Appendix A

A.1 Proof of Theorem 2.30

Theorem 2.30. *The Hilbert modular surface X_D is the moduli space of all pairs (X, ρ), where X is a principally polarized Abelian surface and ρ is a choice of real multiplication on X by $\mathcal{O}_D$.*

Proof. Given $\tau = (\tau_1, \tau_2) \in \mathbb{H} \times \mathbb{H}$ we define a map $\phi_\tau : \mathcal{O}_D \oplus \mathcal{O}_D^\vee \to \mathbb{C}^2$ by

$$\phi_\tau(x, y) = (x + y\tau_1, x^\sigma + y^\sigma \tau_2).$$

Let A_τ be the complex torus $\mathbb{C}^2/\phi_\tau(\mathcal{O}_D \oplus \mathcal{O}_D^\vee)$ with the principal polarization induced by the standard symplectic pairing on $\mathcal{O}_D \oplus \mathcal{O}_D^\vee$, i.e. $\langle (x_1, y_1), (x_2, y_2) \rangle = \mathrm{tr}(x_1 y_2 - x_2 y_1)$. We define real multiplication by $\mathcal{O}_D$ on A_τ by $k(z_1, z_2) = (kz_1, k^\sigma z_2)$. This construction gives a map $\tilde{\Psi}$ from $\mathbb{H} \times \mathbb{H}$ to the set of all triples (X, ν, ϕ) where (X, ν) is a principally polarized Abelian surface $X = \mathbb{C}^2/\Lambda$ with real multiplication by $\mathcal{O}_D$ and ϕ is a choice of an $\mathcal{O}_D$-linear, symplectic isomorphism $\phi : \mathcal{O}_D \oplus \mathcal{O}_D^\vee \to \Lambda$. We need to show that "forgetting" the choice of the isomorphism ϕ means exactly factoring $\tilde{\Psi}$ through $\mathrm{SL}(\mathcal{O}_D \oplus \mathcal{O}_D^\vee)$.

Let $g = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{SL}(\mathcal{O}_D \oplus \mathcal{O}_D^\vee)$. Then also $g^* = \left(\begin{smallmatrix} a & -b \\ -c & d \end{smallmatrix}\right)$ is an automorphism of $\mathcal{O}_D \oplus \mathcal{O}_D^\vee$. Define

$$\chi(g, \tau) = \begin{pmatrix} (c\tau_1 + d)^{-1} & 0 \\ 0 & (c^\sigma \tau_2 + d^\sigma)^{-1} \end{pmatrix}.$$

This yields the following commutative diagram:

$$\begin{tikzcd} \mathcal{O}_D \oplus \mathcal{O}_D^\vee \arrow[r, "\phi_\tau"] \arrow[d, "g^*"'] & \mathbb{C}^2 \arrow[d, "\chi(g,\tau)"] \\ \mathcal{O}_D \oplus \mathcal{O}_D^\vee \arrow[r, "\phi_{g\cdot\tau}"] & \mathbb{C}^2 \end{tikzcd}$$

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Hence $\chi(g,\tau)$ induces an isomorphism between A_τ and $A_{g\cdot\tau}$, that preserves polarizations and commutes with the action of real multiplication. Therefore, we have a well-defined map Ψ from X_D to the set of all principally polarized Abelian surfaces with a choice of real multiplication. This map will be shown to be a bijection.

Let us first show that Ψ is injective: assume that there exists an isomorphism $f : X = \mathbb{C}^2/\Lambda \to X' = \mathbb{C}^2/\Lambda'$. Let us choose two isomorphisms $\phi_\tau : \mathcal{O}_D \oplus \mathcal{O}_D^\vee \to \Lambda = \Lambda_\tau$ and $\phi_{\tau'} : \mathcal{O}_D \oplus \mathcal{O}_D^\vee \to \Lambda' = \Lambda_{\tau'}$. Let $\gamma := (D + \sqrt{D})/2$. Following [McM07], Chap. 3, $A_\tau := \mathbb{C}^2/\phi_\tau(\mathcal{O}_D \oplus \mathcal{O}_D^\vee)$ is then isomorphic to $\mathbb{C}^2/(\mathbb{Z}^2 \oplus \Pi\mathbb{Z}^2)$ where

$$\Pi = \frac{1}{D}\begin{pmatrix} \tau_1(\gamma^\sigma)^2 + \tau_2(\gamma)^2 & -\tau_1\gamma^\sigma - \tau_2\gamma \\ -\tau_1\gamma^\sigma - \tau_2\gamma & \tau_1 + \tau_2 \end{pmatrix}$$

and similarly for $A_{\tau'}$. However, two such tori with corresponding matrices Π and Π' are isomorphic if and only if Π and Π' are equivalent modulo the action of $\mathrm{Sp}_4(\mathbb{Z})$ on $\mathbb{H}_2$. It can be checked that this implies that $\tau' = g\tau$ for some $g \in \mathrm{SL}(\mathcal{O}_D \oplus \mathcal{O}_D^\vee)$. It follows that Ψ is injective.

The longest step is to show that Ψ is surjective. To do this, we consider an arbitrary complex torus $X = \mathbb{C}^2/\Lambda$ with principal polarization given by an alternating form $E : \Lambda \times \Lambda \to \mathbb{Z}$ and real multiplication by $\rho : \mathcal{O}_D \to \mathrm{End}(X)$. For dimension reasons Λ is a projective rank 2 $\mathcal{O}_D$-module. As first step let us show that $\Lambda \cong \mathcal{O}_D \oplus \mathcal{I}$ for some ideal $\mathcal{I}$. This is an immediate consequence of the following two lemmas (compare [May09]).

Lemma A.1. *Let R be a Dedekind domain and $A_1, \ldots, A_n$ be fractional ideals. Then*

$$A_1 \oplus \ldots \oplus A_n \cong R^{n-1} \oplus A_1 \cdots A_n.$$

Proof. By induction, it suffices to consider the case $n = 2$. So we are dealing with fractional ideals A and B. Multiplying by suitable elements x and y of R we may assume that A and B are relatively prime ideals in R. Define $\pi : A \oplus B \to R$ by $\pi(a,b) = a + b$. The kernel of π is $A \cap B = AB$ since A and B are relatively prime. The short exact sequence

$$0 \longrightarrow AB \longrightarrow A \oplus B \xrightarrow{\pi} R \longrightarrow 0$$

splits since R is (as Dedekind domain) free, and this yields the claim. □

Lemma A.2. *For a finitely generated projective $\mathcal{O}_D$-module M of rank n*

$$M = \mathcal{O}_D^{n-1} \oplus I$$

where I is an ideal.

Proof. As M is projective, it is torsion free. Now we proceed the proof by induction. If $n = 1$, then M is a $\mathcal{O}_D$-submodule of $M \otimes_{\mathcal{O}_D} K \cong K$ and is therefore isomorphic to a fractional ideal. By choosing $n - 1$ elements of M that span a vector space of that dimension in $M \otimes_{\mathcal{O}_D} K$, we can construct an $\mathcal{O}_D$-submodule N of rank $n - 1$. The exact sequence

$$0 \longrightarrow N \longrightarrow M \longrightarrow M/N \longrightarrow 0$$

remains exact upon tensoring with K, hence M/N has rank 1. Thus M/N is projective and the sequence splits. Since any fractional ideal is isomorphic to an ideal, the conclusion follows from the inductive hypothesis and the last lemma. □

Hence we have that as $\mathcal{O}_D$-modules $\Lambda \cong \mathcal{O}_D \oplus \mathcal{I}$ for some ideal $\mathcal{I}$. So we may assume that $X = \mathbb{C}^2/\varphi(\mathcal{O}_D \oplus \mathcal{I})$ with real multiplication ρ by $\mathcal{O}_D$ for some embedding $\varphi : \mathcal{O}_D \oplus \mathcal{I} \to \mathbb{C}^2$.

Lemma A.3. *There exists a symplectic isomorphism $\Theta : (\mathcal{O}_D \oplus \mathcal{I}, E, \rho) \to (\mathcal{O}_D \oplus \mathcal{O}_D^\vee, \langle , \rangle, \rho')$, where $\langle , \rangle$ is the symplectic pairing on $\mathcal{O}_D \oplus \mathcal{O}_D^\vee$ that is compatible with the real multiplications ρ and ρ'.*

Proof. We can always choose a symplectic basis with $a_1, a_2 \in \mathcal{O}_D$, $b_1, b_2 \in \mathcal{I}$ for Λ, i.e. $E(a_i, b_j) = \delta_{ij}$ and $E(a_i, a_j) = 0$ and $E(b_i, b_j) = 0$. The form E extends to $E : K^2 \times K^2 \to \mathbb{Q}$ such that $E(kx, y) = E(x, ky)$ for all $x, y \in \Lambda$ and $k \in K$. Let $\langle , \rangle$ be the standard symplectic form on $\mathcal{O}_D \oplus \mathcal{O}_D^\vee$ with its standard basis c_1, c_2, d_1, d_2 and together with its standard real multiplication ρ' (see e.g. [McM07]). We choose a $\mathbb{Z}$-linear symplectic isomorphism $\Theta : \mathcal{O}_D \oplus \mathcal{I} \to \mathcal{O}_D \oplus \mathcal{O}_D^\vee$ with $\Theta(a_i) = c_i$ and $\Theta(b_i) = d_i$ for $i = 1, 2$. Then we may identify the a_i with the c_i since the c_i are a basis of $\mathcal{O}_D$ as $\mathbb{Z}$-module. We now want to prove that this symplectic isomorphism is $\mathcal{O}_D$ linear. By tensoring with K we also extend this map to $\Theta : K \times K \to K \times K$ and verify that Θ is indeed K-linear. Since Θ is $\mathbb{Q}$-linear, it suffices to show that $\Theta(kx) = k\Theta(x)$ for a fixed $k \in K \setminus \mathbb{Q}$ and for all $x \in K \times K$. We now choose $k = \frac{b_1}{b_2}$. By definition we have $\Theta(ka_1) = k\Theta(a_1)$ and $\Theta(ka_2) = k\Theta(a_2)$. We now show that $\Theta(kb_1) = k\Theta(b_1)$ for this fixed $k \in K \setminus \mathbb{Q}$. This is equivalent to showing that $kd_1 = d_2$. By definition of k we have

$$1 = E(a_2, b_2) = E(a_2, kb_1) = E(ka_2, b_1),$$

which yields

$$1 = \langle \Theta(ka_2), \Theta(b_1) \rangle = \langle k\Theta(a_2), d_1 \rangle = \langle c_2, kd_1 \rangle . \tag{A.1}$$

Similarly we have

$$0 = E(a_1, b_2) = E(a_1, kb_1) = E(ka_1, b_1)$$

which yields

$$0 = \langle \Theta(ka_1), \Theta(b_1) \rangle = \langle k\Theta(a_1), d_1 \rangle = \langle c_1, kd_1 \rangle \,. \tag{A.2}$$

Equations (A.1) and (A.2) imply that $kd_1 = d_2$. By considering appropriate equations it follows similarly as above that $\Theta(kb_2) = k\Theta(b_2)$, which implies K-linearity of Θ. This means that the following diagram commutes:

$$\begin{tikzcd}
\mathcal{O}_D \oplus \mathcal{I} \arrow[r, "\Theta"] \arrow[d, "\rho"'] & \mathcal{O}_D \oplus \mathcal{O}_D^\vee \arrow[d, "\rho'"] \\
\mathcal{O}_D \oplus \mathcal{I} \arrow[r, "\Theta"] & \mathcal{O}_D \oplus \mathcal{O}_D^\vee
\end{tikzcd}$$

Or in other words the chosen symplectic isomorphism respects the real multiplication. □

We may therefore assume that $\Lambda = \varphi(\mathcal{O}_D \oplus \mathcal{O}_D^\vee)$ where $\varphi : \mathcal{O}_D \oplus \mathcal{O}_D^\vee \to \mathbb{C}^2$ is an embedding. It remains to show that there exists a $\tau \in \mathbb{H}^2$ with $\phi_\tau = \varphi$.

Recall that

$$(a_1, a_2, b_1, b_2) = ((1, 0), (\gamma, 0), (0, -\gamma^\sigma/\sqrt{D}), (0, 1/\sqrt{D}))$$

is the standard basis of $\mathcal{O}_D \oplus \mathcal{O}_D^\vee$ with respect to the standard symplectic form, where $\gamma = (D + \sqrt{D})/2$. With respect to this basis $\phi_\tau : \mathcal{O}_D \oplus \mathcal{O}_D^\vee \to \mathbb{C}^2$ is given by the matrix

$$\phi_\tau = \begin{pmatrix} 1 & \gamma & -\tau_1\gamma^\sigma/\sqrt{D} & \tau_1/\sqrt{D} \\ 1 & \gamma^\sigma & \tau_2\gamma/\sqrt{D} & -\tau_2/\sqrt{D} \end{pmatrix}$$

or equivalently

$$\phi_\tau = (B, D_\tau (B^t)^{-1})$$

where $B = \left(\begin{smallmatrix} 1 & \gamma \\ 1 & \gamma^\sigma \end{smallmatrix}\right)$ and $D_\tau = \left(\begin{smallmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{smallmatrix}\right)$. Consequently we have $\Psi(\tau) \cong \mathbb{C}^2/(\mathbb{Z}^2 \oplus \Pi\mathbb{Z}^2)$, where

$$\Pi = B^{-1} D_\tau (B^t)^{-1} = \frac{1}{D} \begin{pmatrix} \tau_1(\gamma^\sigma)^2 + \tau_2\gamma^2 & -\tau_1\gamma^\sigma - \tau_2\gamma \\ -\tau_1\gamma^\sigma - \tau_2\gamma & \tau_1 + \tau_2 \end{pmatrix}.$$

Let us now consider φ. As $\rho(\gamma)$ has two distinct eigenvalues, namely γ and γ^σ, we may choose a basis of $\mathbb{C}^2$ such that the action of $\rho(\gamma)$ is given by $\left(\begin{smallmatrix} \gamma & 0 \\ 0 & \gamma^\sigma \end{smallmatrix}\right)$.

One easily calculates that the action of $\rho(\gamma)$ on the lattice $\mathcal{O}_D \oplus \mathcal{O}_D^\vee$ with respect to the basis (a_1, a_2, b_1, b_2) is given by

$$\rho(\gamma)(a_1, a_2) = C \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\rho(\gamma)(b_1, b_2) = C^t \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

where $C = \left(\begin{smallmatrix} 0 & 1 \\ (D-D^2)/4 & D \end{smallmatrix} \right)$.

Since the real multiplication commutes with φ, we have

$$\varphi(\rho(\gamma)a_i) = \begin{pmatrix} \gamma & 0 \\ 0 & \gamma^\sigma \end{pmatrix} \varphi(a_i)$$

$$\varphi(\rho(\gamma)b_i) = \begin{pmatrix} \gamma & 0 \\ 0 & \gamma^\sigma \end{pmatrix} \varphi(b_i).$$

Setting $\varphi(a_1) = \left(\begin{smallmatrix} r \\ s \end{smallmatrix} \right)$ and $\varphi(b_2) = \left(\begin{smallmatrix} v/\sqrt{D} \\ -w/\sqrt{D} \end{smallmatrix} \right)$ with $r, s, v, w \in \mathbb{C}$ we get that $\varphi : \mathcal{O}_D \oplus \mathcal{O}_D^\vee \to \mathbb{C}^2$ is given by the matrix

$$\varphi = (\begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix} B, \begin{pmatrix} v & 0 \\ 0 & w \end{pmatrix} (B^t)^{-1}).$$

Consequently $\mathbb{C}^2/\varphi(\mathcal{O}_D \oplus \mathcal{O}_D^\vee) \cong \mathbb{C}^2/(\mathbb{Z} \oplus \Pi'\mathbb{Z})$, where

$$\Pi' = \begin{pmatrix} \tau_1(\gamma^\sigma)^2 + \tau_2\gamma^2 & -\tau_1\gamma^\sigma - \tau_2\gamma \\ -\tau_1\gamma^\sigma - \tau_2\gamma & \tau_1 + \tau_2 \end{pmatrix}.$$

with $\tau_1, \tau_2 \in \mathbb{C}$. Since $\mathbb{C}^2/(\mathbb{Z} \oplus \Pi'\mathbb{Z})$ is a principal polarized Abelian variety we have in fact that $\Pi' \in \mathbb{H}_2$ and hence $\tau_1, \tau_2 \in \mathbb{H}$. This shows that Ψ is indeed surjective. So Ψ is a bijection and we have finally proven the assertion of the theorem. □

A.2 Elements of Non-cocompact Cofinite Fuchsian Groups

Let $\Gamma \subset \mathrm{PSL}_2(\mathbb{R})$ be an arbitrary Fuchsian group. It is in general a very hard problem to decide whether a given element $N \in \mathrm{PSL}_2(\mathbb{R})$ is in Γ or not. For example one would like to check if a certain matrix lies inside a Veech group $\mathrm{SL}(X, \omega)$ or not. In fact, one is often even interested in writing N as a word

in given generators if possible. These problems are very much reminiscent of the famous—generally unsolvable—word problem (see e.g. [Bau93]). For non-cocompact, cofinite Fuchsian groups we are able to give an algorithm which does solve both of the above problems—at least if the Dirichlet fundamental domain of the Fuchsian group is known sufficiently well.

Let Γ until the end of Appendix A.2 be a non-cocompact cofinite Fuchsian groups. Then Γ contains at least one parabolic element (see [Kat92], Corollary 4.2.7). Before we explain the algorithm, let us collect a bunch of well-known facts. The first one describes the Dirichlet fundamental domain of a Fuchsian group in Euclidean metric (compare [Kat92], Chap. 3).

Lemma A.4. *The Dirichlet fundamental domain of Γ can be described using Euclidean metric as follows:*

$$D_p(\Gamma) = \left\{ z \in \mathbb{H} \mid \left| \frac{T(z) - p}{z - p} \right| \geq \frac{1}{|cz + d|} \; \forall T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \right\}$$

A long but straightforward calculation then yields:

Corollary A.5. *For* $\lim_{k\to\infty} D_{ki}(\Gamma) =: D_\infty(\Gamma)$ *all bounding geodesics of the Dirichlet fundamental domain which are not vertical lines are given by*

$$\left| z - \left(-\frac{d}{c} \right) \right| = \frac{1}{|c|}$$

for some $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma$. *The interior of this limit fundamental domain is*

$$\operatorname{int}(D_\infty(\Gamma)) = \left\{ z \in \mathbb{H} \mid \left| z - \left(-\frac{d}{c} \right) \right| > \frac{1}{|c|} \right\}.$$

Furthermore we remind the reader of the following fact (see [FB06], Hilfssatz V.7.1):

Lemma A.6. *If* $M = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in SL_2(\mathbb{R})$, *then for all* $z \in \mathbb{H}$

$$\operatorname{Im}(Mz) = \frac{\operatorname{Im}(z)}{|cz + d|^2}$$

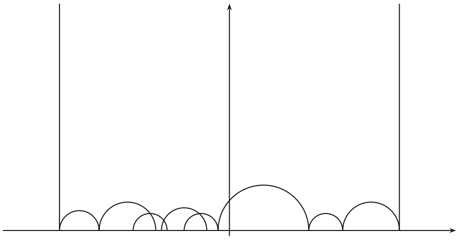
holds.

The algorithm is based on the following lemma:

Lemma A.7. *Let Γ be a Fuchsian group which is a lattice in* $\mathrm{PSL}_2(\mathbb{R})$*. Then we have:*

(i) For all $z \in \mathbb{H}$ there are only finitely many $a_1, \ldots, a_n \in \mathbb{R}$ with $a_i \geq \operatorname{Im}(z)$ which appear as imaginary parts in the elements of Γz.
(ii) Every orbit Γz contains points of maximal imaginary part.

Fig. A.1 A typical fundamental domain



(iii) The points $z \in \mathbb{H}$ *which have maximal imaginary part in their* Γ*-orbit are those with*

$$|cz + d| \geq 1 \ \forall c, d \text{ with } \begin{pmatrix} * & * \\ c & d \end{pmatrix} \in \Gamma.$$

Proof. Let $z = x + iy$. The last lemma gives

$$\mathrm{Im}(Mz) \geq \mathrm{Im}(z) \text{ if and only if } |cz + d| \leq 1.$$

This proves (iii). The inequality $|cz + d| \leq 1$ has only finitely many solutions since Γ is discrete. This yields (i) and (ii). □

In [Koh06] one finds an algorithm which decomposes each element $\mathrm{SL}_2(\mathbb{Z})$ into a word in the standard generators of $\mathrm{SL}_2(\mathbb{Z})$. Simultaneously it gives a possibility to decide whether a given matrix lies in $\mathrm{SL}_2(\mathbb{Z})$, although this is of course trivial. We now imitate this algorithm.

Recall that Γ is a Fuchsian group containing at least one parabolic element. By conjugation we may assume that Γ contains $A = \left(\begin{smallmatrix} 1 & s \\ 0 & 1 \end{smallmatrix}\right)$ with $s \in \mathbb{R}$. Furthermore we may assume that $|s|$ is minimal, i.e. there is no matrix of this form in Γ with an upper right entry of smaller absolute value. The Dirichlet fundamental domain $D_\infty(\Gamma)$ then looks like:

The left and right boundary of $D_\infty(\Gamma)$ are the geodesics joining $-s/2$ (respectively $s/2$) and ∞. The other boundary geodesics fill the gap between $-s/2$ and $s/2$. Now let $X \in \mathrm{PSL}_2(\mathbb{R})$. Let us try to write X as a word in the generators of Γ. Choose an arbitrary point $z_0 \in \mathrm{int}(D_\infty(\Gamma))$ and

(1) look at $y_0 = Xz_0$ and apply k-times (with $k \in \mathbb{Z}$) the matrix A until $|\mathrm{Re}(A^k y_0)| \leq \frac{s}{2}$.
(2) If $y_1 := A^k y_0 \in \mathrm{int}(D_\infty(\Gamma))$, then check if $y_1 = z_0$. If $y_1 \neq z_0$ then $X \notin \Gamma$ since $\mathcal{F}$ is a fundamental domain.
(3) If $y_1 = z_0$, then check if $X = A^{-k}$ in $\mathrm{PSL}_2(\mathbb{R})$. In either case we are finished.
(4) If $y_1 \notin \mathrm{int}(D_\infty(\Gamma))$, then y_1 lies beneath (at least) one of the boundary geodesics and above the x-axis. Then find a matrix $B = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma$ which generates this geodesic in the sense of Corollary A.5. And set $y_2 = By_1$.

Since B generates the geodesic

$$\left|y_1 - \left(-\frac{d}{c}\right)\right| < \frac{1}{|c|}$$

or since equivalently $|cy_1 + d| < 1$ holds, we must have that $\mathrm{Im}(y_2) > \mathrm{Im}(y_1)$. If $y_2 \in \mathrm{int}(D_\infty(\Gamma))$, then we are again finished (after checking if $y_2 = z_0$ and if $A^{-k}B^{-1} = X$). Otherwise we continue with step (I) with y_2 instead of y_0.

As the imaginary part grows with every iteration, the algorithms really determines by Lemma A.7 (iii). We have thus shown:

Theorem A.8. *Let Γ be a non-cocompact, cofinite Fuchsian group. If $A_1, \dots, A_n$ generate Γ in the sense of Corollary A.5 then the algorithm above decides whether an arbitrary element $X \in \mathrm{PSL}_2(\mathbb{R})$ lies in Γ. If so, the algorithm returns X as a word in the A_i.*

A.3 Checking Commensurability

If we knew that in genus 2 all Veech groups of Teichmüller curves were maximal and if we wanted to check if a curve $\mathbb{H}/\Gamma$ is a twisted Teichmüller curve then Proposition 9.2 and Lemma 5.29 make it necessary to find a sufficiently general algorithm which decides if a Fuchsian group $H \subset \mathrm{SL}_2(\mathbb{R})$ is conjugated to a subgroup of another Fuchsian group $G \subset \mathrm{SL}_2(\mathbb{R})$.

Since the signature of a Fuchsian group does not change under conjugation it is necessary that G contains a subgroup of the same signature as H. This is of course a much stronger criterion than just looking at the Euler characteristics. In [Sin70], D. Singerman gave a criterion which decides whether a given group G contains a subgroup of the same signature as H.

Definition A.9. If Γ is of signature $(g; m_1, \dots, m_r; s; t)$ then the integers $m_1, \dots, m_r$ are called the **periods** of Γ.

Let n now be a period of $\Gamma' \subset \Gamma$. Then n is by definition the order of an elliptic element $y \in \Gamma'$ and y is a power of a conjugate of one of the elliptic generators $x_j \in \Gamma$ with order m_j. We shall then say that n has been induced by m_j. Of course, this implies that $n | m_j$.

Theorem A.10 (Singerman, 1970). *Let Γ be a Fuchsian group of signature $(g; m_1, \dots, m_r; s; t)$. Then Γ contains a subgroup Γ_1 of index N with signature $(g' n_{11}, n_{12}, \dots, n_{1p_1}, \dots, n_{r1}, \dots, n_{rp_r}; s'; t')$ if and only if*

(I) There exists a finite permutation group G transitive on N points and an epimorphism $\theta : \Gamma \to G$ satisfying the following properties

(i) The permutation $\theta(x_j)$ has precisely p_j cycles of lengths less than m_j, the lengths of these cycles being $m_j/n_{j1}, \ldots, m_j/n_{jp_j}$,

(ii) If we denote the number of cycles in the permutation $\theta(\gamma)$ by $\delta(\gamma)$ then

$$s' = \sum_{k=1}^{s} \delta(p_k), \quad t' = \sum_{l=1}^{t} \delta(h_l)$$

where p_k are the parabolic and h_l are the hyperbolic generators of Γ.

(II) $vol(\Gamma_1)/vol(\Gamma) = N$.

Nevertheless, there may exist non-conjugated groups of the same signature (see e.g. [SW00]). So Theorem A.10 only gives necessary conditions for H and G. Hence we really need an algorithm to decide, whether there exists an $M \in SL_2(\mathbb{R})$ with $MHM^{-1} \leq G$. For the rest of the section we will as two assumptions suppose that G is a cofinite Fuchsian group and H (and thus necessarily also G) contains elliptic elements. Note that both assumptions are fulfilled for the Veech groups of Teichmüller curves of genus 2 and 4. The idea of the algorithm is that, if $MHM^{-1} \leq G$, then there must exist a fundamental domain $\mathcal{F}_H$ of H and a fundamental domain $\mathcal{F}_G$ of G such that the tessellation of $\mathbb{H}$ by copies of $\mathcal{F}_G$ is a refinement of the tessellation of $\mathbb{H}$ by copies of $M\mathcal{F}_H M^{-1}$. The algorithm now works in the following way:

(1) Choose an arbitrary elliptic fixed point x of H of period n_x.
(2) For each conjugacy class of elliptic fixed points of period m_i with $n_x|m_i$ in G, choose an arbitrary elliptic fixed point z_i and calculate all matrices M_{x,z_i} which send x to z_i.

Of course, elliptic fixed points of H must be sent by M_{x,z_i} to elliptic fixed points of G of suitable period. Thus the upper matrices M_{x,z_i} are the only possible candidates for M. Note that the choices of the elliptic fixed points z_i and x are really free since choosing another representative than x means multiplying M_{x,z_i} by a matrix in H from the right and choosing another representative than z_i means multiplying M_{x,z_i} by a matrix in G from the left.

(3) Choose another arbitrary elliptic fixed point y of H of period n_y and calculate $d(x, y)$.

For computational reasons it is convenient to choose y with minimal distance $d(x, y)$.

(4) Calculate parts of the tessellation of $\mathbb{H}$ by copies of $\mathcal{F}_G$ until $B_{d(x,y)}(z_i)$ is completely covered.

Note that step (4) can be performed in finite time since G is a cofinite Fuchsian group. Since M_{x,z_i} is an isometry the next step of the algorithm is naturally given by:

(5) For all elliptic fixed points z_k of G of period m_k with $z_k \in \delta B_{d(x,y)}(z_i)$ and $n_y|m_k$ calculate if there exists a matrix M_{x,z_i} with $M_{x,z_i} y = z_k$. If so this matrix is unique and called M_{x,y,z_i,z_k}.

This yields a complete list of possible candidates M_{x,y,z_i,z_k}. For each of these matrices now perform step (6).

(6) Set $M := M^{-1}_{x,y,z_i,z_k}$. Then for all generators u_j of H decide whether $M u_j M^{-1} \in G$. If so (for all u_j), then $MHM^{-1} \leq G$.

Step (6) can for example be done by the algorithm given in Sect. A.2.

In the following example we now show how the algorithm explicitly works:

Examples A.11. Let $H = \left\langle S, \begin{pmatrix} 1+2/9\sqrt{8} & 2/9 \\ -16/9 & 1-2/9\sqrt{8} \end{pmatrix} \right\rangle$ and let $G = \mathrm{SL}_2(\mathbb{Z})$. Both groups H and G have only one conjugacy class of elliptic fixed points of order 2. Thus we may choose $x = z_i = i$ and it is well-known that $M_{x,z_i} = \begin{pmatrix} c & d \\ -d & c \end{pmatrix}$ with $c^2 + d^2 = 1$. Another elliptic fixed point of H is

$$\begin{pmatrix} 1+2/9\sqrt{8} & 2/9 \\ -16/9 & 1-2/9\sqrt{8} \end{pmatrix} i = \underbrace{\frac{1}{1553}\left(-702 - 220\sqrt{8} + i(369 + 36\sqrt{8})\right)}_{=:y}.$$

$d(x, y) = d(i, i + 2)$ and $i + 2$ is an elliptic fixed point of G. Solving the equation $M_{x,z_i} y = i + 2$ yields

$$M := M^{-1}_{i,y,i,i+2} = \begin{pmatrix} 1/3 & -1/3\sqrt{8} \\ 1/3\sqrt{8} & 1/3 \end{pmatrix}.$$

We have

$$M \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} M^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

and

$$M \begin{pmatrix} 1+2/9\sqrt{8} & 2/9 \\ -16/9 & 1-2/9\sqrt{8} \end{pmatrix} M^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

and therefore $MHM^{-1} \leq G$.

Appendix B
Tables

Tables with some numerical data for the volume of diagonal twisted Teichmüller curves in the cases $D = 13$ and $D = 17$ can be found on the following pages. As we have seen in Chap. 6 diagonal twisted Teichmüller curves carry the information about the volume of almost all twisted Teichmüller curves in the case $h_D = 1$. Therefore, diagonal twisted Teichmüller curves are especially interesting from a numerical point of view. More precisely, the tables on the next two pages contain the indexes $[\mathrm{SL}(L_D) : \mathrm{SL}(L_D) \cap \Gamma_0^D(m) \cap \Gamma^{D,0}(n)]$ for many $m, n \in \mathcal{O}_D$. From this data one can calculate the volume of the corresponding twisted Teichmüller curves (Theorem 6.25). All calculations were done with PARI/GP (Tables B.1, B.2).

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Table B.1 Index of $\mathrm{SL}(L_{13}) \cap \Gamma_0^{13}(m) \cap \Gamma^{13,0}(n)$ in $\mathrm{SL}(L_{13})$ if $(m,n)=1$

n/m	1	2	3	4	w	$w+1$	$w+2$	$w+3$	$2w$	$2w+1$	$2w+2$	$2w+3$	$3w$	$3w+1$	$3w+2$	$3w+3$
1	**1**	**5**	**16**	**20**	**4**	**1**	**4**	**12**	**20**	**12**	**5**	**4**	**48**	**24**	**18**	**16**
2	**5**	–	**80**	–	**20**	**5**	**20**	**60**	–	**60**	–	**20**	**240**	**120**	**90**	**80**
3	**16**	**80**	–	**320**	–	**16**	–	–	–	–	**80**	–	–	**384**	**288**	–
4	**20**	–	**320**	–	**80**	**20**	**80**	**240**	–	**240**	–	**80**	**960**	**480**	**360**	**320**
w	**4**	**20**	–	**80**	–	**4**	**16**	–	–	**48**	**20**	–	–	**96**	**72**	–
$w+1$	**1**	**5**	**16**	**20**	**4**	**1**	**4**	**12**	**20**	**12**	**5**	**4**	**48**	**24**	**18**	**16**
$w+2$	**4**	**20**	–	**80**	**16**	**4**	–	**48**	**80**	–	**20**	**16**	–	**96**	**72**	–
$w+3$	**12**	**60**	–	**240**	–	**12**	**48**	–	–	**144**	**60**	–	–	**288**	**216**	–
$2w$	**20**	–	–	–	–	**20**	**80**	–	–	**240**	–	–	–	**480**	**360**	–
$2w+1$	**12**	**60**	–	**240**	**48**	**12**	–	**144**	**240**	–	**60**	**48**	**48**	**288**	**216**	–
$2w+2$	**5**	–	**80**	–	**20**	**5**	**20**	**60**	–	**60**	–	**20**	**240**	**120**	**90**	**80**
$2w+3$	**4**	**20**	–	**80**	–	**4**	**16**	–	–	**48**	**20**	–	–	**96**	**72**	–
$3w$	**48**	**240**	–	**960**	–	**48**	–	–	–	**48**	**240**	–	–	**1152**	**864**	–
$3w+1$	**24**	**120**	**384**	**480**	**96**	**24**	**96**	**288**	**480**	**288**	**120**	**96**	**1152**	–	**432**	**384**
$3w+2$	**18**	**90**	**288**	**360**	**72**	**18**	**72**	**216**	**360**	**216**	**90**	**72**	**864**	**432**	–	**288**
$3w+3$	**16**	**80**	–	**320**	–	**16**	–	–	–	–	**80**	–	–	**384**	**288**	–

It is equal to the index of $\Gamma_0^{13}(m) \cap \Gamma^{13,0}(n)$ in $\mathrm{SL}(\mathcal{O}_{13})$ if $(n,m)=1$

Table B.2 Index of $\mathrm{SL}(L^1_{17}) \cap \Gamma^{17}_0(m) \cap \Gamma^{17,0}(n)$ in $\mathrm{SL}(L^1_{17})$

n/m	1	2	3	4	w	$w+1$	$w+2$	$w+3$	$2w$	$2w+1$	$2w+2$	$2w+3$	$3w$	$3w+1$	$3w+2$	$3w+3$
1	**1**	6	**10**	24	4	**3**	2	**12**	24	**14**	12	**1**	40	**48**	28	**30**
2	6	–	60	–	–	–	–	–	–	84	–	6	–	–	–	–
3	**10**	60	–	240	40	**30**	20	**120**	240	**140**	120	**10**	–	**480**	280	–
4	24	–	240	–	–	–	–	–	–	336	–	24	–	–	–	–
w	4	–	40	–	–	12	–	48	–	56	–	4	–	192	–	120
$w+1$	**3**	–	**30**	–	12	–	6	–	–	**42**	–	**3**	120	–	84	–
$w+2$	2	–	20	–	–	6	–	24	–	28	–	2	–	96	–	60
$w+3$	**12**	–	**120**	–	48	–	24	–	–	**168**	–	**12**	480	–	336	–
$2w$	24	–	240	–	–	–	–	–	–	336	–	24	–	–	–	–
$2w+1$	**14**	84	**140**	336	56	**42**	28	**168**	336	–	168	**14**	560	**672**	392	**420**
$2w+2$	12	–	120	–	–	–	–	–	–	168	–	12	–	–	–	–
$2w+3$	**1**	6	**10**	24	4	**3**	2	**12**	24	**14**	12	**1**	40	**48**	28	**30**
$3w$	40	–	–	–	–	120	–	480	–	560	–	40	–	1920	–	–
$3w+1$	**48**	–	**480**	–	192	–	96	–	–	**672**	–	**48**	1920	–	1344	–
$3w+2$	28	–	280	–	–	84	–	336	–	392	–	28	–	1344	–	840
$3w+3$	**30**	–	–	–	120	–	60	–	–	**420**	–	**30**	–	–	840	–

The index is either the same (bold face) or $2/3$ as large (normal face) as the index of $\Gamma^{17}_0(m) \cap \Gamma^{17,0}(n)$ in $\mathrm{SL}(\mathcal{O}_{17})$

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