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## Glossary

### Chapter 1. Orderings

$\leq$	quasi-ordering . . . . .	12
$\equiv$	equivalent elements w.r.t. quasi-ordering . . . . .	12
$E \amalg F$	disjoint union of $E$ and $F$ . . . . .	12
$E \dot{\amalg} F$	ordered union of $E$ and $F$ . . . . .	13
$E \times F$	Cartesian product of $E$ and $F$ . . . . .	13
$E \dot{\times} F$	anti-lexicographic product of $E$ and $F$ . . . . .	13
$E^*$	set of words over $E$ . . . . .	13
$E^+$	set of non-empty words over $E$ . . . . .	13
$\leq^!$	ordering defined by $x \leq^! y \Leftrightarrow x = y \vee x < y$ . . . . .	14
$E^\diamond$	set of commutative words over $E$ . . . . .	15
$\omega$	the ordinal number $\omega = \{0, 1, 2, \dots\}$ . . . . .	16
$\text{fin}(A)$	final segment generated by $A$ . . . . .	17
$\text{in}(A)$	initial segment generated by $A$ . . . . .	17
$\text{root}(T)$	root of $T$ . . . . .	19
$E^\top$	set of finite $E$ -labeled trees . . . . .	20
$S^\geq$	subset of positive or zero elements in $S$ . . . . .	22
$S^>$	subset of strictly positive elements in $S$ . . . . .	22
$S^\neq$	subset of non-zero elements in $S$ . . . . .	22
$S^\leq$	subset of negative or zero elements in $S$ . . . . .	22
$S^<$	subset of strictly negative elements in $S$ . . . . .	22
$ x $	absolute value of $x$ . . . . .	22
$R^*$	subset of non-zero-divisors in $R$ . . . . .	25
$x \preceq y$	$x$ is dominated by $y$ . . . . .	25
$x = O(y)$	Landau's notation for $x \preceq y$ . . . . .	25
$x \asymp y$	$x$ is asymptotic to $y$ . . . . .	25
$x \prec y$	$x$ is negligible w.r.t. $y$ . . . . .	25
$x = o(y)$	Landau's notation for $x \prec y$ . . . . .	25
$x \sim y$	$x$ is equivalent to $y$ . . . . .	25

$A^\times$	elements of $A$ which can be raised to a power . . . . .	30
$\ x\ $	$\ x\  = x$ if $x \succcurlyeq 1$ and $\ x\  = x^{-1}$ otherwise . . . . .	30
$x \preccurlyeq y$	$x$ is flatter than or as flat as $y$ . . . . .	30
$x \prec y$	$x$ is flatter than $y$ . . . . .	30
$x \succcurlyeq y$	$x$ is as flat as $y$ . . . . .	30
$x \approx y$	$x$ and $y$ are similar modulo flatness . . . . .	30
$A^\flat$	flat subring of $A$ . . . . .	31
$x \preccurlyeq^\# y$	$x \preccurlyeq y$ modulo flatness . . . . .	31
$x \prec^\# y$	$x \prec y$ modulo flatness . . . . .	31
$x \preccurlyeq_\varphi y$	$x \preccurlyeq y$ modulo elements flatter than $\varphi$ . . . . .	31
$x \prec_\varphi y$	$x \prec y$ modulo elements flatter than $\varphi$ . . . . .	31
$x \preccurlyeq_\varphi^* y$	$x \preccurlyeq y$ modulo elements flatter than or as flat as $\varphi$ . . .	31
$x \prec_\varphi^* y$	$x \prec y$ modulo elements flatter than or as flat as $\varphi$ . . .	31

## Chapter 2. Grid-based series

$\mathfrak{M}$	quasi-ordered monomial monoid . . . . .	34
$\mathfrak{m}, \mathfrak{n}, \mathfrak{v}, \dots$	monomials . . . . .	34
$C$	ring of coefficients . . . . .	36
$\text{supp } f$	support of a series $f$ . . . . .	36
$C[[\mathfrak{M}]]$	ring of grid-based series in $\mathfrak{M}$ with coefficients in $C$ . .	36
$f_{\mathfrak{m}}$	coefficient of $\mathfrak{m}$ in $f$ . . . . .	36
$C[z_1, \dots, z_n]$	natural multivariate series in $z_1, \dots, z_n$ . . . . .	38
$C[[z_1; \dots; z_n]]$	recursive multivariate series in $z_1, \dots, z_n$ . . . . .	38
$\mathfrak{d}_f$	dominant monomial of $f$ . . . . .	40
$c_f$	dominant coefficient of $f$ . . . . .	40
$\tau_f$	dominant term of $f$ . . . . .	40
$\delta_f$	series with $f = \tau_f(1 + \delta_f)$ . . . . .	40
$f_{\succ}$	purely infinite part of $f$ . . . . .	41
$f_{\prec}$	constant part of $f$ . . . . .	41
$f_{\prec}$	infinitesimal part of $f$ . . . . .	41
$f_{\preccurlyeq}$	bounded part of $f$ . . . . .	41
$C[[\mathfrak{M}]]_{\succ}$	set of purely infinite elements in $C[[\mathfrak{M}]]$ . . . . .	41
$C[[\mathfrak{M}]]_{\prec}$	set of infinitesimal elements in $C[[\mathfrak{M}]]$ . . . . .	41
$C[[\mathfrak{M}]]_{\preccurlyeq}$	set of bounded elements in $C[[\mathfrak{M}]]$ . . . . .	41
$\pi_\lambda$	the series $(1+z)^\lambda \in C[[z]]$ . . . . .	42
$f_{\mathfrak{S}}$	restriction of $f$ to $\mathfrak{S}$ . . . . .	42
$f \trianglelefteq g$	$f$ is a truncation of $g$ . . . . .	43
$\mathcal{F}(S)$	collection of families with values in $S$ . . . . .	44
$\mathcal{F}, \mathcal{G}, \dots$	families . . . . .	44
$(f_i; i \in I)$	alternative notation for family $(f_i)_{i \in I}$ . . . . .	44
$\mathcal{F} \amalg \mathcal{G}$	disjoint union of two families $\mathcal{F}$ and $\mathcal{G}$ . . . . .	44
$\mathcal{F} \times \mathcal{G}$	Cartesian product of families $\mathcal{F}$ and $\mathcal{G}$ . . . . .	44

$\varphi(\mathcal{F}_1, \dots, \mathcal{F}_n)$	application of a function $\varphi$ on families $\mathcal{F}_1, \dots, \mathcal{F}_n$ . . . .	45
$\mathcal{F} \approx \mathcal{G}$	$\mathcal{F}$ and $\mathcal{G}$ coincide modulo renaming indexes . . . . .	45
$\mathcal{F} \subsetneq \mathcal{G}$	$\mathcal{F}$ is contained in $\mathcal{G}$ modulo renaming indexes . . . . .	45
$\mathcal{S}(A)$	collection of strongly summable families over $A$ . . . .	46
$\sum \mathcal{F}$	strong sum of $\mathcal{F}$ . . . . .	46
term $\mathcal{F}$	family of terms occurring in $\mathcal{F}$ . . . . .	49
mon $\mathcal{F}$	family of monomials occurring in $\mathcal{F}$ . . . . .	49
$\mathcal{F}^*$	family of power products of elements in $\mathcal{F}$ . . . . .	50
$\hat{\varphi}$	extension of $\varphi$ by strong linearity . . . . .	50
$g \circ (f_1, \dots, f_k)$	composition of $g \in C[[z_1, \dots, z_k]]$ with $f_1, \dots, f_k \prec 1$ . . .	53

### Chapter 3. The Newton polygon method

$N_{P, \mathfrak{m}}$	Newton polynomial associated to $\mathfrak{m}$ . . . . .	65
$C[[\mathfrak{M}]]_A$	series in $C[[\mathfrak{M}]]$ with Cartesian representations in $A$ . .	71
$C\{\{\mathfrak{M}\}\}$	subset of convergent grid-based series in $C[[\mathfrak{M}]]$ . . . .	73

### Chapter 4. Transseries

$\log_n$	the $n$ -th iterate of $\log$ with $n \in \mathbb{Z}$ . . . . .	80
$\exp_n$	the $n$ -th iterate of $\exp$ with $n \in \mathbb{Z}$ . . . . .	80
$\mathbb{T}$	field of grid-based transseries . . . . .	84
$\mathfrak{T}^b$	flat subset of $\mathfrak{T}$ . . . . .	87
$\mathfrak{T}^\sharp$	steep complement of $\mathfrak{T}^b$ . . . . .	87
$\mathbb{T} = C[[x]]$	field of grid-based transseries in $x$ . . . . .	87
$\mathbb{L}$	field of logarithmic transseries in $x$ . . . . .	88
$\mathbb{T}_{\exp}$	exponential extension of $\mathbb{T}$ . . . . .	88
$C_p^q[[x]]$	transseries of exp. height $\leq p$ and log. depth $\leq q$ . . .	89
$\mathbb{E}$	field of exponential transseries . . . . .	90
$f \uparrow$	upward shifting $f \circ \exp$ of $f$ . . . . .	90
$f \downarrow$	downward shifting $f \circ \log$ of $f$ . . . . .	90
$\text{con } f$	contraction $\log \circ f \circ \exp$ of $f$ . . . . .	91
$\text{dil } f$	dilatation $\exp \circ f \circ \log$ of $f$ . . . . .	91
$C\{\{\{x\}\}\}$	field of convergent transseries . . . . .	94

### Chapter 5. Operations on transseries

$f'$	derivative of $f$ . . . . .	98
$f^\dagger$	logarithmic derivative of $f$ . . . . .	98
$\int f$	distinguished integral of $f$ . . . . .	103
$\circ_g$	right composition with $g$ . . . . .	106
$f \circ g$	composition of $f$ and $g$ . . . . .	106
$g^{\text{inv}}$	functional inverse of $g$ . . . . .	111
$\langle f, g \rangle$	scalar product $(fg)_\prec$ of $f$ and $g$ . . . . .	112



**Chapter 6. Grid-based operators**

$\Psi \circ (\Phi_1, \dots, \Phi_n)$	composition of multilinear grid-based operators . . . . .	117
$\text{supp } \Phi$	operator support of $\Phi$ . . . . .	117
$\mathcal{D}_{\mathbb{T}_{\leftarrow e^x}}, \mathcal{D}_{\mathbb{T}_{\rightleftarrows e^x}}$	sets of operators $\sum_{n \in \mathbb{N}} L_n \partial^n$ on $\mathbb{T}_{\leftarrow e^x}$ resp. $\mathbb{T}_{\rightleftarrows e^x}$ . .	118
$\Phi_i$	homogeneous part of degree $i$ of $\Phi$ . . . . .	122
$\Psi \circ \Phi$	composition of grid-based operators $\Psi$ and $\Phi$ . . . . .	124
$\mathcal{L}(M_1, \dots, M_m, N)$	set of strongly multilinear $\Phi: M_1 \times \dots \times M_m \rightarrow N$ . . . .	124
$\mathcal{G}(C[\mathbb{M}], C[\mathbb{N}])$	set of grid-based operators from $\Phi: C[\mathbb{M}] \rightarrow C[\mathbb{N}]$ . . . .	125
$\Omega_{m_1, \dots, m_m, n}$	atomic $m$ -linear grid-based operator . . . . .	125
$ \alpha $	arity of an atomic operator $\alpha$ . . . . .	126
$i_{\alpha, 1}, \dots, i_{\alpha,  \alpha }$	inputs of an atomic operator $\alpha$ . . . . .	126
$o_\alpha$	output of an atomic operator $\alpha$ . . . . .	126
$\mathcal{D}_{\mathbb{M}}$	atomic decomposition of $\text{Id}_{C[\mathbb{M}]}$ . . . . .	126
$\mathcal{A} \circ \mathcal{B}$	composition of atomic families $\mathcal{A}$ and $\mathcal{B}$ . . . . .	126

**Chapter 7. Linear differential equations**

$L_*, f_*$	alternative notation for dominant coefficient $c_L, c_f$ . . . .	136
$\text{supp}_{\text{ser}} L$	serial support of $L$ . . . . .	136
$L \times h$	multiplicative conjugate of $L$ by $h$ . . . . .	137
$L \uparrow, L \downarrow$	upward and downward shiftings of $L$ . . . . .	137
$s_{i,j}$	Stirling numbers of the first kind . . . . .	137
$S_{i,j}$	Stirling numbers of the second kind . . . . .	137
$\uparrow_l, \downarrow_l$	$l$ -th iterates of $\uparrow$ and $\downarrow$ . . . . .	138
$\mathcal{D}_{\mathbb{T}^b}$	space of operators of the form $\sum_{n \in \mathbb{N}} L_n \partial^n$ on $\mathbb{T}^b$ . . .	138
$U_i$	differential polynomial with $f^{(i)} = U_i(f^\dagger) f$ . . . . .	139
$R_L$	differential Riccati polynomial associated to $L$ . . . . .	139
$R_{L, \text{alg}}$	algebraic part of $R_L$ . . . . .	140
$P_{+\varphi}$	additive conjugate of $P$ by $\varphi$ . . . . .	140
$L'$	derivative of differential operator $L$ . . . . .	140
$\mathfrak{R}_L$	set of regular monomials for $L$ . . . . .	141
$\mathfrak{H}_L$	set of irregular monomials for $L$ . . . . .	141
$T_L$	trace of $L$ . . . . .	141
$T_{L; \mathfrak{B}}$	trace of $L$ w.r.t. $\mathfrak{B}$ . . . . .	141
$L^{-1}$	distinguished right inverse of $L$ . . . . .	146
$H_L$	set of solutions to $Lh = 0$ . . . . .	146
$\tilde{\mathbb{T}}$	complexification $\mathbb{T} \oplus i\mathbb{T}$ of $\mathbb{T}$ . . . . .	158
$\mathbb{O}$	field of oscillating transseries . . . . .	159
$f; \psi$	spectral part of $f$ associated to $e^{i\psi}$ . . . . .	159
$L; \psi$	action of $L$ on the spectral component $\tilde{\mathbb{T}} e^{i\psi}$ . . . . .	159

**Chapter 8. Algebraic differential equations**

$P_{\mathfrak{m}}$	coefficient of $\mathfrak{m}$ in differential polynomial $P$ . . . . .	166
$D_P$	dominant coefficient of $P$ . . . . .	167
$f^i$	shorthand for $f^{i_0} \dots (f^{(r)})^{i_r}$ . . . . .	167
$P_i$	coefficient of $f^i$ in $P$ . . . . .	167
$P_i$	$i$ -th homogeneous part of $P$ . . . . .	167
$\deg P$	(total) degree of $P$ . . . . .	167
$\text{val } P$	differential valuation of $P$ . . . . .	167
$f^{[\omega]}$	shorthand for $f^{(\omega_1)} \dots f^{(\omega_l)}$ . . . . .	168
$P_{[\omega]}$	(normalized) coefficient of $f^{[\omega]}$ in $P$ . . . . .	168
$P_{[\omega]}$	$\omega$ -th isobaric part of $P$ . . . . .	168
$\text{wt } P$	weight of $P$ . . . . .	168
$\text{wv } P$	weighted differential valuation of $P$ . . . . .	168
$f^{\langle i \rangle}$	$i$ -th iterated logarithmic derivative $f^{\dagger \dots \dagger}$ . . . . .	168
$f^{\langle i \rangle}$	shorthand for $f^{i_0} (f^{\dagger})^{i_1} \dots (f^{\langle r \rangle})^{i_r}$ . . . . .	168
$P_{\langle i \rangle}$	coefficient of $f^{\langle i \rangle}$ in $P$ . . . . .	168
$P_{+h}$	additive conjugation of $P$ with $h$ . . . . .	169
$P_{\times h}$	multiplicative conjugation of $P$ with $h$ . . . . .	170
$P \uparrow, P \downarrow$	upward and downward shiftings of $P$ . . . . .	170
$s_{\tau, \omega}$	generalized Stirling number of the first kind . . . . .	170
$S_{\tau, \omega}$	generalized Stirling number of the second kind . . . . .	171
$N_P$	differential Newton polynomial for $P$ . . . . .	173
$\deg_{\mathfrak{N}} P$	Newton degree of $P(f) = 0$ ( $f \in C[[\mathfrak{N}]]$ ) . . . . .	174
$\mathfrak{e}_{P, i, j}$	$(i, j)$ -equalizer for $P$ . . . . .	178
$R_{P_i}$	differential Riccati polynomial associated to $P_i$ . . . . .	179
$\mathfrak{E}$	set of exponential transmonomials . . . . .	184
$\mathfrak{L}$	set of logarithmic transmonomials . . . . .	197

**Chapter 9. The intermediate value theorem**

$M_x$	maximal open interval containing $x$ . . . . .	203
$(\leftarrow, z)$	the interval $\{u \in E: u < z\}$ . . . . .	203
$(z, \rightarrow)$	the interval $\{u \in E: u > z\}$ . . . . .	203
$\overline{E}$	compactification of a total ordering $E$ . . . . .	203
$\bar{f}$	a typical element of $\overline{E}$ . . . . .	203
$\perp_{\overline{E}}$	minimal element of $\overline{E}$ . . . . .	203
$\top_{\overline{E}}$	maximal element of $\overline{E}$ . . . . .	203
$\overline{\phantom{x}}$	shorthand for $\sup C$ . . . . .	206
$\mathbb{S}$	totally ordered grid-based algebra $C[[\mathfrak{M}]]$ . . . . .	208
$\overline{\phantom{x}}$	shorthand for $\inf C^>$ . . . . .	208
$\overline{\phantom{x}}$	shorthand for $\sup \mathbb{S}$ . . . . .	208
$\overline{C}^\#$	set $\overline{C} \setminus (C \cup \{-\overline{\phantom{x}}, \overline{\phantom{x}}\})$ of finite cuts in $C$ . . . . .	208

$\overline{\mathfrak{M}}$	set of monomials and monomial cuts . . . . .	208
$\overline{\mathfrak{d}}_{\overline{f}}$	dominant monomial of $\overline{f}$ . . . . .	209
$\overline{\mathfrak{w}}_{\overline{f}}$	width of $\overline{f}$ . . . . .	209
$\varphi_{\overline{f}}$	initializer of $\overline{f}$ . . . . .	210
$\hat{\$}$	serial completion of $\$$ . . . . .	210
$\hat{f}$	typical serial cut . . . . .	210
$\overline{\mathfrak{x}}_k$	shorthand for $\sup \{f \in \mathbb{T}; \exp(f) = k\}$ . . . . .	212
$\overline{\gamma}$	the cut $(x \log x \log_2 x \cdots)^{-1}$ . . . . .	219
$\mathcal{L}^{\varphi_0, \dots, \varphi_{k-1}, g, h}$	basic integral neighbourhood . . . . .	220
$f_0, \dots, f_k$	integral coordinates of $f$ . . . . .	221
$\sigma_P$	sign of $P$ . . . . .	227
$\sigma_P^+$	sign of $P$ at the right of a point . . . . .	227
$\sigma_P^-$	sign of $P$ at the left of a point . . . . .	227
$\sigma_{P, \mathfrak{W}}$	sign of $P$ modulo $\mathfrak{W}$ . . . . .	228
$\sigma_{P, \mathfrak{W}}^+$	sign of $P$ at the right of a point modulo $\mathfrak{W}$ . . . . .	228
$\sigma_{P, \mathfrak{W}}^-$	sign of $P$ at the left of a point modulo $\mathfrak{W}$ . . . . .	228

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