Conclusion and Open Problems

A class of second order evolution equations which serves as model for linear control problems arising from elasticity are considered. In this book we have characterized a class of unbounded feedback operators for which an affirmative answer to the question of exact controllability by collocated actuators and sensors implies uniform stability. This is the converse of the well known result of Russell which states that uniform stabilization implies exact controllability for linear, time-reversible systems. This is achieved by combining an observability estimate for the undamped evolution equation with the boundedness of a transfer function and leads to different kinds of stability results. The second part of this book focuses on the problem of feedback stabilization for a class of evolution equations with delay. The same method, as the first part, was used to investigate the stabilization problem of the delay system. Again the stability of the closed-loop system is reduced to the existence of an observability estimate for the corresponding uncontrolled system combined with a boundedness property of the transfer function of the associated open-loop system. We finally have given numerous concrete examples of systems (with or without delay) that enter into our abstract framework.

Possible future developments in connection with problems addressed in this book can be summarized as follows:

1. Stabilization by nonlinear feedbacks

   The stabilization for some nonlinear infinite dimensional systems is of great interest. In [8, 9], the authors show that if the linear system is observable through a locally distributed observation, then any dissipative nonlinear bounded and locally distributed feedback stabilizes the system, a general simply computable energy decay formula is further given. In this way, the authors show that for the locally distributed case, one can combine the optimal geometric optics conditions of Bardos-Lebeau-Rauch and the optimal-weight convexity method of Alabau-Boussouira [6, 7] based on nonlinear Gronwall inequalities with optimal weight to deduce sharp easily computable energy decay rates for nonlinear damped systems. Using recent results of Alabau-Boussouira [7], a very simple upper

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estimate is given for feedbacks with general growth close to the origin (not close to a linear behavior) and linear at infinity. Optimality of these estimates has been proved in the finite dimensional case in [7] and in certain infinite dimensional situations [6] using optimality results by Vancostenoble and Martinez [128] (see also [127]). The case of unbounded nonlinear feedbacks merits to be studied. Similarly the question of nonlinear feedback stability via weak observability estimates for the linear uncontrolled system is largely open.

2. Stabilization of C_0-semigroup operators by observability strategy.

   Consider the same problem as (2.1) where we do not suppose that the generator of the free dynamic, i.e., the operator A, is skew adjoint. The main question is to give the same characterization of the uniform or non uniform stability by observability, as in Theorems 2.2.3 and 2.2.5.

3. Determination of the best decay rate by observability techniques

   The fastest decay rate can be estimated by the observability (or controllability) cost and open-loop admissibility cost. A first development in this direction can be found in [20] where the authors propose a numerical strategy to give a good estimation for the best decay rate and optimal location of the actuator for a large class of evolution systems.

4. Inverse problems via observability strategy

   In many applications the control do not affect the complete state of the dynamical system but only a part of it or the observation is only performed on a part of the complete state of the dynamical system. So in [13], by an observability strategy (in the sense that it is possible to recover the initial state of the dynamical system from knowledge of the input and output), some inverse problems, like the determination of both the potential and the damping coefficient in a dissipative wave equation from boundary measurements, can be tackled. They can establish some stability estimates of logarithmic type when the measurements are given by the operator who maps the initial conditions to the Neumann boundary trace of the solution of the corresponding initial-boundary value problem. The method combines an observability inequality together with a spectral decomposition.

5. Stabilization by indirect damping

   The notion of indirect damping mechanisms has been introduced by Russell in [119] and since that time it retains the attention of many authors, because several models from acoustic theory enter in this framework, see [2–4,32,33,94,103,104] and the references therein for more details. Recently in [1] second order evolution equations with unbounded dynamic feedbacks (in particular coupled systems or hybrid systems) are considered. Under a regularity assumption the authors show that observability properties for the undamped problem imply decay estimates for the damped problem and obtain both uniform and non uniform decay properties, see [1] for more details.

6. Stabilization by switching time delay

   Delay effects arise in many applications and practical problems and as we already explained before an arbitrarily small delay may destabilize a system which is uniformly asymptotically stable in absence of delay. In this book we have shown that if the delay term is small enough, then the new system keeps

the same properties than without delay. Since this condition is quite strong, other approaches can be proposed. In [21], the strategy consists in stabilizing the wave system by a control law that uses information from the past (by switching or not). This means that the stabilization is obtained by a control method (called switching control method by time-delay) and not by a feedback law. This strategy can provide a guide to the time-delay compensation scheme, known as Smith predictor control, which uses feedback loop for controlling any system, where the Smith predictor control is devised to remove the delay effect from the closed-loop design.

7. Stabilization of elastic multistructures by observability

   The field of partial differential equations set on multistructures is extremely rich and provides a large number of interesting problems of quite complex mathematical nature. We have seen that the methodology presented in this book allows to prove the stabilization of some evolution problems on one-dimensional networks that are very special elastic multistructures. But many questions remain open, for instance the optimization problem on the number and on the position of actuators is widely open.

8. Numerical approximations issues.

   Consider a semi-discrete time approximation of the same class of exponentially stable infinite-dimensional systems with unbounded feedbacks as in Chap. 2. It has recently been proved that for temporal semi-discrete systems, due to the high frequency spurious modes, the exponential decay property may be lost as the time step tends to zero. One open problem is to prove that, by adding a suitable numerical viscosity term in the numerical scheme, one can recover a uniformly exponentially stable system. In this way this result would generalize the result obtained by Ervedoza and Zuazua in [57] for bounded feedback operators.

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LECTURE NOTES IN MATHEMATICS

Edited by J.-M. Morel, B. Teissier; P.K. Maini

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