1\ldots X_p)$$

by remark 2.2. Thus $F^*G_1^*\ldots G_p^*$ is the required number m.

Theorem 2.2. For $r = 1, 2$, let $\underline{N}_r$ denote the set of functionals X of type r such that $\forall Z(XZ \leq 1)$. Let F be a functional of type $(2)(1)(1)0$ such that

$$(2.2) \qquad \exists m(\forall Y \in \underline{N}_2)(\forall \alpha \in \underline{N}_1)[FY\alpha(\lambda u.0) \leq m].$$

Then F does not satisfy the functional interpretation (1.3) of the axiom of extensionality.

Proof. It is easy to define a primitive recursive functional $\lambda n.Y_n$ such that, for all α,

$$Y_n\alpha = \begin{cases} 1 & \text{if } (\forall u < n)(\alpha u = 0) \text{ and } \alpha n = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Also it is easy to define a functional $\lambda n.\alpha_n$ such that

$$\alpha_n u = \begin{cases} 0 & \text{if } u < n \\ 1 & \text{if } u \geq n. \end{cases}$$

Assume (1.3) and take β to be $\lambda u.0$. Denote $\lambda Y\alpha.FY\alpha(\lambda u.0)$ by G.

Clearly $Y_n(\lambda u.0) = 0$. Hence $Y_n\alpha_n \neq 0 \rightarrow \alpha_n(GY_n\alpha_n) \neq 0$ by (1.3). But $Y_n\alpha_n = 1$. Hence $\alpha_n(GY_n\alpha_n) \neq 0$. Hence $GY_n\alpha_n \geq n$ by the definition of α_n. But $Y_n \in \underline{N}_2$ and $\alpha_n \in \underline{N}_1$. Thus the hypothesis (2.2) has been contradicted, so (1.3) has been refuted. Q. E. D.

Theorem 2.3. Suppose a functional of type $(2)(1)(1)0$ is hereditarily majorizable. Then F does not satisfy the functional interpretation (1.3) of the axiom of extensionality.

Proof. By theorem 2.1 (for $k = 1$) and theorem 2.2.

Remark 2.4. By inspection of theorems 2.1 - 2.3 we see that actually the following sharp form of theorem 2.3 has been proved. There are primitive recursive functionals $\lambda n.Y_n$ and $\lambda n.\alpha_n$ such that, for all F and F^*: if F^* maj F, then

$$\neg[Y_d\alpha_d \neq Y\beta_0 \rightarrow \alpha_d(Y_d\alpha_d\beta_0) \neq \beta_0(Y_d\alpha_d\beta_0)],$$

where β_0 is $\lambda u.0$ and $d = F^*(\lambda\alpha.1)(\lambda u.0)(\lambda u.0)$.

Indeed, the above proofs go through for relative hereditary majorization: all that is assumed of the set $\underline{A}$ of majorizing functionals and the set $\underline{B}$ of functionals being majorized is that $\underline{A}$ and $\underline{B}$ are closed under application and contain certain simple primitive recursive functionals.

Construction 2.1. Given F and F^* of type $(0)\sigma$ such that $\forall n(F^*n \text{ maj } Fn)$, to find H^* such that H^* maj F. Solution: take H^* to be $(\lambda X_1 \ldots X_k).\sum_{m \leq n} F^*mX_1\ldots X_k$, where $X_1,\ldots,X_k$ are variables of types such that $F^*mX_1\ldots X_k$ has type 0. We denote this H^* by $(F^*)^+$.

§ 3. Primitive recursive functionals.

We indicate extensional equality of F and G by $F =_e G$ as in § 1. By applying universal quantifiers to the appropriate axioms for $\underline{HA}^\omega$ we obtain:

(3.1) $\quad (\forall XY)[\Pi XY =_e X]$,

(3.2) $\quad (\forall XYZ)[\Sigma XYZ =_e XZ(YZ)]$,

(3.3) $\quad (\forall XY)[RXY0 =_e X]$,

(3.4) $\quad (\forall XYu)[RXY(Su) =_e Y(RXYu)u]$.

The set $\underline{P}$ of all primitive recursive functionals is defined with reference to a given notion of functional. In the case of the set-theoretic notion (2.4.6) there is no problem since in this case the equations (3.1) - (3.4) pick out functionals Π, Σ and R of all appropriate types from the

supply in a unique way. Hence we define $\underline{P}$ inductively by the following two clauses:

(a) $\underline{P}$ contains zero, the successor function, and the functionals Π, Σ and R of all appropriate types.

(b) If $F \in \underline{P}$ and $G \in \underline{P}$, then $FG \in \underline{P}$.

In the case of the model ICF there is some choice as to which associate is to be named by the constant Π: the equation (3.1) does not pick out an associate from the supply ICF in a unique way. Similarly for the constants Σ and R. Also, the operation of application does not determine uniquely the corresponding operation on associates. Similar remarks apply to the case of HRO. Fortunately these sources of nonuniqueness do not cause any trouble in the present paper. We merely add the following clause to (a) and (b):

(c) If $F \in \underline{P}$ and $H =_e F$, then $H \in \underline{P}$.

<u>Theorem 3.1</u>. Every primitive recursive functional has a primitive recursive hereditarily majorizing functional.

<u>Proof</u>. We shall indicate, for each of the clauses (a) - (c), how the majorizing functional H^* for H is obtained when H arises by use of the given clause. In the case of clause (b), H is FG so we can take H^* to be F^*G^* by remark 2.1. The case of clause (c) is handled by the observation that if $H =_e F$ and F^* maj F, then F^* maj H. It remains to treat the case of clause (a); namely, we must verify theorem 3.1 for each of the generating functionals 0, S, Π, Σ and R. Obviously we can take 0^* and S^* to be 0 and S, respectively. By remarks (2.2) - (2.3) and equations (3.1) - (3.2) we can take Π^* and Σ^* to be Π and Σ, respectively. Similarly, by remarks (2.2) - (2.3), equations (3.3) - (3.4) and induction on n, we find RX^*Y^*n maj $RXYn$ whenever X^* maj X, Y^* maj Y. Hence we can take R^* to be $\lambda XY.(RXY)^+$ as in construction 2.1.

<u>Theorem 3.2</u>. There is no Dialectica interpretation of the axiom of extensionality (1.2) by a primitive recursive functional.

<u>Proof</u>. By theorems 2.3 and 3.1.

<u>Corollary</u>. $\underset{\sim}{E}$ - $\underset{\sim}{HA}^{\omega}$ does not have a Dialectica interpretation in itself.

This corollary follows from the fact that $\underset{\sim}{E}$ - $\underset{\sim}{HA}^{\omega}$ can be axiomatized as $\underset{\sim}{HA}^{\omega} + \forall y E_{\sigma}(y)$ for all σ, and it has a model by primitive recursive functionals. For such a model we can use any of various classes of functionals mentioned in § 1: the set-theoretic functionals, the extensional continuous functionals, or the extensional effective operations. We can even use the <u>minimal term model</u> consisting of the closed terms of $\underset{\sim}{HA}^{\omega}$ since these terms act extensionally on themselves.

Remark 3.1. To treat theorem 3.1 by use of $\underset{\sim}{\mathrm{HA}}^{\omega}$ let us consider first the case of a primitive recursive functional defined by use of clauses (a) and (b) alone. Then the functional will be represented by a closed term F. Inspection of the proof of theorem 3.1 provides a corresponding term F^* together with a derivation, in $\underset{\sim}{\mathrm{HA}}^{\omega}$, of the formula F^* maj F. In the case of a primitive recursive functional defined by the use of clause (c) as well as clauses (a) - (b), the functional will be described in $\underset{\sim}{\mathrm{HA}}^{\omega}$ by the use of variables. For illustration suppose that clause (c) has been applied only at the last step of the definition. Then the formula to be derived in $\underset{\sim}{\mathrm{HA}}^{\omega}$ is $\forall X(X =_e F \to F^* \text{ maj } X)$. (This shows, by the way, that the majorizing functionals in theorem 3.1 can be chosen from the functionals generated by clauses (a) - (b) alone.) Thus each instance of theorem 3.1 is derivable in $\underset{\sim}{\mathrm{HA}}^{\omega}$.

A similar remark applies to theorem 3.2. Also, remark 2.4 and the proof of theorem 3.2 provide an effective procedure which when applied to the definition of F by use of clauses (a) - (c) yields primitive recursive Y_n, α_n and β_0 such that

$$\neg[Y\alpha_n \neq Y\beta_0 \to \alpha_n(FY_n\alpha_n\beta_0) \neq \beta(FY_n\alpha_n\beta_0)].$$

Thus if a functional H agrees with a primitive recursive functional at all primitive recursive arguments, then H does not satisfy the functional interpretation of the axiom (1.2) of extensionality.

Remark 3.2. There exists a functional F satisfying the functional interpretation (1.3) of the axiom $\forall Y \underset{\sim}{E}_2(Y)$ of extensionality which is general recursive in the sense of Kleene 1959 (cf. 2.8.2), where it is understood that Y, α and β range over all set-theoretic functionals of types 2, 1 and 1, respectively. Namely, the instructions for calculating $FY\alpha\beta$ are as follows. If $Y\alpha = Y\beta$, take $FY\alpha\beta$ to be 0. If $Y\alpha \neq Y\beta$, examine αn and βn successively for $n = 0,1,2,\ldots$, and take $FY\alpha\beta$ to be the least n such that $\alpha n \neq \beta n$. Since Y is extensional, the required n exists. Thus F is μ-recursive (Kleene 1959, p. 45).

Exactly the same definition yields F satisfying (1.3) when Y, α and β are understood to range over extensional continuous functionals. Hence, by theorem 4 of Kleene 1959A, p. 94, the functional F is itself continuous (which is also easy to see directly).

If Y, α and β are understood to range over (extensional) effective operations, then clearly the above definition yields an effective operation F satisfying (1.3).

§ 4. Discussion of $\forall y\, E_3(y)$.

The axiom $\forall y\, E_3(y)$ is

$$(4.1) \qquad \forall YXW(\forall\alpha[X\alpha = Y\alpha] \rightarrow YX = YW),$$

where X and W have type 2, and Y has type 3 (i.e., $(2)0$). The functional interpretation of (4.1) is

$$(4.2) \qquad \forall YXW[YX \neq YW \rightarrow X(FYXW) \neq W(FYXW)].$$

If Y, X and W are taken to range over extensional continuous functionals, then there exists a Kleene general recursive functional F satisfying (4.2): we merely generalize the definition of F given in remark 3.2, using the fact that the extensional continuous functionals of a given type have a recursively dense base (2.6.16). Let $h_0, h_1, \ldots, h_n, \ldots$ be a recursively dense base for the functionals of type 1. If $YX = YW$, take $FYXW$ to be h_0. If $YX \neq YW$, take $FYXW$ to be h_k, where $k = \min_n(Xh_n \neq Wh_n)$.

These considerations obviously generalize to the case of $\forall y\, E_\sigma(y)$ for arbitrary σ, where all variables are taken to range over extensional continuous functionals.

In the following theorem, Y, X and W are taken to range over set-theoretic functionals.

Theorem 4.1. From a functional F satisfying (4.2) we can construct, in set theory, a function ψ defined on all sets $\underline{M}$ of functions of type 1, such that $\psi\underline{M} \in \underline{M}$ for all nonempty $\underline{M}$.

Proof. Let $\varphi\underline{M}$ denote the characteristic function of $\underline{M}$. That is to say: for all α of type 1, $\varphi\underline{M}\alpha$ is 1 or 0 according as to whether α is in $\underline{M}$ or not. Let H of type 3 be defined by the condition that HX is 0 or 1 according as to whether $\forall\alpha(X\alpha = 0)$ or not. Define $\psi\underline{M}$ to be $FH(\varphi\underline{M})(\lambda\alpha.0)$. Then clearly $\psi\underline{M} \in \underline{M}$ for every nonempty $\underline{M}$.

Let ZF denote Zermelo - Fraenkel set theory and let ZFC denote ZF extended by the addition of the axiom of choice. In Rosser 1969, pp. 113 - 115, it is shown that there are models of ZF in which there is no well-ordering of the real numbers. On the other hand, a well-ordering of the real numbers can be defined in ZF with the help of the choice function ψ of theorem 4.1. Hence we obtain:

Corollary 1. There are models of ZF in which there is no functional satisfying the functional interpretation (4.2) of $\forall y\, E_3(y)$.

Similarly, from the fact that there are models of ZFC in which no formula well-orders the real numbers (Rosser 1969, p. 89), we obtain:

Corollary 2. There are models of ZFC in which no functional definable by a formula of ZF satisfies the functional interpretation (4.2) of $\forall y\, E_3(y)$.

Of course the existence of a functional satisfying the functional interpretation of $\forall y\, E_3(y)$ follows immediately from the axiom of choice.

From corollary 2 and Kleene 1959, p. 32 we obtain:

Corollary 3. There are models of ZFC in which no Kleene general recursive functional satisfies the functional interpretation (4.2) of $\forall y\, E_3(y)$.

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LMPS	for :	"Logic, Methodology and Philosophy of Science".
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Some corrections: page 41, line 13, replace $\leqq$ by $\geqq$; line 15 must read: $\Lambda n(\mathrm{lth}(n) + k = en \dot{-} 1 \wedge en \neq 0 \rightarrow Yn)$; line 16, replace K by N. Page 51: formula (10) is not correct as it stands. See instead the techniques developed in Kreisel-Troelstra 1970, § 5 for dealing with schemata with parameters. Page 79, the definition 14.2.4 is misstated and should read: φ_u is a mapping defined as follows. Let $u = \langle x_o, \ldots, x_n \rangle$, $v = \langle y_o, \ldots, y_m \rangle$, $w = (\overline{\lambda x.1})(n+1)$. Then (a) If $v \leq u$ we take $\varphi_u v = w$; (b) If $y_o > x_o$ we take $\varphi_u v = \langle y_o - x_o, y_1, \ldots, y_m \rangle$; (c) If $y_o = x_o, \ldots, y_i = x_i$, $y_{i+1} > x_{i+1}$ we take $\varphi_u v = \langle 0, \ldots, 0, y_{i+1} - x_{i+1}, y_{i+2}, \ldots, y_m \rangle$; (d) In all other cases we take e.g. $\varphi_u v = w$. φ_u is an order-isomorphism when restricted to species $F[u)$, $F[u,v)$; φ_u maps $F[u)$, $F[u,v)$ onto elements of WO.
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(A.S. Troelstra, continued)
page 398, line 6, replace " HRE " by " HEO " ; line -3, read "8.2" for "7.2".
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Correction : in the proof of 2.7, under "Extensions", the analysis should not be applied to a path, but to a spine as defined in § 4.2 of this volume. (The reason is that our permutative reductions did not reckon with permutation with UP, IP - rules.)
The change does not affect the applications.
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INDEX

Notions and notations are listed only when they have more than local significance in the text. The references given refer to definitions or special conventions regarding the notion or notations listed.

I. List of symbols

A) Formal systems, arranged primarily in alphabetic order

For some general conventions see 1.1.2 (viii). For any formal system $\mathbf{H}$ based on intuitionistic logic, $\mathbf{H}^c$ indicates the corresponding classical system.

B) Schemata and rules, arranged primarily in alphabetic order

For &I, $\vee I_r$, $\vee I_l$, $\rightarrow I$, $\forall I$, $\exists I$, $\bigwedge_I$, &E_r, &E_l, $\rightarrow E$, $\forall E$, $\exists E$ see 1.1.7; for $\forall_2 I$, $\forall_2 E$, λI, λE see 4.5.2.

C) Syntactical variables (in order of appearance)

There are many local deviations in the use of variables. Often new variables are made by adding sub- or super-scripts reserved for variables of a certain category.

x, y, z, u, v, w 1.1.2 (ii); cf. 6.2.1, 6.3.3
a, b, c 1.1.2 (ii); cf. 6.2.1
$A, B, C, \ldots$ 1.1.2 (ii)
t, s 1.1.2 (iii), 6.2.1.; cf. 1.6.5, 6.3.3
$\bar{n}, \bar{m}, \bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v}, \bar{w}$ 1.3.9 D; cf. 5.2.3
n, m 5.2.3
$x^\sigma, y^\sigma, z^\sigma, u^\sigma, v^\sigma, w^\sigma$ 1.6.3
$\underline{\underline{x}}, \underline{\underline{y}}, \underline{\underline{z}}, \underline{\underline{u}}, \underline{\underline{v}}, \underline{\underline{w}}$ 1.6.5
$\underline{\underline{X}}, \underline{\underline{Y}}, \underline{\underline{Z}}, \underline{\underline{U}}, \underline{\underline{V}}, \underline{\underline{W}}$ 1.6.5
s, t, T 1.6.5
s^σ, t^σ 1.6.5
$\underline{\underline{s}}, \underline{\underline{t}}, \underline{\underline{T}}$ 1.6.5, 6.5.5
X^n, Y^n, Z^n 1.9.3
$x^1, y^1, z^1, u^1, v^1, w^1$ 1.9.10
$\alpha, \beta, \gamma, \ldots$ 1.9.10; cf. 6.3.3
$e, f, e', e'', e_1, \ldots, f', f'', \ldots$ 1.9.25; cf. 6.2.1
$\Pi, \Pi', \Pi'', \Pi_1, \ldots$ 4.1.2
$\Sigma, \Sigma', \Sigma'', \ldots, \Sigma_0, \Sigma_1, \ldots$ 4.1.2
a, b, e, k, m, n 6.2.1
$\alpha, \beta, \alpha^1, \beta^1$ 6.3.3
α^2, β^2 6.3.3
$f^{(0)1}, f^{(1)2}$ 6.3.3
r, s, t, u, v, t_1, t_1' 6.3.3
Θ, Θ' 6.4.1
$\Phi, \Psi, \Psi', \ldots$ 6.8.1.

D) Other symbols

The symbols are primarily listed in order of appearance, some very similar ones are grouped together.

$\&, \vee, \exists, \forall, \rightarrow, \curlywedge$ 1.1.2 (i)
$\Rightarrow, \Leftrightarrow, \underset{\sim}{\forall}, \underset{\sim}{\exists}, \in, \subseteq$ 1.1.2 (i)
$=_{def}$ 1.1.2 (i)
$\equiv$ 1.1.2 (i)
$t[x], t[x,y]$ 1.1.2 (iii)
$\neg$ 1.1.2 (vi)
$\longleftrightarrow$ 1.1.2 (vi)
$[x/t]E$ 1.1.2 (vii)

II. List of notions