Appendix A

A.1 Brunn–Minkowski Inequality

The most important integral inequality in convexity is Hölder's inequality which states that for any two non-negative, measurable functions $f, g : \mathbb{R}^n \to \mathbb{R}_+$ and $0 \leq \lambda \leq 1$ we have

$$\int_{\mathbb{R}^n} f(x)^{1-\lambda} g(x)^{\lambda} dx \leq \left(\int_{\mathbb{R}^n} f(x) dx \right)^{1-\lambda} \left(\int_{\mathbb{R}^n} g(x) dx \right)^{\lambda}.$$

In the case that $f = \chi_A$ and $g = \chi_B$ are the characteristic functions of A, B, measurable sets in $\mathbb{R}^n$, Hölder's inequality reads

$$|A \cap B|_n \leq |A|_n^{1-\lambda} |B|_n^{\lambda} \qquad \forall\, 0 \leq \lambda \leq 1,$$

which is equivalent to

$$|A \cap B|_n \leq \min\{|A|_n, |B|_n\}.$$

It is clear that these inequalities cannot be reversed at all. However, we can give a kind of reverse inequality, due to Prékopa and Leindler, using one type of convex convolution of functions.

Given two non-negative measurable functions f, g and $0 \leq \lambda \leq 1$ defined on $\mathbb{R}^n$, we consider the function $f^{1-\lambda} *_{\sup} g^{\lambda}$ defined by

$$f^{1-\lambda} *_{\sup} g^{\lambda}(z) := \sup_{z=(1-\lambda)x+\lambda y} f(x)^{1-\lambda} g(y)^{\lambda}.$$

D. Alonso-Gutiérrez, J. Bastero, *Approaching the Kannan-Lovász-Simonovits and Variance Conjectures*, Lecture Notes in Mathematics 2131,
DOI 10.1007/978-3-319-13263-1

This function is not necessarily measurable, but we can consider its exterior Lebesgue integral

$$\int_{\mathrm{R}^n}^{*} f^{1-\lambda} *_{\sup} g^{\lambda}(z)dz = \inf\left\{ \int_{\mathrm{R}^n} h(z)dz : f^{1-\lambda} *_{\sup} g^{\lambda}(z) \leq h(z) \right\}.$$

Then we have

Theorem A.1 *Let f, g be two non-negative measurable functions defined on* R^n *and* $0 \leq \lambda \leq 1$. *Then*

$$\left(\int_{\mathrm{R}^n} f(x)dx \right)^{1-\lambda} \left(\int_{\mathrm{R}^n} g(x)dx \right)^{\lambda} \leq \int_{\mathrm{R}^n}^{*} f^{1-\lambda} *_{sup} g^{\lambda}(x)dx.$$

This result is a consequence of the following inequality:

Theorem A.2 (Prékopa–Leindler's Inequality) *Let f, g, h be three non-negative measurable functions defined on* R^n *and* $0 \leq \lambda \leq 1$ *such that*

$$f(x)^{1-\lambda} g(y)^{\lambda} \leq h(z) \qquad \textit{whenever} \qquad z = (1-\lambda)x + \lambda y.$$

Then

$$\left(\int_{\mathrm{R}^n} f(x)dx \right)^{1-\lambda} \left(\int_{\mathrm{R}^n} g(y)dy \right)^{\lambda} \leq \int_{\mathrm{R}^n} h(z)dz.$$

Proof We present the approach learned from K. Ball (see [3] and [9]). We can assume, by homogeneity, that $\|f\|_\infty = \|g\|_\infty = 1$. In dimension $n = 1$, let A, B be two non-empty compact sets in R. It is clear that

$$A + B \supseteq (\min A + B) \cup (A + \max B)$$

and

$$(\min A + B) \cap (A + \max B) = \min A + \max B,$$

which has volume 0, so we have

$$|A + B|_1 \geq |A|_1 + |B|_1$$

for compact sets in R and by an approximation procedure, for any couple of Borel sets in R. Then for any $0 \leq t < 1$, since

$$\{x \in \mathrm{R} : h(x) \geq t\} \supseteq (1-\lambda)\{x \in \mathrm{R} : f(x) \geq t\} + \lambda\{x \in \mathrm{R} : g(x) \geq t\},$$

we have

$$\begin{aligned}\int_{\mathrm{R}} h(x)dx &\geq \int_0^1 |\{x \in \mathrm{R} \,:\, h(x) \geq t\}|dt \\ &\geq (1-\lambda)\int_0^1 |\{x \in \mathrm{R} \,:\, f(x) \geq t\}|dt + \lambda \int_0^1 |\{x \in \mathrm{R} \,:\, g(x) \geq t\}|dt \\ &\geq \text{ (by the arithmetic-geometric mean inequality)} \\ &\geq \left(\int_{\mathrm{R}} f(x)dx\right)^{1-\lambda} \left(\int_{\mathrm{R}} g(x)dx\right)^{\lambda}.\end{aligned}$$

The case $n > 1$ is deduced by induction. Fix $x_1 \in \mathrm{R}$, define $f_{x_1} : \mathrm{R}^{n-1} \to [0, \infty)$ by $f_{x_1}(x_2, \ldots, x_n) = f(x_1, \ldots, x_n)$. By assumption we have

$$h_{z_1}((1-\lambda)(x_2, \ldots, x_n) + \lambda(y_2, \ldots, y_n)) \geq f_{x_1}(x_2, \ldots, x_n)^{1-\lambda} g_{y_1}(y_2, \ldots, y_n)^{\lambda}$$

for any $(x_2, \ldots, x_n), (y_2, \ldots, y_n) \in \mathrm{R}^{n-1}$, whenever $z_1 = (1-\lambda)x_1 + \lambda y_1$. By the induction hypothesis,

$$\int_{\mathrm{R}^{n-1}} h_{z_1}(\overline{z})d\,\overline{z} \geq \left(\int_{\mathrm{R}^{n-1}} f_{x_1}(\overline{x})d\,\overline{x}\right)^{1-\lambda} \left(\int_{\mathrm{R}^{n-1}} g_{y_1}(\overline{y})d\,\overline{y}\right)^{\lambda}.$$

Applying again the inequality for $n = 1$ and Fubini's theorem we obtain the result.

If we apply this inequality to $f = \chi_A$ and $g = \chi_B$, characteristic functions of A and B Borel sets in R^n, we obtain

Theorem A.3 (Brunn–Minkowski Inequality) *Let A, B two Borel sets in R^n. For any $0 \leq \lambda \leq 1$*

$$|A|^{1-\lambda}|B|^{\lambda} \leq |(1-\lambda)A + \lambda B|, \tag{A.1}$$

or, equivalently,

$$|A|^{\frac{1}{n}} + |B|^{\frac{1}{n}} \leq |A + B|^{\frac{1}{n}}, \tag{A.2}$$

whenever $A \neq \emptyset$ and $B \neq \emptyset$.

Inequality (A.1) is a dimension-free version of Brunn–Minkowski inequality (note that in this case the set $(1-\lambda)A + \lambda B$ is measurable).

Inequalities (A.1) and (A.2) are equivalent. By taking $A' = \dfrac{A}{|A|^{\frac{1}{n}}}$, $B' = \dfrac{B}{|B|^{\frac{1}{n}}}$ and $\lambda = \dfrac{|B|^{\frac{1}{n}}}{|A|^{\frac{1}{n}} + |B|^{\frac{1}{n}}}$ in (A.1), we get (A.2).

The reverse implication is easy to obtain, given any $0 \le \lambda \le 1$,

$$\begin{aligned}|(1-\lambda)A + \lambda B|^{\frac{1}{n}} &\ge (1-\lambda)|A|^{\frac{1}{n}} + \lambda|B|^{\frac{1}{n}} \\ &\ge \text{ (by the arithmetic-geometric mean inequality)} \\ &\ge |A|^{\frac{1-\lambda}{n}}|B|^{\frac{\lambda}{n}}.\end{aligned}$$

A.2 Consequences of Brunn–Minkowski Inequality

Proposition A.1 *Any log-concave probability μ on $\mathbb{R}^n$ satisfies Brunn–Minkowski inequality, i.e.,*

$$\mu((1-\lambda)A + \lambda B) \ge \mu(A)^{1-\lambda}\mu(B)^{\lambda}$$

for any $A, B \subseteq \mathbb{R}^n$ non-empty Borel sets and any $0 \le \lambda \le 1$.

Proof We have that $d\mu(x) = \mathrm{e}^{-V(x)}dx$ where $V : \mathbb{R}^n \to (-\infty, \infty]$ is a convex function. We take $f(x) = \chi_A(x)\mathrm{e}^{-V(x)}$, $g(y) = \chi_B(y)\mathrm{e}^{-V(y)}$ and

$$h(z) = \chi_{(1-\lambda)A+\lambda B}(z)\mathrm{e}^{-V(z)}.$$

Then we apply Prékopa–Leindler's inequality.

Proposition A.2 (Isoperimetric Inequality in $\mathbb{R}^n$) *Let A be a bounded Borel set in $\mathbb{R}^n$ then*

$$\frac{|\partial A|_{n-1}^{\frac{1}{n-1}}}{|A|_n^{\frac{1}{n}}} \ge \frac{|S^{n-1}|_{n-1}^{\frac{1}{n-1}}}{|B_2^n|_n^{\frac{1}{n}}} = n^{\frac{1}{n-1}}|B_2^n|^{\frac{1}{n(n-1)}},$$

where

$$|\partial A|_{n-1} = \liminf_{t\to 0}\frac{|A^t|_n - |A|_n}{t}$$

and $A^t = \{x \in \mathbb{R}^n; d(x, A) \le t\} = A + tB_2^n$.

Proof It is clear that

$$\begin{aligned}|A^t|_n - |A|_n = |A + tB_2^n|_n - |A|_n &\ge \left(|A|_n^{\frac{1}{n}} + t|B_2^n|^{\frac{1}{n}}\right)^n - |A|_n \\ &\ge nt|A|_n^{\frac{n-1}{n}}|B_2^n|^{\frac{1}{n}}.\end{aligned}$$

Hence,

$$|\partial A|_{n-1} \geq n|A|_n^{\frac{n-1}{n}} |B_2^n|^{\frac{1}{n}}$$

and the result follows.

Proposition A.3 (Isoperimetric Inequality in S^{n-1} by P. Levy) *Let A be any Borel set in $A \subseteq S^{n-1}$ such that $\sigma(A) \geq 1/2$, where σ is the uniform probability on S^{n-1}. Then*

$$\sigma(A^\varepsilon) \geq 1 - 2\mathrm{e}^{-C\varepsilon^2 n} \qquad \textit{for any} \qquad 0 < \varepsilon < 1,$$

where $C > 0$ is an absolute constant. (Caps are the minimizers for measuring the boundary).

We show a proof given by Arias de Reyna et al. [2]. The original proof gives better constants:

$$\sigma(A^\varepsilon) \geq 1 - \sqrt{\frac{\pi}{8}}\mathrm{e}^{-\frac{\varepsilon^2 n}{2}}.$$

We begin showing the following lemma:

Lemma A.1 *Let μ be the uniform probability on B_2^n, i.e, $\mu(A) = \dfrac{|A \cap B_2^n|}{|B_2^n|}$. Let A, B be Borel sets in B_2^n. Then*

$$\min\{\mu(A), \mu(B)\} \leq \mathrm{e}^{-\dfrac{d(A,B)^2 n}{8}}.$$

Proof Assume that A, B are closed in B_2^n and let $\alpha = \min\{\mu(A), \mu(B)\}$, $\rho = d(A, B)$. By Brunn–Minkowski inequality

$$\left|\frac{1}{2}A + \frac{1}{2}B\right|^{\frac{1}{n}} \geq \frac{1}{2}|A|^{\frac{1}{n}} + \frac{1}{2}|B|^{\frac{1}{n}} \Longrightarrow \mu\left(\frac{A+B}{2}\right) \geq \alpha.$$

If $a \in A, b \in B, \Longrightarrow |a+b|^2 = 2|a|^2 + 2|b|^2 - |a-b|^2 \leq 4 - \rho^2$ and then

$$\frac{A+B}{2} \subseteq \sqrt{1 - \frac{\rho^2}{4}} B_2^n.$$

Thus,

$$\alpha \leq \mu\left(\frac{A+B}{2}\right) = \left(1 - \frac{\rho^2}{4}\right)^{\frac{n}{2}} \leq \mathrm{e}^{-\frac{\rho^2 n}{8}}.$$

Proof (of Proposition A.3)

Let $A \subseteq S^{n-1}$ with $\sigma(A) \geq 1/2$. Given $\varepsilon > 0$ denote $B = (A^\varepsilon)^c$. Fix $0 < \lambda < 1$. Define $\bar{A} = \{ta \in \mathbb{R}^n \ : \ \lambda \leq t \leq 1, a \in A\}$ and $\bar{B} = \{sb \in \mathbb{R}^n \ : \ \lambda \leq s \leq 1, b \in B\} \subseteq B_2^n$.

If $\bar{a} = ta \in \bar{A}, \bar{b} = sb \in \bar{B}$, we have

$$|\bar{a} - \bar{b}| = |ta - sb| \geq |\lambda a - \lambda b| = \lambda |a - b| \geq \lambda \varepsilon.$$

Besides, notice that

$$\mu(\bar{A}) = \frac{1}{|B_2^n|} \int_\lambda^1 r^{n-1} dr \sigma(A) = (1 - \lambda^n)\mu(A)$$

and, in the same way, $\mu(\bar{B}) = (1 - \lambda^n)\mu(B)$.

Since $\sigma(A) \geq 1/2$ we have that $\sigma(B) \leq 1/2$ and then, by the previous lemma,

$$(1 - \lambda^n)\mu(B) = \alpha \leq \mathrm{e}^{-\dfrac{n\lambda^2\varepsilon^2}{8}}.$$

Hence, taking $\lambda = 2^{-\frac{1}{n}}$, we obtain

$$\sigma((A^\varepsilon)^c) \leq 2\mathrm{e}^{-C\varepsilon^2 n}$$

and the result follows.

A.3 Borell's Inequality and Concentration of Mass

The following inequality, proved by C. Borell [4], is verified by every log-concave probability measure. As a consequence, we show the exponential decay of the distribution function and the equivalence of all the p-th moments of the Euclidean norm.

Proposition A.4 *Let μ be a log-concave probability on $\mathbb{R}^n$. Let $\frac{1}{2} \leq \theta < 1$. Then, for every symmetric convex set $A \subseteq \mathbb{R}^n$ with $\mu(A) \geq \theta$, we have that*

$$\mu((tA)^c) \leq \theta \left(\frac{1-\theta}{\theta}\right)^{\frac{1+t}{2}}$$

for every $t > 1$.

Proof It is clear that

$$A^c \supseteq \frac{2}{t+1}(tA)^c + \frac{t-1}{t+1}A.$$

Then, by Brunn–Minkowski inequality,

$$1-\theta \geq \mu(A^c) \geq \mu((tA)^c)^{\frac{2}{t+1}}\mu(A)^{\frac{t-1}{t+1}},$$

which implies the result.

Proposition A.5 (Reverse Hölder's Inequality and Exponential Decay) *There exist absolute constants* $C_1, C_2 = 2C_1\mathrm{e} > 0$ *such that for any log-concave probability on* R^n *and for any semi-norm* $f : \mathrm{R}^n \to [0,\infty)$ *we have*

(i) $\left(\mathrm{E}_\mu f^p\right)^{\frac{1}{p}} \leq C_1 p\, \mathrm{E}_\mu f, \qquad \forall\, p > 1,$

(ii) $\mathrm{E}_\mu \mathrm{e}^{\frac{f}{C_2 \mathrm{E}_\mu f}} \leq 2$, *and*

(iii) $\mu\left\{x \in \mathrm{R}^n : f(x) \geq C_2 t \mathrm{E}_\mu f\right\} \leq 2\mathrm{e}^{-t}, \qquad \forall\, t > 0.$

Proof (i) Since any semi-norm is integrable we can assume $\mathrm{E}_\mu f = 1$. Let A be the set $A = \{x \in \mathrm{R}^n \;:\; f(x) < 3\}$. By Markov's inequality $\mu(A) \geq \frac{2}{3}$. Then,

$$\mu\{x \in \mathrm{R}^n : f(x) \geq 3t\} = \mu((tA)^c) \leq \frac{2}{3}\left(\frac{1}{2}\right)^{\frac{1+t}{2}} \leq 2^{-\frac{t}{2}} \leq \mathrm{e}^{-t\frac{\log 2}{2}}$$

whenever $t > 1$. Let $p > 1$. Then

$$\begin{aligned}
\mathrm{E}_\mu f^p &= \int_0^3 p\, t^{p-1}\mu\{x \in \mathrm{R}^n \;:\; f(x) > t\}dt \\
&\quad + \int_3^\infty p\, t^{p-1}\mu\{x \in \mathrm{R}^n \;:\; f(x) > t\}dt \\
&\leq 3^p + 3^p\int_1^\infty p\, s^{p-1}\mathrm{e}^{-2s}ds \leq (C_1 p)^p
\end{aligned}$$

for some absolute constant $C_1 > 0$ and *i)* follows.

(ii) Assume again that $\mathrm{E}_\mu f = 1$. Let $A > 0$ to be fixed later.

$$\begin{aligned}
\mathrm{E}_\mu \mathrm{e}^{\frac{f}{A}} &= 1 + \sum_{p=1}^{\infty}\frac{1}{p!}\mathrm{E}_\mu\left(\frac{f}{A}\right)^p \\
&\leq 1 + \sum_{p=1}^{\infty}\frac{1}{p!}\frac{C_1^p p^p}{A^p} \leq 1 + \sum_{p=1}^{\infty}\left(\frac{C_1 \mathrm{e}}{A}\right)^p.
\end{aligned}$$

The result follows choosing $A = 2C_1\mathrm{e}$.

(iii) By Markov's inequality, we have

$$\mu\{x \in \mathrm{R}^n : f(x) > t\mathrm{E}_\mu f\} = \mu\left\{x \in \mathrm{R}^n : \frac{f(x)}{C_2 \mathrm{E}_\mu f} > \frac{t}{C_2}\right\}$$
$$\leq \mathrm{e}^{-t/C_2} \mathrm{E}_\mu \mathrm{e}^{\frac{f}{C_2 \mathrm{E}_\mu f}} 2\mathrm{e}^{-\frac{t}{C_2}}$$

and (iii) follows.

Remark A.1 In the case that $f(x) = |x|$, by repeating the arguments and taking $A = \{x : |x| \leq \mathrm{e}^3 \left(\mathrm{E}_\mu |x|^p\right)^{\frac{1}{p}}\}$ in the proof before, $p \geq 2$, we obtain

$$\mu\left\{x \in \mathrm{R}^n : |x| \geq t\mathrm{e}^3 \left(\mathrm{E}_\mu |x|^p\right)^{\frac{1}{p}}\right\} \leq (1 - \mathrm{e}^{-3p}) \left(\frac{\mathrm{e}^{-3p}}{1 - e^{-3p}}\right)^{\frac{1+t}{2}} \leq \mathrm{e}^{-3pt}$$

for every $t \geq 1$.

Remark A.2 Alesker (see [1]) proved that for isotropic convex bodies,

$$|\{x \in K; |x| > ct\mathrm{E}|x|\}|_n \leq 2\mathrm{e}^{-t^2} \qquad \forall t > 0$$

for some absolute $c > 0$.

Remark A.3 Paouris [7] proved the following strong inequality:

There exists an absolute constant $C > 0$ such that for every log-concave probability μ in R^n we have

$$\left(\mathrm{E}_\mu |x|^p\right)^{\frac{1}{p}} \leq C \max\left\{\left(\mathrm{E}_\mu |x|^2\right)^{\frac{1}{2}}, \sup_{\theta \in S^{n-1}} \left(\mathrm{E}_\mu |\langle x, \theta\rangle|^p\right)^{\frac{1}{p}}\right\}$$

for any $p \geq 1$. Using Borell's inequality we have

$$\left(\mathrm{E}_\mu |\langle x, \theta\rangle|^p\right)^{\frac{1}{p}} \leq C_1 p \left(\mathrm{E}_\mu |\langle x, \theta\rangle|^2\right)^{\frac{1}{2}} \leq C_1 p \lambda_\mu$$

for some absolute constant $C_1 > 0$ and for any $\theta \in S^{n-1}$. Hence

$$\left(\mathrm{E}_\mu |x|^p\right)^{\frac{1}{p}} \leq C \max\left\{\left(\mathrm{E}_\mu |x|^2\right)^{\frac{1}{2}}, p\lambda_\mu\right\}$$

for any $p \geq 1$. If we take $p = \frac{t\left(\mathrm{E}_\mu|x|^2\right)^{\frac{1}{2}}}{\lambda_\mu}$, with $t \geq \max\left\{1, \frac{\lambda_\mu}{\left(\mathrm{E}_\mu|x|^2\right)^{\frac{1}{2}}}\right\}$ we obtain, by (i) in the latter proposition,

$$\mu\left\{|x| > Ct\mathrm{e}^3\left(\mathrm{E}_\mu|x|^2\right)^{\frac{1}{2}}\right\} \leq \mu\left\{|x| > \mathrm{e}^3\left(\mathrm{E}_\mu|x|^p\right)^{\frac{1}{p}}\right\}$$

$$\leq \mathrm{e}^{-3p} = \mathrm{e}^{-3\frac{t\left(\mathrm{E}_\mu|x|^2\right)^{\frac{1}{2}}}{\lambda_\mu}}.$$

In particular when μ is isotropic and $t \geq 1$

$$\mu\{x \in \mathrm{R}^n : |x| \geq C_1 t\sqrt{n}\} \leq \mathrm{e}^{-C_2 t\sqrt{n}}.$$

This inequality can also be expressed in the following way:

$$\mu\{x \in \mathrm{R}^n : |x| \geq (1+s)\sqrt{n}\} \leq \mathrm{e}^{-C_3 s\sqrt{n}}$$

for some $s \geq s_0$. Indeed, we take $t = \frac{1+s}{C_1} \geq 1$ (so, $s \geq C_1 - 1$) and $C_2 t \geq C_3 s$.

This inequality had been previously obtained by Bobkov and Nazarov in the case of isotropic unconditional log-concave probabilities.

Remark A.4 R. Latała and O. Guédon (see [6] and [5]) extended previous results for the range $-1 < p \leq 1$, since they proved the following "small-ball probability" result:

Proposition A.6 *There exists an absolute constant* $C > 0$ *such that for any norm* $f : \mathrm{R}^n \to [0, \infty)$ *and for any log-concave probability on* R^n *we have*

(i)

$$\mu\{x \in \mathrm{R}^n; f(x) \leq t\mathrm{E}_\mu f\} \leq Ct \qquad \forall t > 0,$$

(ii) For any $-1 < p \leq 1$ *we have*

$$\left(\mathrm{E}_\mu f^p\right)^{\frac{1}{p}} \leq \mathrm{E}_\mu f \leq \frac{C}{p+1}\left(\mathrm{E}_\mu f^p\right)^{\frac{1}{p}}.$$

Proof (Only of (ii))

We can assume $-1 < p < 0$. Let $q = -p \in (0,1)$. Then

$$\mathrm{E}_\mu f^p = \int_0^{\frac{1}{\mathrm{E}_\mu f}} qt^{q-1}\mu\left\{x \in \mathrm{R}^n : \frac{1}{f(x)} > t\right\} dt$$
$$+ \int_{\frac{1}{\mathrm{E}_\mu f}}^{\infty} qt^{q-1}\mu\left\{x \in \mathrm{R}^n : \frac{1}{f(x)} > t\right\} dt$$

$$\leq \left(\mathrm{E}_\mu f\right)^p + \int_0^1 \left(\mathrm{E}_\mu f\right)^p q\mu\left\{x \in \mathrm{R}^n \,:\, f(x) < s\mathrm{E}_\mu f\right\} \frac{ds}{s^{q+1}}$$

$$\leq \left(\mathrm{E}_\mu f\right)^p \left(1 + Cq\int_0^1 \frac{ds}{s^q}\right) \leq \left(\mathrm{E}_\mu f\right)^p \frac{1+Cq}{1-q} \leq \frac{\mathrm{e}^{-Cp}}{1+p}\left(\mathrm{E}_\mu f\right)^p.$$

Thus,

$$(\mathrm{E}f^p)^{\frac{1}{p}} \geq \left(\frac{1+p}{\mathrm{e}^{-Cp}}\right)^{-\frac{1}{p}} \mathrm{E}f \geq C_1(1+p)\mathrm{E}f.$$

Remark A.5 Paouris [8] extended this result showing that for any $1 \leq q \leq C_3\sqrt{n}$ we have

$$\left(\mathrm{E}_\mu |x|^{-q}\right)^{-\frac{1}{q}} \sim \left(\mathrm{E}_\mu |x|^q\right)^{\frac{1}{q}}$$

for some absolute constant $0 < C_3 < 1$. Furthermore a small-ball probability estimate can be deduced for every isotropic measure since, in that case, one has

$$\mu\{x \in \mathrm{R}^n \,:\, |x| \leq \varepsilon\sqrt{n}\} \leq \varepsilon^{c_4\sqrt{n}}$$

for any $0 < \varepsilon < \varepsilon_0$, where ε_0, c_4 are absolute constants.

References

1. S. Alesker, ψ_2-Estimate for the Euclidean norm on a convex body in isotropic position, in *Geometric Aspects of Functional Analysis*, ed. by J. Lindenstrauss, V.D. Milman. Operator Theory: Advances and Applications, vol. 77 Birkhäuser, Basel (1995), pp. 1–4
2. J. Arias-de-Reyna, K. Ball, R. Villla, Concentration of the distance in finite-dimensional normed spaces. Mathematika **45**(2), 245–252 (1998)
3. K. Ball, Logarithmic concave functions and sections of convex bodies in R^n. Stud. Math. **88**, 69–84 (1988)
4. C. Borell, Convex measures on locally convex spaces. Ark. Math. **12**, 239–252 (1974)
5. O. Guédon, Kahane-Khinchine type inequalities for negative exponent. Mathematika **46**, 165–173 (1999)
6. R. Latała, On the equivalence between geometric and arithmetic means for log-concave measures, in *Convex Geometric Analysis* (Berkeley, CA, 1996), Mathematical Sciences Research Institute Publications, vol. 34 (Cambridge University Press, Cambridge, 1999), pp. 123–127
7. G. Paouris, Concentration of mass on convex bodies. Geom. Funct. Anal. (GAFA) **16**, 1021–1049 (2006)
8. G. Paouris, Small ball probability estimates for log-concave measures. Trans. Am. Math. Soc. **364**(1), 287–308 (2012)
9. G. Pisier, *The Volume of Convex Bodies and Banach Space Geometry*, vol 94 (Cambridge University Press, Cambridge, 1989)

Index

D. Alonso-Gutiérrez, J. Bastero, *Approaching the Kannan-Lovász-Simonovits and Variance Conjectures*, Lecture Notes in Mathematics 2131,
DOI 10.1007/978-3-319-13263-1

LECTURE NOTES IN MATHEMATICS

Springer

Edited by J.-M. Morel, B. Teissier; P.K. Maini

Editorial Policy (for the publication of monographs)

1. Lecture Notes aim to report new developments in all areas of mathematics and their applications - quickly, informally and at a high level. Mathematical texts analysing new developments in modelling and numerical simulation are welcome.
   Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They may be based on specialised lecture courses. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character. For similar reasons it is unusual for doctoral theses to be accepted for the Lecture Notes series, though habilitation theses may be appropriate.

2. Manuscripts should be submitted either online at www.editorialmanager.com/lnm to Springer's mathematics editorial in Heidelberg, or to one of the series editors. In general, manuscripts will be sent out to 2 external referees for evaluation. If a decision cannot yet be reached on the basis of the first 2 reports, further referees may be contacted: The author will be informed of this. A final decision to publish can be made only on the basis of the complete manuscript, however a refereeing process leading to a preliminary decision can be based on a pre-final or incomplete manuscript. The strict minimum amount of material that will be considered should include a detailed outline describing the planned contents of each chapter, a bibliography and several sample chapters.
   Authors should be aware that incomplete or insufficiently close to final manuscripts almost always result in longer refereeing times and nevertheless unclear referees' recommendations, making further refereeing of a final draft necessary.
   Authors should also be aware that parallel submission of their manuscript to another publisher while under consideration for LNM will in general lead to immediate rejection.

3. Manuscripts should in general be submitted in English. Final manuscripts should contain at least 100 pages of mathematical text and should always include
   - a table of contents;
   - an informative introduction, with adequate motivation and perhaps some historical remarks: it should be accessible to a reader not intimately familiar with the topic treated;
   - a subject index: as a rule this is genuinely helpful for the reader.

   For evaluation purposes, manuscripts may be submitted in print or electronic form (print form is still preferred by most referees), in the latter case preferably as pdf- or zipped ps-files. Lecture Notes volumes are, as a rule, printed digitally from the authors' files. To ensure best results, authors are asked to use the LaTeX2e style files available from Springer's web-server at:

   ftp://ftp.springer.de/pub/tex/latex/svmonot1/ (for monographs) and
   ftp://ftp.springer.de/pub/tex/latex/svmultt1/ (for summer schools/tutorials).

Additional technical instructions, if necessary, are available on request from lnm@springer.com.

4. Careful preparation of the manuscripts will help keep production time short besides ensuring satisfactory appearance of the finished book in print and online. After acceptance of the manuscript authors will be asked to prepare the final LaTeX source files and also the corresponding dvi-, pdf- or zipped ps-file. The LaTeX source files are essential for producing the full-text online version of the book (see http://www.springerlink.com/openurl.asp?genre=journal&issn=0075-8434 for the existing online volumes of LNM). The actual production of a Lecture Notes volume takes approximately 12 weeks.

5. Authors receive a total of 50 free copies of their volume, but no royalties. They are entitled to a discount of 33.3 % on the price of Springer books purchased for their personal use, if ordering directly from Springer.

6. Commitment to publish is made by letter of intent rather than by signing a formal contract. Springer-Verlag secures the copyright for each volume. Authors are free to reuse material contained in their LNM volumes in later publications: a brief written (or e-mail) request for formal permission is sufficient.

Addresses:

Professor J.-M. Morel, CMLA,
École Normale Supérieure de Cachan,
61 Avenue du Président Wilson, 94235 Cachan Cedex, France
E-mail: morel@cmla.ens-cachan.fr

Professor B. Teissier, Institut Mathématique de Jussieu,
UMR 7586 du CNRS, Équipe “Géométrie et Dynamique”,
175 rue du Chevaleret
75013 Paris, France
E-mail: teissier@math.jussieu.fr

For the “Mathematical Biosciences Subseries” of LNM:

Professor P. K. Maini, Center for Mathematical Biology,
Mathematical Institute, 24-29 St Giles,
Oxford OX1 3LP, UK
E-mail: maini@maths.ox.ac.uk

Springer, Mathematics Editorial, Tiergartenstr. 17,
69121 Heidelberg, Germany,
Tel.: +49 (6221) 4876-8259

Fax: +49 (6221) 4876-8259
E-mail: lnm@springer.com