Bibliographical Remarks

The origins of the method of integral manifolds are found in the works of J. Hadamard [70], A. Lyapunov [100], H. Poincare [139] and O. Perron [136]. The essence of the method of integral manifolds was realized with amazing depth by A. Lyapunov [100] who used order reduction when he investigated the critical cases of one zero, or a pair of purely imaginary, eigenvalues. The possibility of lowering the dimensionality of the system is the essential aspect of the method of integral manifolds. The foundations of the theory were laid by N. Bogolyubov [13] and significant impact on the development of the method was provided by N. Bogolyubov and Yu Mitropolskii [14, 15] and J. Hale [71–73].

As to the singularly perturbed systems, pioneering papers were published during 1957–1970 by such as K. Zadiraka, V. Fodchuk and Ya. Baris from the scientific school headed by N. Bogolyubov and Yu. Mitropolsky in the Institute of Mathematics of Ukrainian Academy of Science, Kiev. The existence of slow integral manifolds, stable [217, 218], unstable and conditionally stable [4, 5], are shown in these papers. Some of these were translated into English on the initiative of Jack Hale and AMS, the main results of [217] can be found in [112]. The authors of [53, 90] gave a short but realistic description of the history of the geometrical theory of singular perturbations. At the same time a series of papers devoted to the existence and asymptotic expansions of integral manifolds for nonautonomous differential systems with slow and fast variables were published by Yu Mitropolskii and O. Lykova. These results can be found in the books [113, 114], see also the books [29, 185, 197, 210]. Various aspects of the theory of slow integral manifolds and the behavior of solutions in their neighborhood are presented in [27, 38, 45, 58, 74, 75, 81, 92, 105, 117, 120, 126, 191, 200], see also references therein.

The concept of a fast integral manifold was introduced in [170]. This allowed the construction of a smooth transformation which reduced the original singularly perturbed differential system to a block-diagonal form. This means that the system under consideration is decomposed into two subsystems, the first of which is independent and regularly perturbed with respect to ε and the second one describes

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fast components of solutions. Some theoretical and applied results along these lines were obtained for ODE's [167, 168, 171–173, 175, 176, 178, 206, 208], for PDE's [12, 174, 177, 181], non-Lipschitzian [147], discontinuous [182], discrete [207, 209] and difference-differential systems [48–50].

In the first papers devoted to canards non-standard analysis was the main tool of investigations [7, 8, 16, 35, 36, 221], matched asymptotics were used in [42, 110], the Gevrey version of matched asymptotic expansions (see references in [51]), the approach based on the blow-up technique in [39], and on the technique of upper and lower solutions in [26, 123].

In many papers devoted to canards the term "canard" is associated with periodic trajectories [7, 8, 20, 21, 35, 110]. In the papers [59, 60] it was suggested a canard is a one-dimensional slow invariant manifold if it contains a stable slow invariant manifold and an unstable one, and a canard is obtained as a result of gluing stable (attractive) and unstable (repulsive) slow invariant manifolds at one point of the breakdown surface due to the availability of an additional scalar parameter which may be considered as a control parameter. This approach was proposed for the first time in [59,60] and was then applied to construct canards in $\mathbb{R}^3$ [56,61,184], canards for PDE [60,61] and canard travelling waves [149,189]. Moreover, the use of control functions instead of control parameters allowed the construction of black swans [61, 155–158, 161–164, 184, 188], and canard cascades [180], the consideration of the effect of delayed loss of stability [124, 125] and [150, 157, 162], and the solution of a number of applied problems [56, 61, 156, 159, 160, 183, 184]. Different kinds of canards, canards in piecewise linear systems, the influence of stochastic perturbations, mixed mode oscillations and miscellaneous applications were considered in [10, 11, 18–20, 23, 24, 30–32, 34, 40, 47, 63, 64, 67, 68, 79, 82, 104, 115, 135, 137, 140–144, 151, 154, 198, 212–215, 220], see also the overview [33].

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École Normale Supérieure de Cachan,
61 Avenue du Président Wilson, 94235 Cachan Cedex, France
E-mail: morel@cmla.ens-cachan.fr

Professor B. Teissier, Institut Mathématique de Jussieu,
UMR 7586 du CNRS, Équipe "Géométrie et Dynamique",
175 rue du Chevaleret
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