Appendix A
Tightness

For convenience, in this section we review the tightness criteria that are needed in this monograph.

We first recall the Joffe–Métivier criterion ([JM]) for weak convergence on $D([0, \infty), \mathbb{R}^d)$. This criterion is concerned with a collection $(X^{(n)}(t))_{t \geq 0}$ of semi-martingales with values in $\mathbb{R}^d$ with càdlàg paths. First observe that by forming

$$(< X^{(n)}(t), \lambda >)_{t \geq 0} \quad , \quad \lambda \in \mathbb{R}^d \tag{A.1}$$

we obtain $\mathbb{R}$-valued semi-martingales. If for every $\lambda \in \mathbb{R}^d$ the laws of these projections are tight on $D([0, \infty), \mathbb{R})$ then this is true for $\{[\mathcal{L}[(X^{(n)}(t))_{t \geq 0}], n \in \mathbb{N}\}$.

The tightness criterion for $\mathbb{R}$-valued semimartingales which is needed after the above reduction, is in terms of the socalled *local characteristics* of the semimartingales.

A.1 The Joffe–Métivier criteria for tightness of D-semimartingales

We recall the Joffe Métivier criterion ([JM]) for tightness of locally square integrable processes.

A càdlàg adapted process X, defined on $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with values in $\mathbb{R}$ is called a *D-semimartingale* if there exists a càdlàg function $A(t)$, a linear subspace $D(L) \subset C(\mathbb{R})$ and a mapping $L : (D(L) \times \mathbb{R} \times [0, \infty) \times \Omega) \to \mathbb{R}$ with the following properties:

1. for every $(x, t, \omega) \in \mathbb{R} \times [0, \infty) \times \Omega$ the mapping $\phi \to L(\phi, x, t, \omega)$ is a linear functional on $D(L)$ and $L(\phi, \cdot, t, \omega) \in D(L)$,
2. for every $\phi \in D(L)$, $(x, t, \omega) \to L(\phi, x, t, \omega)$ is $\mathcal{B}(\mathbb{R}) \times \mathcal{P}$-measurable, where $\mathcal{P}$ is the predictable σ-algebra on $[0, \infty) \times \Omega$, ($\mathcal{P}$ is generated by sets of the form $(s, t] \times F$ where $F \in \mathcal{F}_s$ and s, t are arbitrary),

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3. for every $\phi \in D(L)$ the process M^ϕ defined by

$$M^\phi(t,\omega) := \phi(X_t(\omega) - \phi(X_0(\omega)) - \int_0^t L(\phi, X_{s-}(\omega), s, \omega) dA_s, \tag{A.2}$$

is a locally square integrable martingale on $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$,
4. the functions $\psi(x) := x$ and ψ^2 belong to $D(L)$.

The functions (recall here $\Psi(x) = x$)

$$\beta(x,t,\omega) := L(\psi, x, t, \omega) \tag{A.3}$$

$$\alpha(x,t,\omega) := L((\psi)^2, x, t, \omega) - 2x\beta(x,t,\omega) \tag{A.4}$$

are called the *local characteristics of the first and second order.*

Theorem 15 (Tightness criterion). *Let $X^m = (\Omega^m, \mathcal{F}^m, \mathcal{F}_t^M, P^m)$ be a sequence of D-semimartingales with common $D(L)$ and associated operators L^m, functions A^m, α^m, β^m.*

Then the sequence $\{X^m : m \in \mathbb{N}\}$ is tight in $D_{\mathbb{R}}([0,\infty)$ provided the following conditions are satisfied:

1. $\sup_m E|X_0^m|^2 < \infty$,
2. *there is a $K > 0$ and a sequence of positive adapted processes $\{\{C_t^m : t \geq 0\}$ on $\Omega^m\}_{m \in \mathbb{N}}$ such that for every $m \in \mathbb{N}, x \in \mathbb{R}, \omega \in \Omega^m$,*

$$|\beta_m(x,t,\omega)|^2 + \alpha_m(x,t,\omega) \leq K(C_t^m(\omega) + x^2) \tag{A.5}$$

and for every $T > 0$,

$$\sup_m \sup_{t \in [0,T]} E|C_t^m| < \infty, \text{ and } \lim_{k \to \infty} \sup_m P^m[\sup_{t \in [0,T]} C_t^m \geq k] = 0, \tag{A.6}$$

3. *there exists a positive function γ on $[0,\infty)$ and a decreasing sequence of numbers (δ_m) such that $\lim_{t \to 0} \gamma(t) = 0$, $\lim_{m \to \infty} \delta_m = 0$ and for all $0 < s < t$ and all m,*

$$|(A^m(t) - A^m(s))| \leq \gamma(t-s) + \delta_m, \tag{A.7}$$

4. *if we set*

$$M_t^m := X_t^m - X_0^m - \int_0^t \beta_m(X_{s-}^m, s, \cdot) dA_s^m, \tag{A.8}$$

then for each $T > 0$ there is a constant K_T and m_0 such that for all $m \geq m_0$, then

$$E[\sup_{t \in [0,T]} |X_t^m|^2] \leq K_T(1 + E|X_0^m|^2), \tag{A.9}$$

and

$$E[\sup_{t\in[0,T]} |M_t^m|^2] \leq K_T(1 + E|X_0^m|^2). \qquad \square \tag{A.10}$$

Corollary A.1. *Assume that for $T > 0$ there is a constant K_T such that*

$$\sup_m \sup_{t\leq T, x\in\mathbb{R}} (|\alpha_m(t,x)| + |\beta_m(t,x)|) \leq K_T, \ a.s. \tag{A.11}$$

$$\sum_m (A^m(t) - A^m(s)) \leq K_T(t-s) \ \ if \ 0 \leq s \leq t \leq T, \tag{A.12}$$

and

$$\sup_m E|X_0^m|^2 < \infty, \tag{A.13}$$

and M_t^m is a square integrable martingale with $sup_m E(|M_T^m|^2) \leq K_T$.
Then the $\{X^m : m \in \mathbb{N}\}$ are tight in $D_{\mathbb{R}}([0,\infty)$. $\square$

A.2 Tightness criteria for continuous processes

Now consider the special case of probability measures on $C([0,\infty),\mathbb{R}^d)$. This criterion is concerned with a collection $(X^{(n)}(t))_{t\geq 0}$ of semimartingales with values in $\mathbb{R}^d$ with continuous paths. First observe that by forming

$$(< X^{(n)}(t), \lambda >)_{t\geq 0} \quad , \quad \lambda \in \mathbb{R}^d \tag{A.14}$$

we obtain $\mathbb{R}$-valued semi-martingales. If for every $\lambda \in \mathbb{R}^d$ the laws of these projections are tight on $C([0,\infty),\mathbb{R})$ then this is true for $\{[\mathcal{L}[(X^{(n)}(t))_{t\geq 0}], n \in \mathbb{N}\}$. The tightness criterion for $\mathbb{R}$-valued semimartingales is in terms of the socalled local characteristics of the semimartingales.

For Itô processes the local characteristics can be calculated directly from the coefficients. For example, if we have a sequence of semimartingales X^n that are also a Markov processes with generators:

$$L^{(n)} f = \Big(\sum_{i=1}^d a_i^n(x)\frac{\partial}{\partial x_i} + \sum_{i=1}^d\sum_{j=1}^d b_{i,j}^n(x)\frac{\partial^2}{\partial x_i \partial x_j}\Big) f, \tag{A.15}$$

then the local characteristics are given by

$$a^n = (a_i^n)_{i=1,\cdots,d}, \ \ b^n = (b_{i,j}^n)_{i,j,=1,\cdots,d}. \tag{A.16}$$

The Joffe–Métivier criterion implies that if

$$\sup_n \sup_{0\le t\le T} E[(|a^n(X^{(n)}(t)| + |b^n(X^{(n)}(t)|)^2] < \infty,$$

$$\lim_{k\to\infty} \sup_n P[\sup_{0\le t\le T} (|a^n(X^{(n)})(t)| + |b^n(X^{(n)})(t)|) \ge k] = 0 \tag{A.17}$$

then $\{\mathcal{L}[(X^{(n)}(t))_{t\ge 0}], n \in \mathbb{N}\}$ are tight in $C([0,\infty),\mathbb{R})$. See [JM] for details.

Theorem 16 (Ethier-Kurtz [EK2] Chapt. 3, Theorem 10.2). *Let*

$$J(x) = \int_0^\infty e^{-u}[J(x,u)\wedge 1]du, \quad J(x,u) = \sup_{0\le t\le u} d(x(t), x(t-)). \tag{A.18}$$

Assume that a sequence of processes $X_n \Rightarrow X$ in for a Polish space $ED([0,\infty), E)$. Then X is a.s. continuous if and only if $J(X_n) \Rightarrow 0$. □

Appendix B
Nonlinear Semigroup Perturbations

We use the following result of Marsden [MA], (4.17).

Theorem 17 (Perturbation). *Let $\mathbb{B}$ be a Banach space and let A_S be the infinitesimal generator of a* strongly continuous semigroup*, with $\|S_t\| \leq Me^{Ct}$ for some C. Let $A_T : \mathbb{B} \to \mathbb{B}$ be a vector field on $\mathbb{B}$ such that A_T is of class C^2 with its first and second derivatives uniformly bounded on bounded subsets and let $\{T_t\}$ be the flow of A_T.*

Then $A_S + A_T$ has a unique flow which is Lipschitz for each t, $0 \leq t \leq T$, and

$$V_t x = \lim_{n\to\infty} (S_{t/n} \cdot T_{t/n})^n x \tag{B.1}$$

uniformly in t for each x on bounded sets of t. If $x \in \mathcal{D}(A_S + A_T)$, then

$$\frac{d}{dt} V_t x = (A_S + A_T)U_t \tag{B.2}$$

on $[0, \tau)$ where τ is the exit time from $\mathbb{B}$. □

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Index of Notation and Tables of Basic Objects

- $\mathbb{Z}$—the set of integers
- $\mathbb{N} = \{1, 2, 3, \dots\}$
- $\Omega_N = \otimes_{\mathbb{N}} Z_N \quad , \quad Z_N$ cyclical group of order N
- $\mathcal{M}(E)$ denotes the space of finite Borel measures on a Polish space E
- $\mathcal{P}(E)$ denotes the space of probability measures on the Borel field on a Polish space E.
- Table 9.1: List of Basic Dual Objects
- Table 9.2: State Spaces of Basic Dual Objects
- Table 9.3: Dual State Spaces—Mutation-dominant regime
- Table 9.4: Dual State Spaces—Selection-dominant regime

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   Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They may be based on specialised lecture courses. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character. For similar reasons it is unusual for doctoral theses to be accepted for the Lecture Notes series, though habilitation theses may be appropriate.

2. Manuscripts should be submitted either online at www.editorialmanager.com/lnm to Springer's mathematics editorial in Heidelberg, or to one of the series editors. In general, manuscripts will be sent out to 2 external referees for evaluation. If a decision cannot yet be reached on the basis of the first 2 reports, further referees may be contacted: The author will be informed of this. A final decision to publish can be made only on the basis of the complete manuscript, however a refereeing process leading to a preliminary decision can be based on a pre-final or incomplete manuscript. The strict minimum amount of material that will be considered should include a detailed outline describing the planned contents of each chapter, a bibliography and several sample chapters.
   Authors should be aware that incomplete or insufficiently close to final manuscripts almost always result in longer refereeing times and nevertheless unclear referees' recommendations, making further refereeing of a final draft necessary.
   Authors should also be aware that parallel submission of their manuscript to another publisher while under consideration for LNM will in general lead to immediate rejection.

3. Manuscripts should in general be submitted in English. Final manuscripts should contain at least 100 pages of mathematical text and should always include
   - a table of contents;
   - an informative introduction, with adequate motivation and perhaps some historical remarks: it should be accessible to a reader not intimately familiar with the topic treated;
   - a subject index: as a rule this is genuinely helpful for the reader.

   For evaluation purposes, manuscripts may be submitted in print or electronic form (print form is still preferred by most referees), in the latter case preferably as pdf- or zipped ps-files. Lecture Notes volumes are, as a rule, printed digitally from the authors' files. To ensure best results, authors are asked to use the LaTeX2e style files available from Springer's web-server at:

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4. Careful preparation of the manuscripts will help keep production time short besides ensuring satisfactory appearance of the finished book in print and online. After acceptance of the manuscript authors will be asked to prepare the final LaTeX source files and also the corresponding dvi-, pdf- or zipped ps-file. The LaTeX source files are essential for producing the full-text online version of the book (see http://www.springerlink.com/openurl.asp?genre=journal&issn=0075-8434 for the existing online volumes of LNM). The actual production of a Lecture Notes volume takes approximately 12 weeks.

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